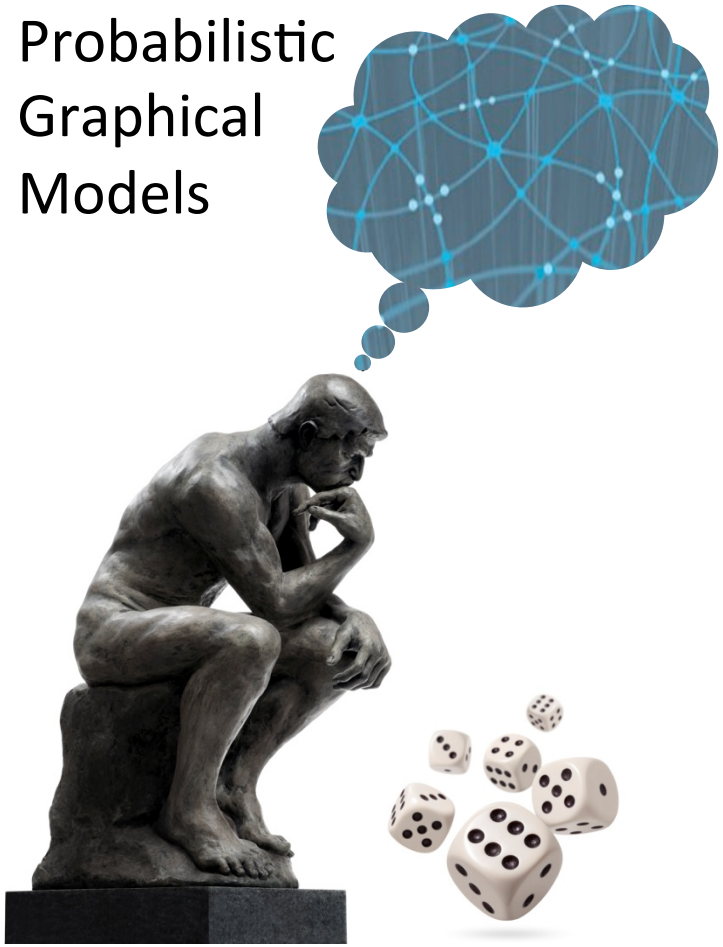
Probabilistic Graphical Models



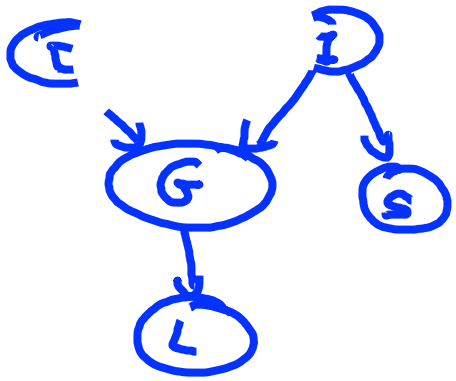
Representation

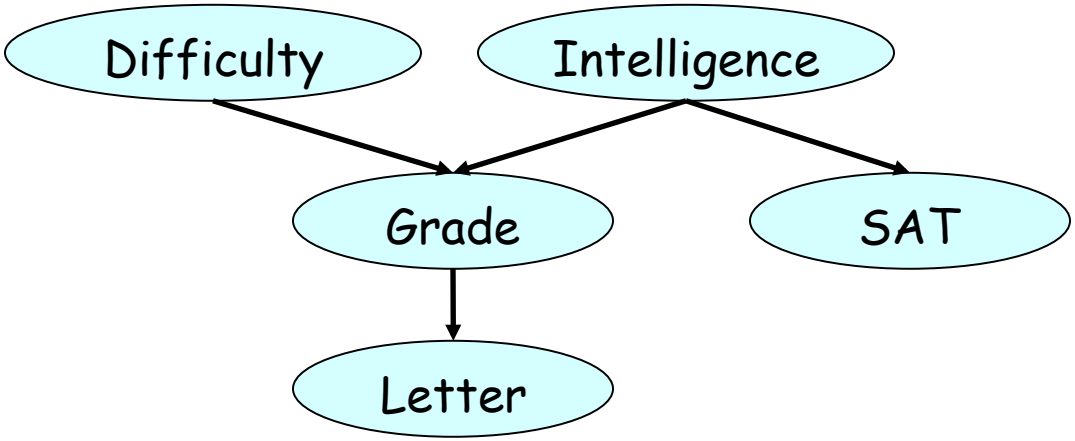
