

Probabilistic  
Graphical  
Models



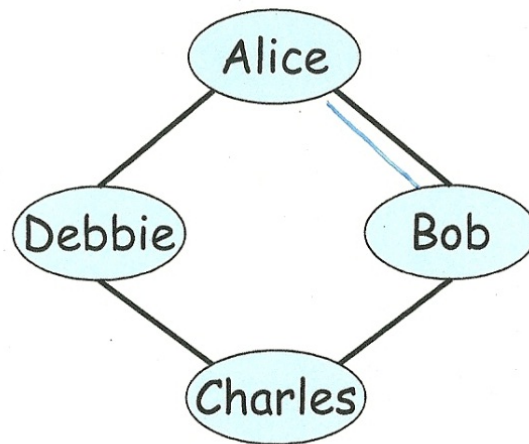
Representation

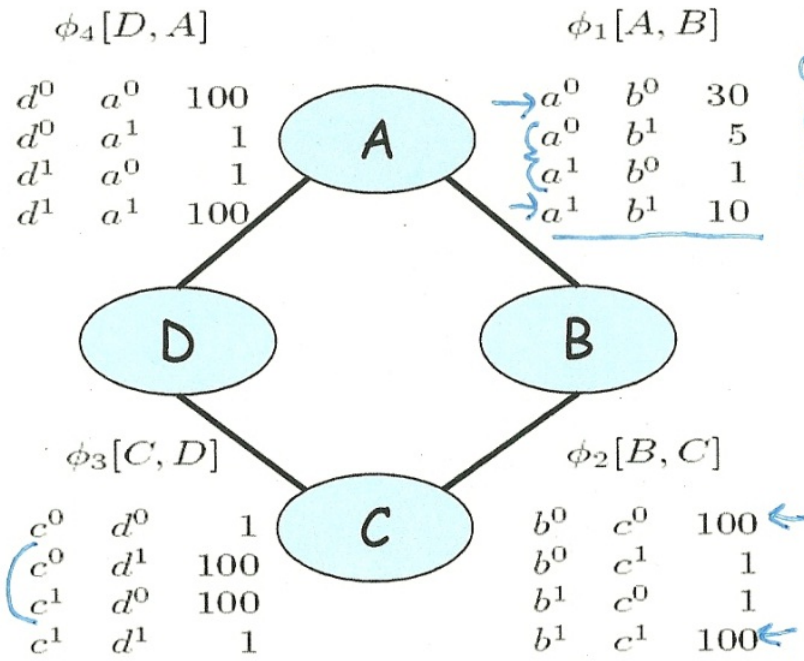
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Markov Networks

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Pairwise  
Markov  
Networks





affinity  
 compatibility  
 soft constraints

$$\rightarrow \tilde{P}(A, B, C, D) = \phi_1(A, B) \times \phi_2(B, C) \times \phi_3(C, D) \times \phi_4(A, D)$$

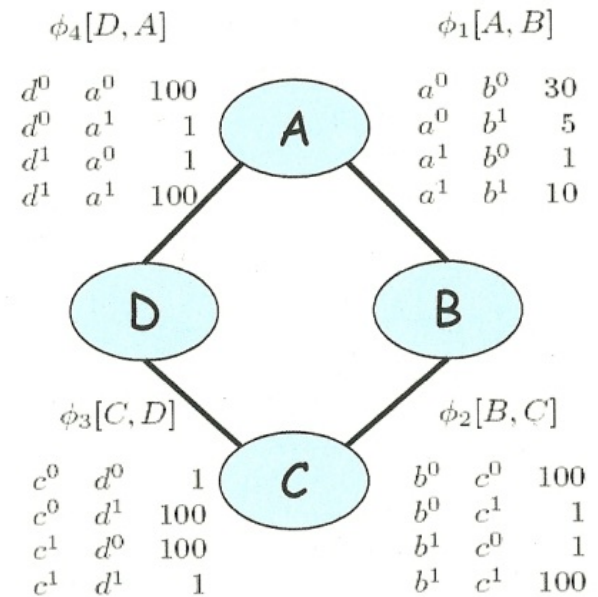
*unnormalized measure*

$$P(A, B, C, D) = \frac{1}{Z} \tilde{P}(A, B, C, D)$$

*partition function*

Assignment				Unnormalized
$a^0$	$b^0$	$c^0$	$d^0$	300000
$a^0$	$b^0$	$c^0$	$d^1$	300000
$a^0$	$b^0$	$c^1$	$d^0$	300000
$a^0$	$b^0$	$c^1$	$d^1$	30
$a^0$	$b^1$	$c^0$	$d^0$	500
$a^0$	$b^1$	$c^0$	$d^1$	500
$a^0$	$b^1$	$c^1$	$d^0$	5000000
$a^0$	$b^1$	$c^1$	$d^1$	500
$a^1$	$b^0$	$c^0$	$d^0$	100
$a^1$	$b^0$	$c^0$	$d^1$	1000000
$a^1$	$b^0$	$c^1$	$d^0$	100
$a^1$	$b^0$	$c^1$	$d^1$	100
$a^1$	$b^1$	$c^0$	$d^0$	10
$a^1$	$b^1$	$c^0$	$d^1$	100000
$a^1$	$b^1$	$c^1$	$d^0$	100000
$a^1$	$b^1$	$c^1$	$d^1$	100000

