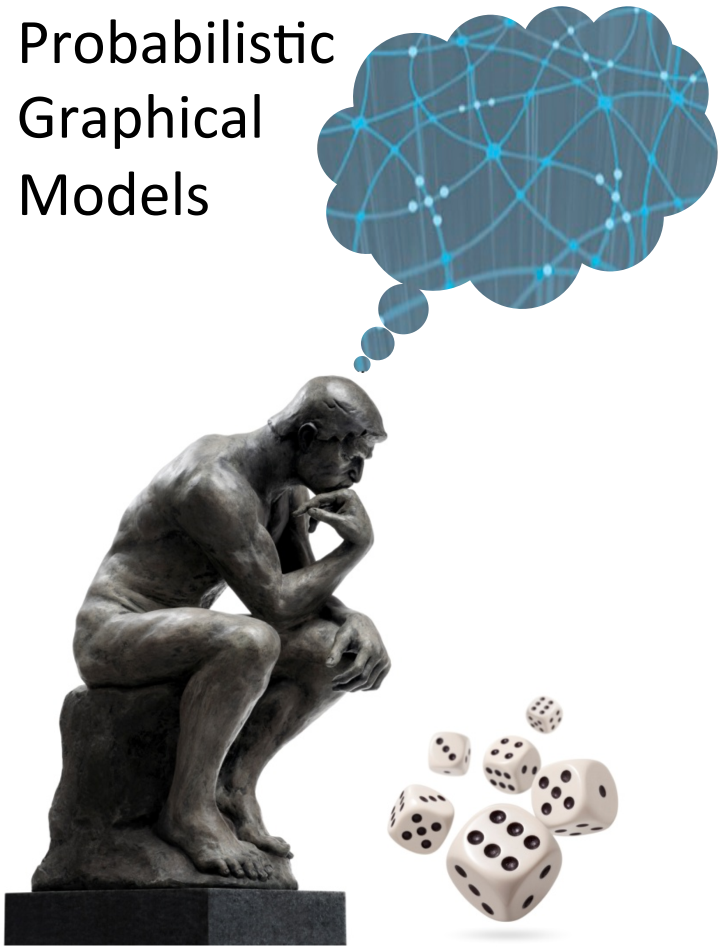
Probabilistic
Graphical
Models

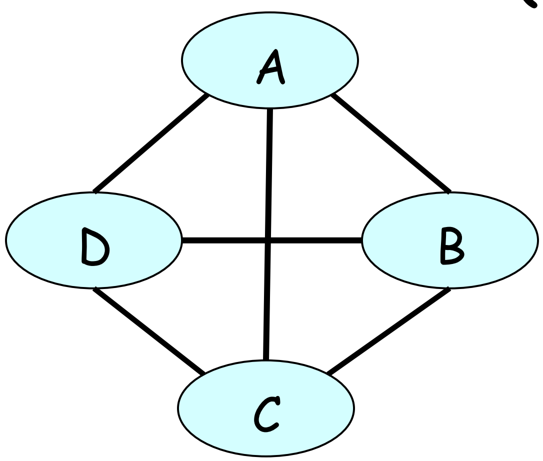


Representation

Markov Networks

General Gibbs Distribution

$P(A,B,C,D)$



Is this fully expressive?

Gibbs Distribution

- Parameters:

  General factors $\phi_i(\mathbf{D}_i)$

  $\Phi = \{\phi_i(\mathbf{D}_i)\}$

a^1	b^1	c^1	0.25
a^1	b^1	c^2	0.35
a^1	b^2	c^1	0.08
a^1	b^2	c^2	0.16
a^2	b^1	c^1	0.05
a^2	b^1	c^2	0.07
a^2	b^2	c^1	0
a^2	b^2	c^2	0
a^3	b^1	c^1	0.15
a^3	b^1	c^2	0.21
a^3	b^2	c^1	0.09
a^3	b^2	c^2	0.18