Bayesian Networks

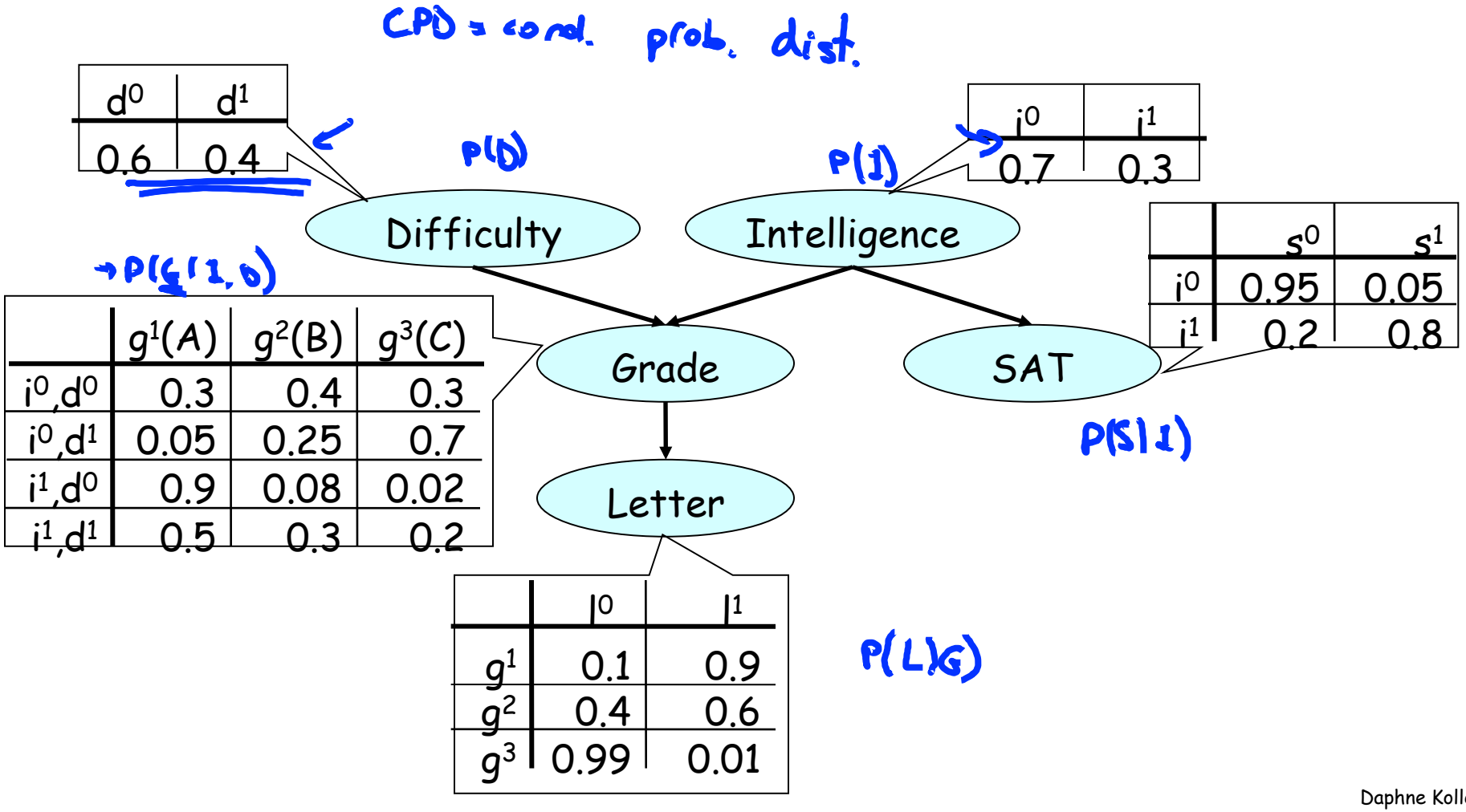
Semantics & Factorization

- Grade
- Course Difficulty
- Student Intelligence
- Student SAT
- Reference Letter

