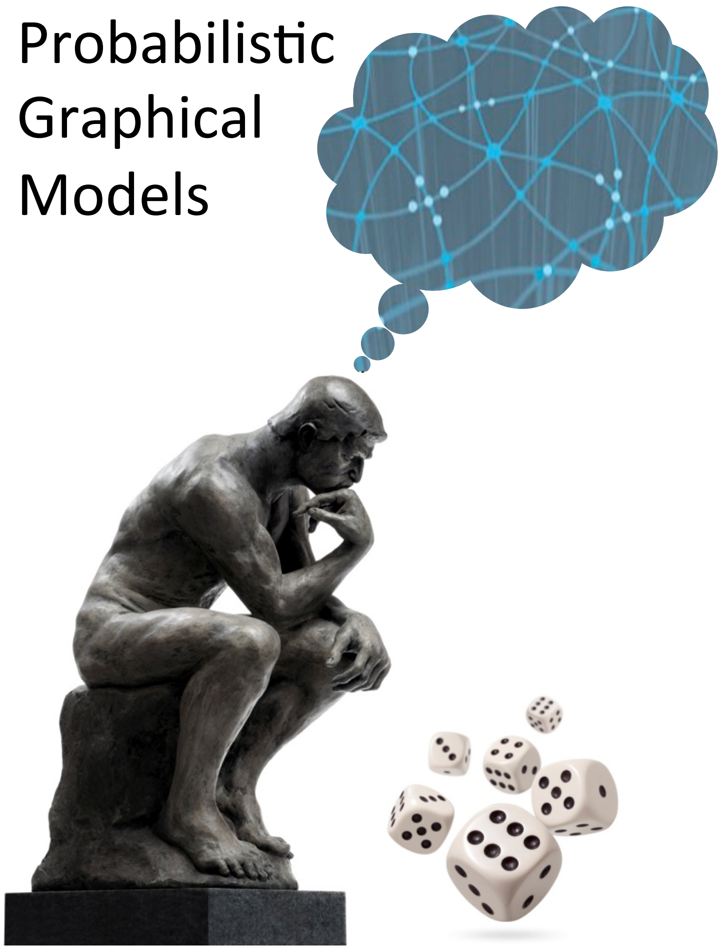
Probabilistic
Graphical
Models



Representation

Independencies

Preliminaries

Independence

- For events α, β, $P \models \alpha \perp \beta$ if:
  - $P(\alpha, \beta) = P(\alpha)\,P(\beta)$ satisfies independence
  - $P(\alpha | \beta) = P(\alpha)$
  - $P(\beta | \alpha) = P(\beta)$
- For random variables X, Y, $P \models X \perp Y$ if:
  - $P(X, Y) = P(X)\,P(Y)$
  - $P(X | Y) = P(X)$
  - $P(Y | X) = P(Y)$

universal

$\forall x, y \quad P(x,y) = P(x) \cdot P(y)$

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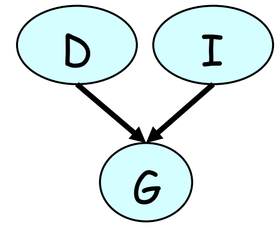
Independence



I	D	G	Prob.
i^0	d^0	g^1	0.126
i^0	d^0	g^2	0.168
i^0	d^0	g^3	0.126
i^0	d^1	g^1	0.009
i^0	d^1	g^2	0.045
i^0	d^1	g^3	0.126
i^1	d^0	g^1	0.252
i^1	d^0	g^2	0.0224
i^1	d^0	g^3	0.0056
i^1	d^1	g^1	0.06
i^1	d^1	g^2	0.036
i^1	d^1	g^3	0.024

P(I,D) = ×

I	D	Prob
i^0	d^0	0.42
i^0	d^1	0.18
i^1	d^0	0.28
i^1	d^1	0.12

P(I)

I	Prob
i^0	0.6
i^1	0.4

P(D)

D	Prob
d^0	0.7
d^1	0.3

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Conditional Independence

- For (sets of) random variables **X**,**Y**,**Z**

  satisfies indep conditioning

  $P \models (\mathbf{X} \perp \mathbf{Y} \mid \mathbf{Z})$ if:

  given

  - $P(X, Y|Z) = P(X|Z)\, P(Y|Z)$
  - $P(X|Y,Z) = P(X|Z)$
  - $P(Y|X,Z) = P(X|Z)$
  - $P(X,Y,Z) \propto \phi_1(X,Z)\, \phi_2(Y,Z)$

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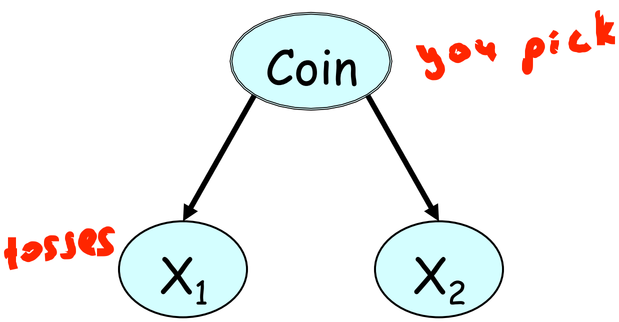
Conditional Independence



$P \not\models X_1 \perp X_2$

$P \models (X_1 \perp X_2 \mid C)$

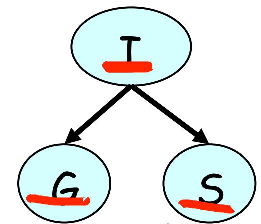
Conditional Independence

$P(I, S, G)$

I	S	G	Prob.
i^0	s^0	g^1	0.114
i^0	s^0	g^2	0.1938
i^0	s^0	g^3	0.2622
i^0	s^1	g^1	0.006
i^0	s^1	g^2	0.0102
i^0	s^1	g^3	0.0138
i^1	s^0	g^1	0.252
i^1	s^0	g^2	0.0224
i^1	s^0	g^3	0.0056
i^1	s^1	g^1	0.108
i^1	s^1	g^2	0.0096
i^1	s^1	g^3	0.0024

$P(S,G \mid i^0)$

S	G	Prob.
s^0	g^1	0.19
s^0	g^2	0.323
s^0	g^3	0.437
s^1	g^1	0.01
s^1	g^2	0.017
s^1	g^3	0.023



$P(S \mid i^0)$

S	Prob
s^0	0.95
s^1	0.05

$P(G \mid i^0)$

G	Prob.
g^1	0.2
g^2	0.34
g^3	0.46

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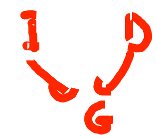
Conditioning can Lose Independences



I	D	G	Prob.
i^0	d^0	g^1	0.126
i^0	d^0	g^2	0.168
i^0	d^0	g^3	0.126
i^0	d^1	g^1	0.009
i^0	d^1	g^2	0.045
i^0	d^1	g^3	0.126
i^1	d^0	g^1	0.252
i^1	d^0	g^2	0.0224
i^1	d^0	g^3	0.0056
i^1	d^1	g^1	0.06
i^1	d^1	g^2	0.036
i^1	d^1	g^3	0.024

$P(I, D \mid g^1)$

I	D	Prob.
i^0	d^0	0.282
i^0	d^1	0.02
i^1	d^0	0.564
i^1	d^1	0.134