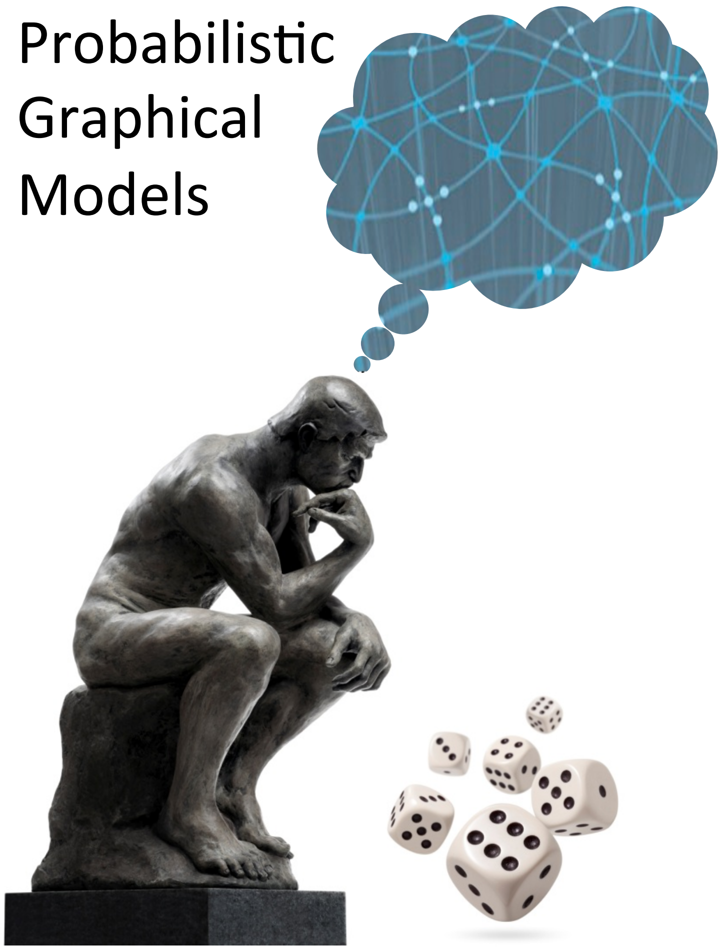
Probabilistic
Graphical
Models



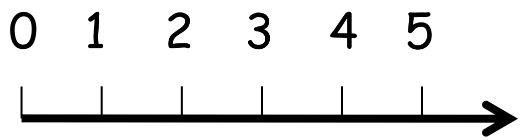
Representation

Template Models

Temporal Models

Distributions over Trajectories

discretize time



- Pick time granularity Δ sensor
- $X^{(t)}$ – variable X at time $t\Delta$
- $X^{(t:t')} = \underbrace{\{X^{(t)}, \ldots, X^{(t')}\}}$ $(t \le t')$

- Want to represent $P(X^{(t:t')})$ for any t, t'

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Markov Assumption

$$\underbrace{P(\boldsymbol{X}^{(0:T)})}_{\text{time flows forward}} = \underbrace{P(\boldsymbol{X}^{(0)})} \prod_{t=0}^{T-1} P(\boldsymbol{X}^{(\underline{t+1})} \mid \boldsymbol{X}^{(\underline{0:t})})$$

chain rule for probabilities

state at t+1 state at 0..t

$$(\underbrace{\boldsymbol{X}^{(t+1)}}_{\text{next step}} \perp \underbrace{\boxed{\boldsymbol{X}^{(0:t-1)}}}_{\text{past}} \mid \underbrace{\boldsymbol{X}^{(t)}}_{\text{present}})$$

forgetting

$$P(\boldsymbol{X}^{(0:T)}) = P(\boldsymbol{X}^{(0)}) \prod_{t=0}^{T-1} P(\boldsymbol{X}^{(t+1)} \mid \boldsymbol{X}^{(t)})$$

Is this true?