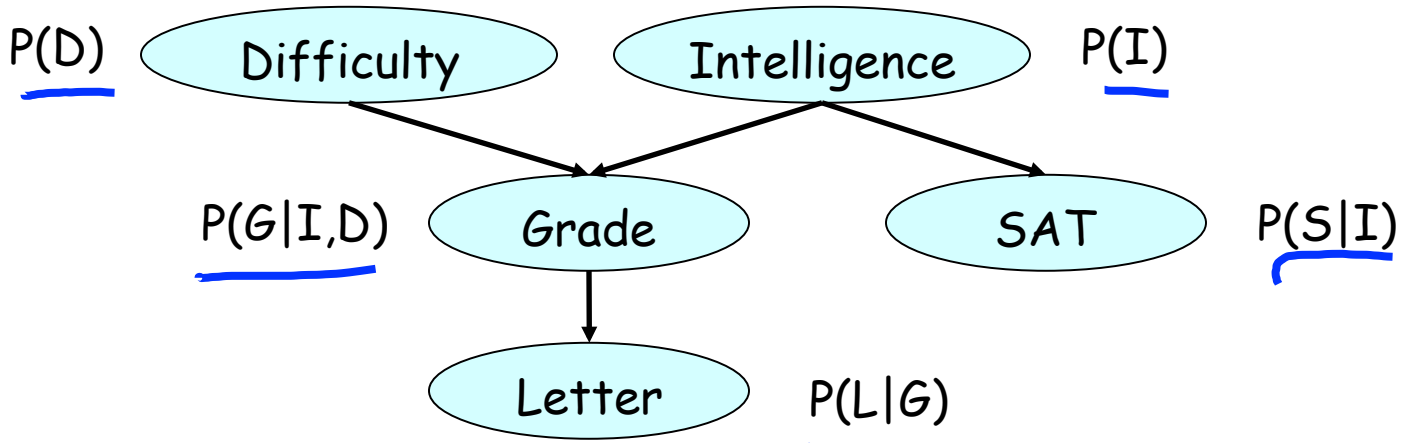
P(G,D,I,S,L)