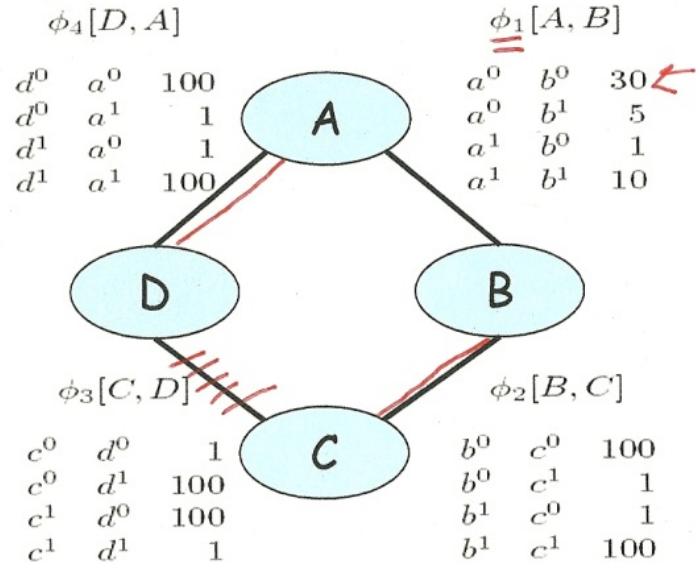
Z



$$\Phi = \{\phi_1, \phi_2, \phi_3, \phi_4\}$$

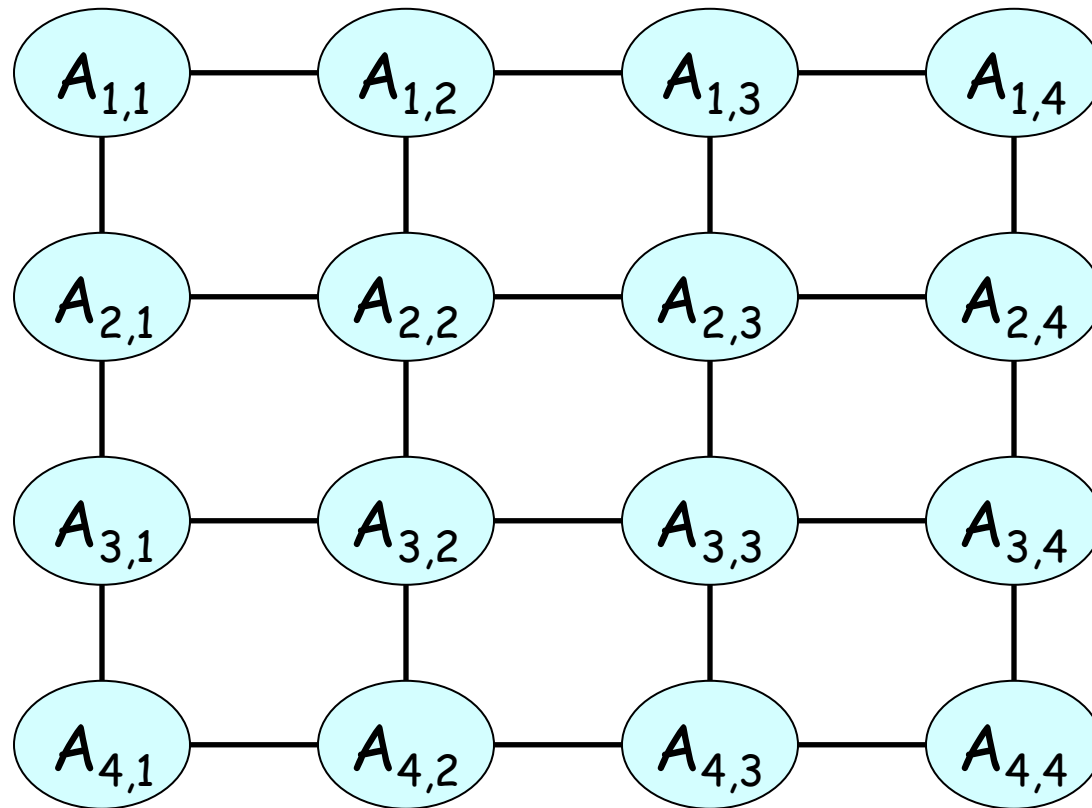
$P(A, B)$

A	B	Prob.
$a^0$	$b^0$	0.13
$a^0$	$b^1$	0.69
$a^1$	$b^0$	0.14
$a^1$	$b^1$	0.04

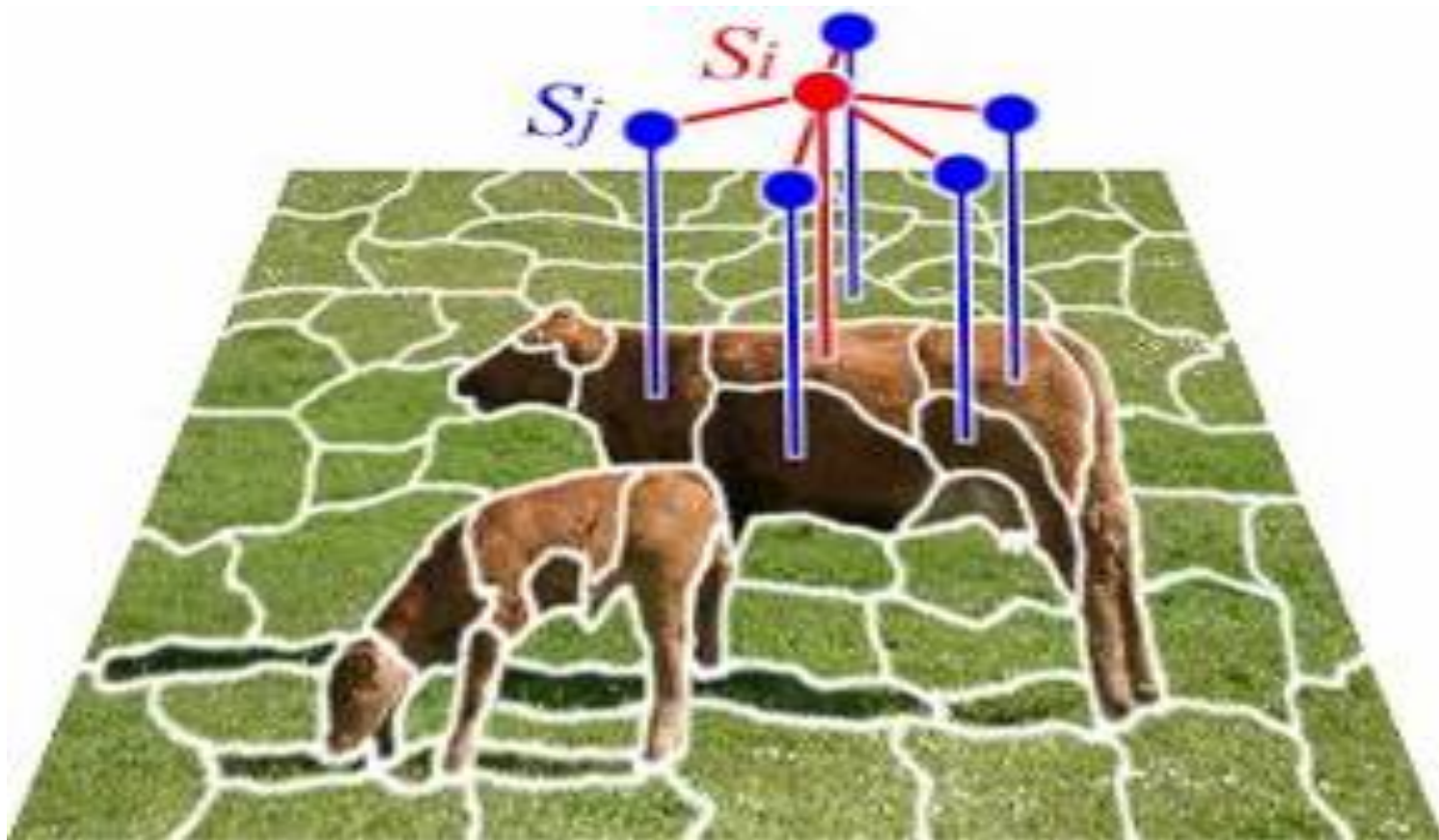


# Pairwise Markov Networks

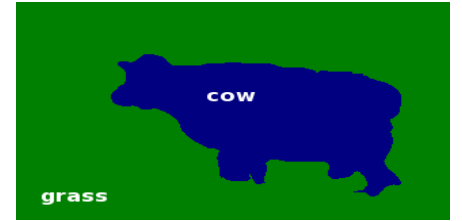
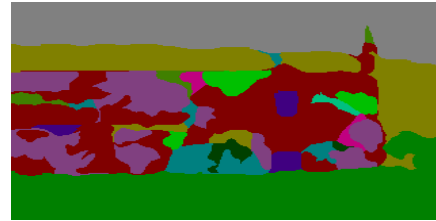
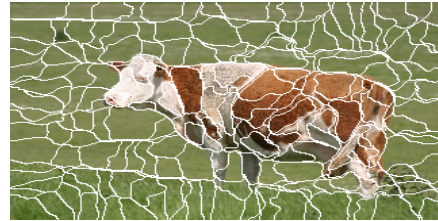
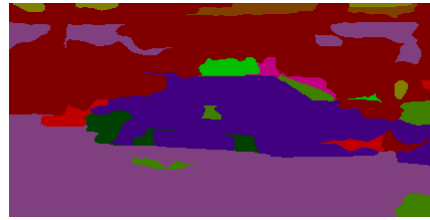
- A pairwise Markov network is an undirected graph whose nodes are  $X_1, \dots, X_n$  <sup>random variables</sup> and each edge  $X_i - X_j$  is associated with a factor (potential)  $\phi_{ij}(X_i, X_j)$











(a)

(b)

(c)

(d)