X = Location of robot

$L^{t+1} \perp L^{t-1} \mid L^{t}$?

probably not

velocity

enrich state by adding V^t and other variables

(adding dependencies back in time - semi Markov)

Time Invariance

next time

current time

- Template probability model $P(\mathbf{X}' \mid \mathbf{X})$
- For all t:

$$P(\boldsymbol{X}^{(t+1)} \mid \boldsymbol{X}^{(t)}) = P(\boldsymbol{X}' \mid \boldsymbol{X})$$

traffic time of day, day of week, football

enrich model by including

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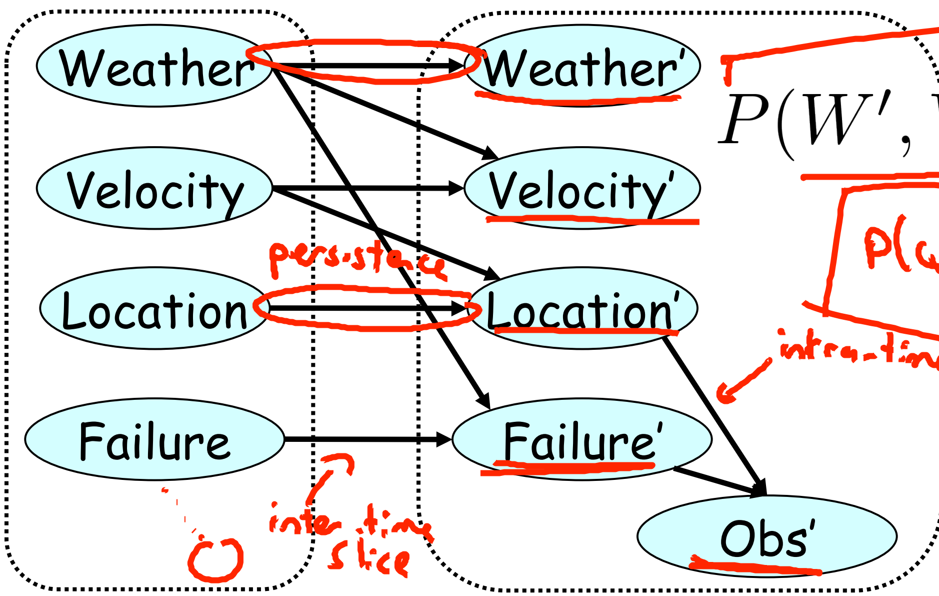
Template Transition Model

network fragment



Time slice *t* Time slice *t+1*

cond dist t+1 given t

$$P(W',V',L',F',O' \mid W,V,L,F) =$$

intra-time-slice

$$P(W'|W) \cdot P(V'|W,V) \cdot P(L'|L,V) \cdot \underbrace{P(F'|F,W)}_{t \text{ to } t+1} \cdot \underbrace{P(O'|L',F')}_{t+1}$$

rapidly-acting

$$\cancel{P(W,V,L,F)}$$

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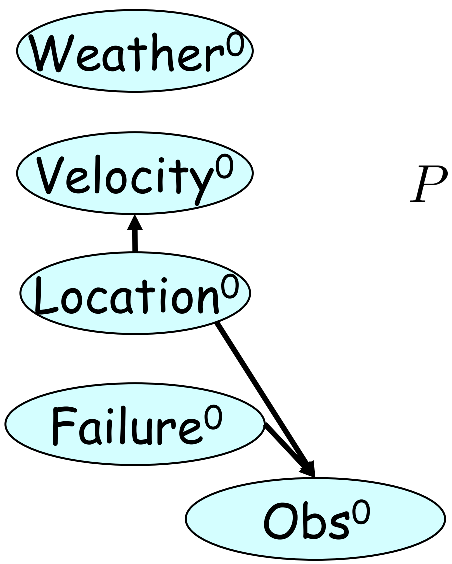
Initial State Distribution