Difficulty
Intelligence
Grade
SAT
Letter

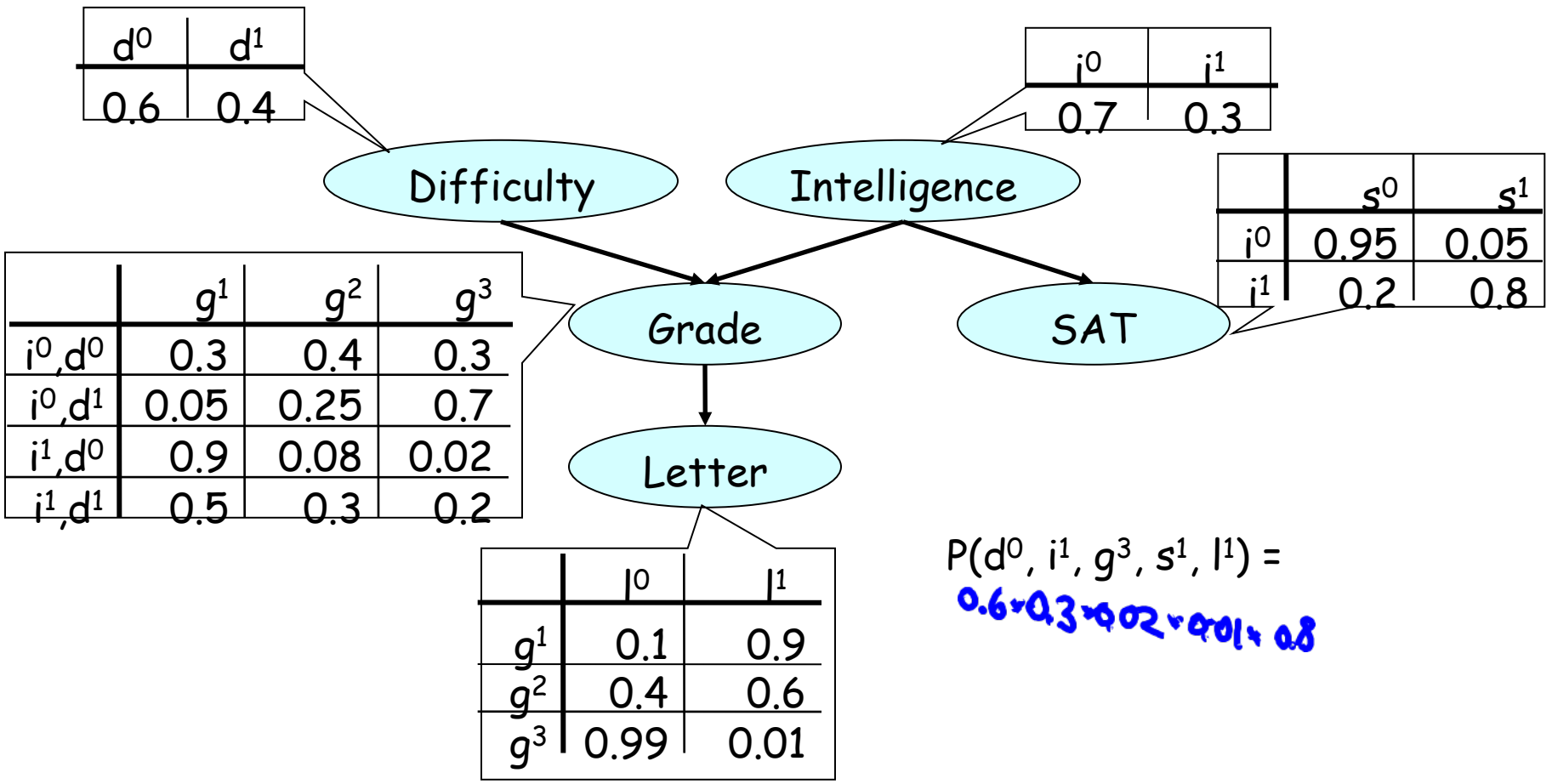

CPD = cond. prob. dist.
d0 d1
0.6 0.4
P(D)
i0 i1
0.7 0.3
P(I)
Difficulty
Intelligence
Grade
SAT
Letter
P(G|I,D)
g1(A) g2(B) g3(C)
i0,d0 0.3 0.4 0.3
i0,d1 0.05 0.25 0.7
i1,d0 0.9 0.08 0.02
i1,d1 0.5 0.3 0.2
s0 s1
i0 0.95 0.05
i1 0.2 0.8
P(S|I)
l0 l1
g1 0.1 0.9
g2 0.4 0.6
g3 0.99 0.01
P(L|G)

Chain Rule for Bayesian Networks



$$P(D,I,G,S,L) = P(D)\,P(I)\,P(G|I,D)\,P(S|I)\,P(L|G)$$

Distribution defined as a product of factors!



d^0	d^1
0.6	0.4

i^0	i^1
0.7	0.3

	g^1	g^2	g^3
i^0,d^0	0.3	0.4	0.3
i^0,d^1	0.05	0.25	0.7
i^1,d^0	0.9	0.08	0.02
i^1,d^1	0.5	0.3	0.2

	s^0	s^1
i^0	0.95	0.05
i^1	0.2	0.8

	l^0	l^1
g^1	0.1	0.9
g^2	0.4	0.6
g^3	0.99	0.01

$P(d^0, i^1, g^3, s^1, l^1) =$

$0.6 \cdot 0.3 \cdot 0.02 \cdot 0.01 \cdot 0.8$

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Bayesian Network

- A Bayesian network is:
  - A directed acyclic graph (DAG) G whose nodes represent the random variables $X_1,\ldots,X_n$
  - For each node X_i a CPD $P(X_i \mid Par_G(X_i))$
- The BN represents a joint distribution via the chain rule for Bayesian networks