Gibbs Distribution

Set of factors

$$\Phi = \{\phi_1(\boldsymbol{D}_1), \ldots, \phi_k(\boldsymbol{D}_k)\}$$

unnormalized measure

$$\tilde{P}_\Phi(X_1, \ldots, X_n) = \prod_{i=1}^{k} \phi_i(\boldsymbol{D}_i)$$

factor product

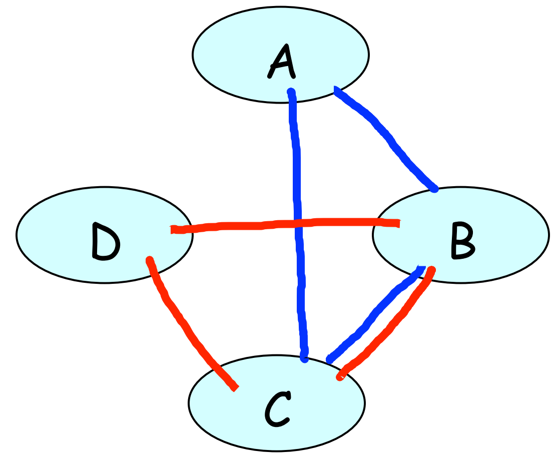
partition function

$$Z_\Phi = \sum_{X_1,\ldots,X_n} \tilde{P}_\Phi(X_1, \ldots, X_n)$$

$$P_\Phi(X_1, \ldots, X_n) = \frac{1}{Z_\Phi} \tilde{P}_\Phi(X_1, \ldots, X_n)$$

Induced Markov Network

$\phi_1(A, B, C), \phi_2(B, C, D)$



$$\Phi = \{\phi_1(\boldsymbol{D}_1), \ldots, \phi_k(\boldsymbol{D}_k)\}$$

Induced Markov network H_Φ has an edge X_i—X_j whenever

there exists $\phi_m \in \Phi$ s.t. $X_i, X_j \in D_m$

Factorization

P factorizes over H if

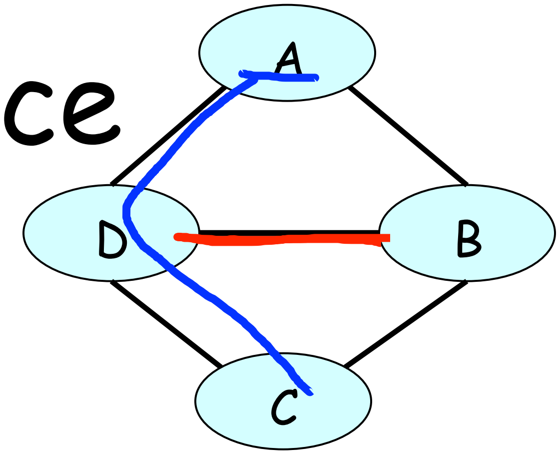
there exist $\underline{\Phi} = \{\phi_1(\boldsymbol{D}_1), \ldots, \phi_k(\boldsymbol{D}_k)\}$

such that

$\underline{P = P_\Phi}$ normalized product of factors in Φ

~~H is the~~ induced graph for Φ

Flow of Influence



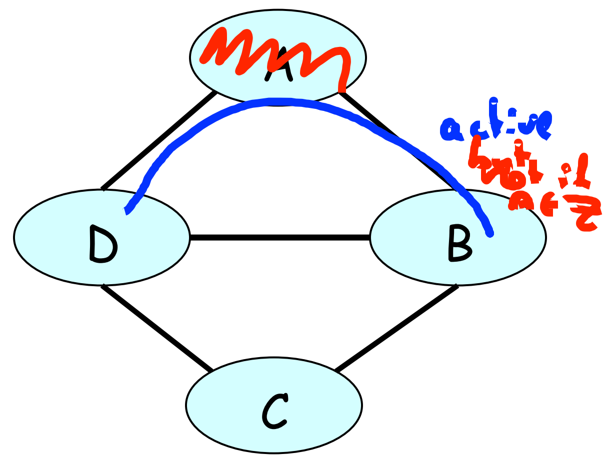
$\phi_1(A, B, D), \phi_2(B, C, D)$

$\phi_1(A, B), \phi_2(B, C), \phi_3(C, D), \phi_4(A, D), \phi_5(B, D)$

- Influence can flow along any trail, regardless of the form of the factors

Active Trails

- A trail $X_1 — \ldots — X_n$ is active given Z if no X_i is in Z



Summary

- Gibbs distribution represents distribution as a product of factors
- Induced Markov network connects every pair of nodes that are in the same factor
- Markov network structure doesn't fully specify the factorization of P
- But active trails depend only on graph structure