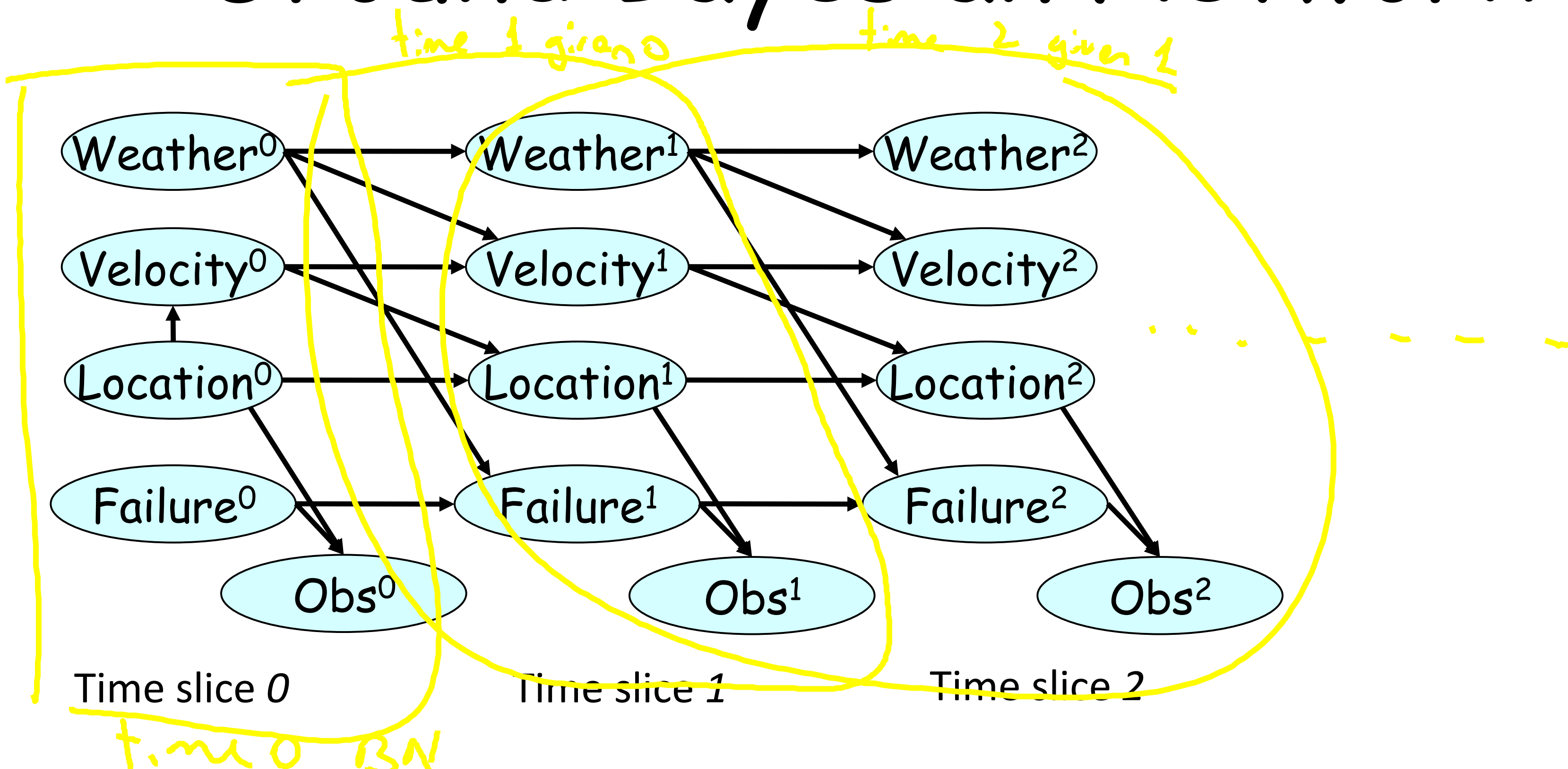
Time slice *0*

$$P(W^{(0)}, V^{(0)}, L^{(0)}, F^{(0)}, O^{(0)}) =$$

$$P(W^{(0)})P(V^{(0)} \mid L^{(0)})P(L^{(0)})P(F^{(0)})P(O^{(0)} \mid F^{(0)}, L^{(0)})$$

chain rule

Ground Bayesian Network



2-time-slice Bayesian Network

template

- A transition model (2TBN) over $X_1,\ldots,X_n$ is specified as a BN fragment such that:
  - The nodes include $X_1',\ldots,X_n'$ and a subset of $X_1,\ldots,X_n$

    the time t vars that directly affect state at t+1
  - Only the nodes $X_1',\ldots,X_n'$ have parents and a CPD
- The 2TBN defines a conditional distribution

$$P(\boldsymbol{X}' \mid \boldsymbol{X}) = \prod_{i=1}^{n} P(X_i' \mid \mathbf{Pa}_{X_i'})$$

chain rule

Dynamic Bayesian Network

- A dynamic Bayesian network (DBN) over $X_1,\ldots,X_n$ is defined by a
  - $\underbrace{\text{2TBN } BN_{\rightarrow}}$ over $X_1,\ldots,X_n$ dynamics
  - time 0 – a Bayesian network $BN^{(0)}$ over $X_1^{(0)},\ldots,X_n^{(0)}$