$$P(X_1,\ldots,X_n) = \Pi_i P(X_i \mid Par_G(X_i))$$

BN Is a Legal Distribution: $P \geq 0$

P is a product of CPDs

CPDs are non-negative

BN Is a Legal Distribution: $\sum P = 1$

$$\sum_{D,I,G,S,L} P(D,I,G,S,L) = \sum_{D,I,G,S,L} P(D)\,P(I)\,P(G|I,D)\,P(S|I)\,P(L|G)$$

chain rule

$$= \sum_{D,I,G,S} P(D)\,P(I)\,P(G|I,D)\,P(S|I) \sum_{L} P(L|G)$$

$$= \sum_{D,I,G,S} P(D)\,P(I)\,P(G|I,D)\,P(S|I)$$

$$= \sum_{D,I,G} P(D)\,P(I)\,P(G|I,D) \sum_{S} P(S|I)$$

$$= \sum_{D,I} P(D)\,P(I) \sum_{G} P(G|I,D)$$

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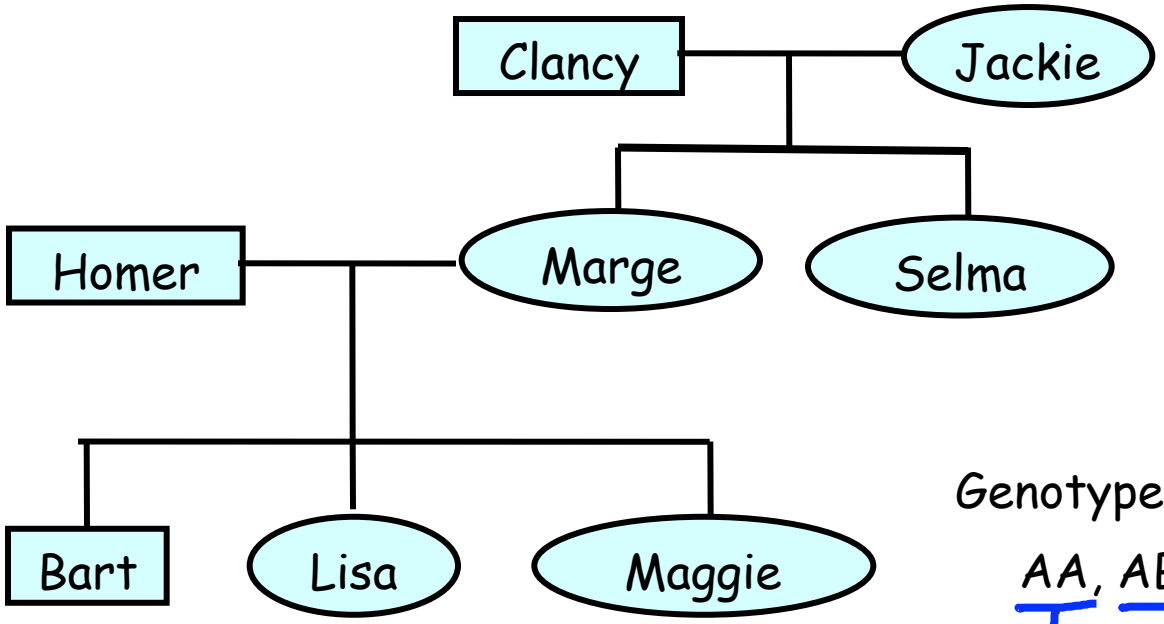
P Factorizes over G

- Let G be a graph over $X_1,\ldots,X_n$.
- P factorizes over G if

$$P(X_1,\ldots,X_n) = \Pi_i P(X_i \mid Par_G(X_i))$$

Genetic Inheritance



Genotype

AA, AB, AO, BO, BB, OO

Phenotype

A, B, AB, O

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BNs for Genetic Inheritance