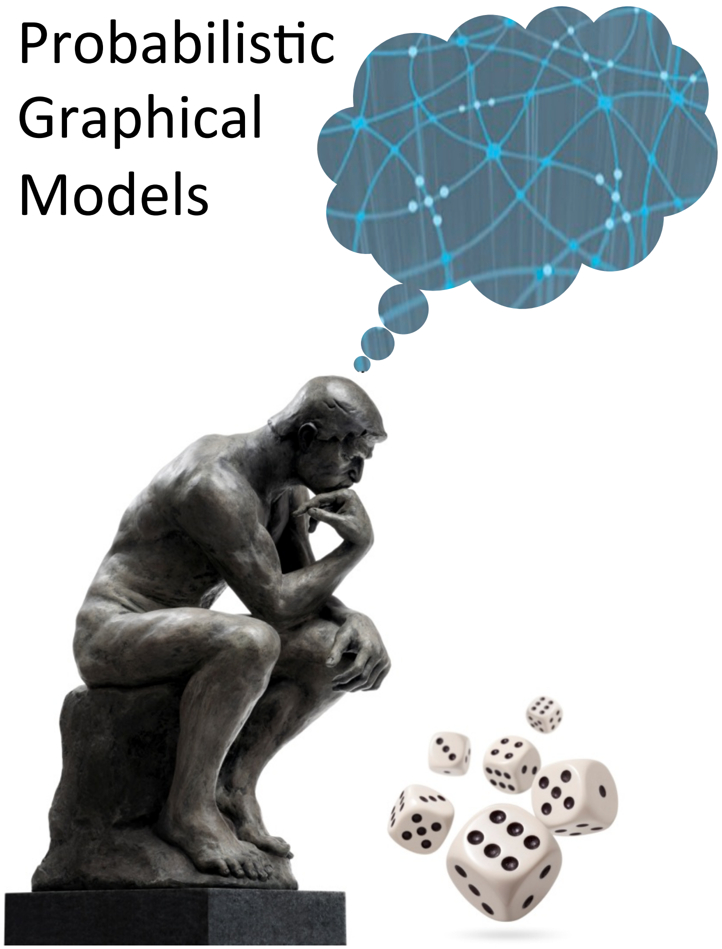
Probabilistic
Graphical
Models



Representation

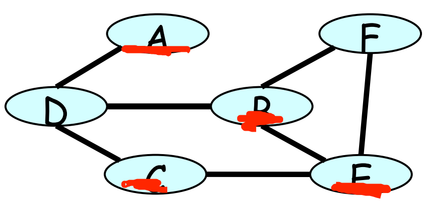
Independencies

Markov Networks

Separation in MNs

Definition:

X and **Y** are *separated* in H given **Z** if there is no active trail in H between **X** and **Y** given **Z** no node along trail is in Z

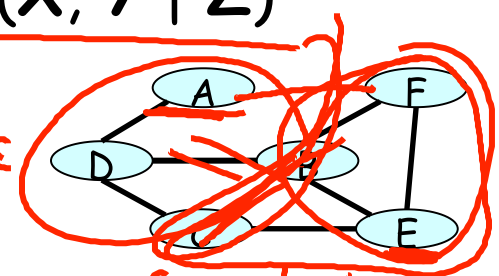


A,E separated given B,D
given D
given B,C

Factorization ⇨ Independence: MNS

Theorem: If P factorizes over H, and $\text{sep}_H(\mathbf{X}, \mathbf{Y} \mid \mathbf{Z})$ then P satisfies $(\mathbf{X} \perp \mathbf{Y} \mid \mathbf{Z})$

A sep. from E
given B, C



respect H

$$\underbrace{\prod \phi}_{\text{on A side}} \cdot \underbrace{\prod \phi}_{\text{on E side}} =$$

cannot involve E

cannot involve A

Factorization ⇨ Independence: MNs

$$I(H) = \{(X \perp Y \mid Z) \; : \; \boxed{sep_H(X, Y \mid Z)}\}$$

If P satisfies I(H), we say that H is an I-map (independency map) of P

Theorem: If P factorizes over H, then H is an I-map of P

Independence ⇨ Factorization

- **Theorem** (Hammersley Clifford):

  For a positive distribution P, if H is an I-map for P, then P factorizes over H

$P(\bar{x}) > 0 \quad \forall \bar{x}$

Summary

Two equivalent* views of graph structure:

- Factorization: H allows P to be represented
- I-map: Independencies encoded by H hold in P

If P factorizes over a graph H, we can read from the graph independencies that must hold in P (an independency map)

* for positive distributions