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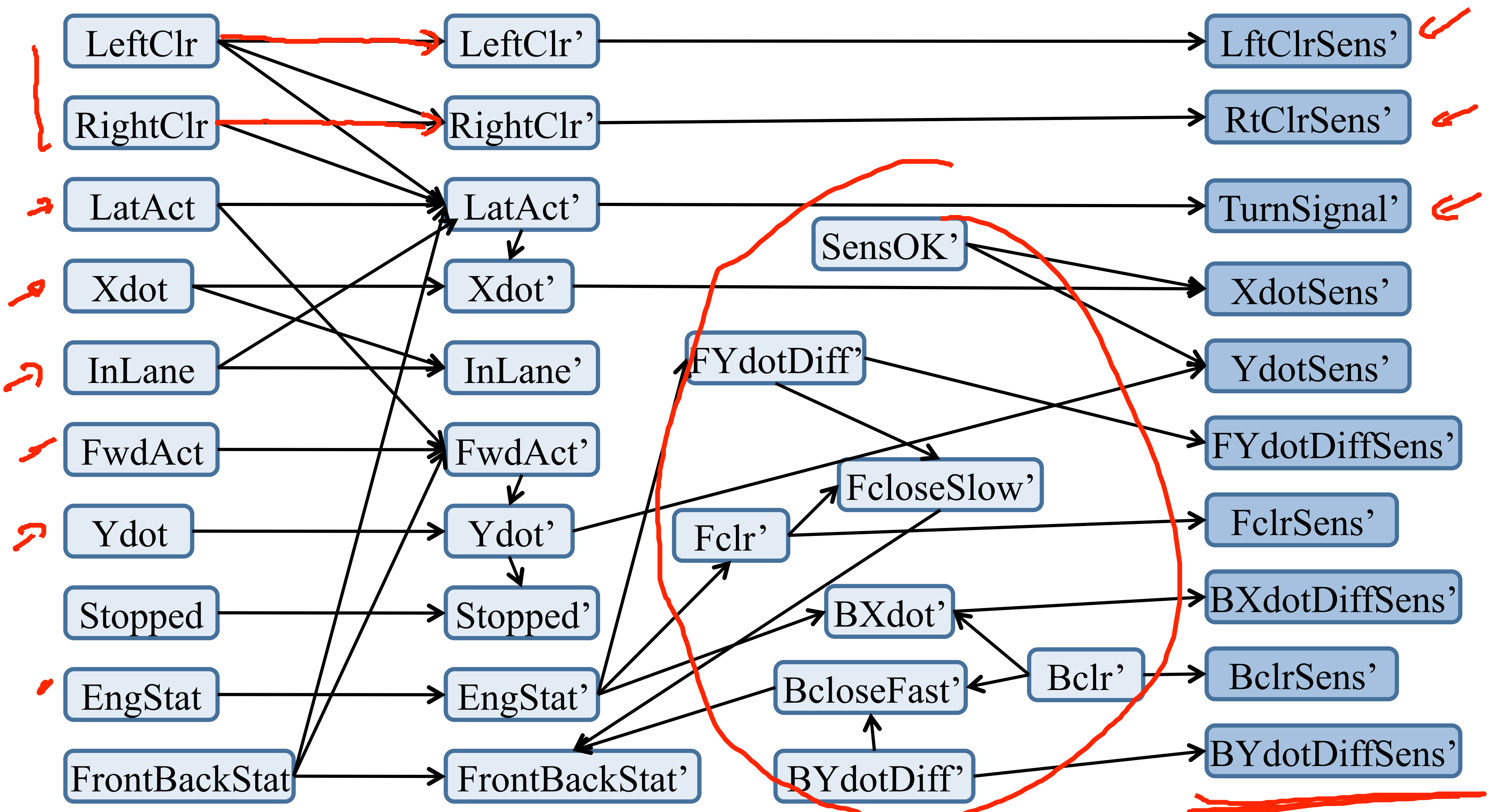
Ground Network

- For a trajectory over $\underbrace{0,\ldots,T}$ we define a <u>ground</u> (<u>unrolled</u> network) such that

  time 0
  - The dependency model for $X_1^{(0)},\ldots,X_n^{(0)}$ is copied from $BN^{(0)}$

  trans.t.
  - The dependency model for $X_1^{(t)},\ldots,X_n^{(t)}$ for all $t > 0$ is copied from $BN_{\rightarrow}$



Huang, Koller, Malik, Ogasawara, Rao, Russell, Weber, AAAI 94

Summary

- DBNS are a compact representation for encoding structured distributions over arbitrarily long temporal trajectories
- They make assumptions that may require appropriate model (re)design:
  - Markov assumption
  - Time invariance