Probabilistic
Graphical
Models

Acting

Decision Making

Maximum Expected Utility

Simple Decision Making

A simple decision making situation $\mathcal{D}$:

- A set of possible actions $Val(A)=\{a^1,\ldots,a^K\}$
- A set of states $Val(\mathbf{X}) = \{\mathbf{x}^1,\ldots,\mathbf{x}^N\}$
- A distribution $P(\mathbf{X} \mid A)$
- A utility function $U(\mathbf{X}, A)$

Expected Utility

$$\mathrm{EU}[\mathcal{D}[a]] = \sum_{\boldsymbol{x}} P(\boldsymbol{x} \mid a) U(\boldsymbol{x}, a)$$

- Want to choose action a that maximizes the expected utility

Max. expected utility

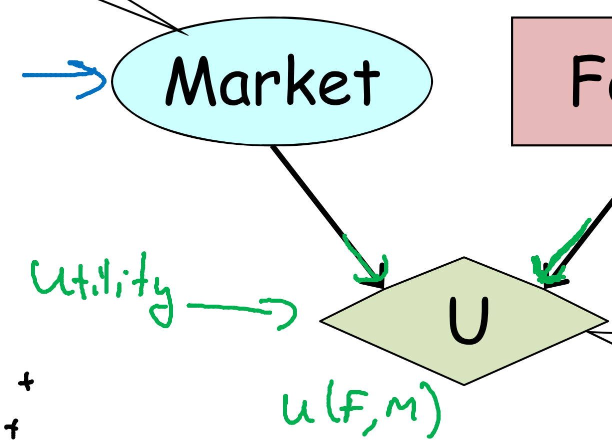
$$\boxed{a^* = \mathrm{argmax}_a \mathrm{EU}[\mathcal{D}[a]]}$$

Simple Influence Diagram

poor moderate great

m^0	m^1	m^2
0.5	0.3	0.2

random variables



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	f^0	f^1
m^0	0	-7
m^1	0	5
m^2	0	20

$EU(f^0) = 0$

$EU(f^1) = 0.5 \times -7 + 0.3 \times 5 + 0.2 \times 20 = 2$

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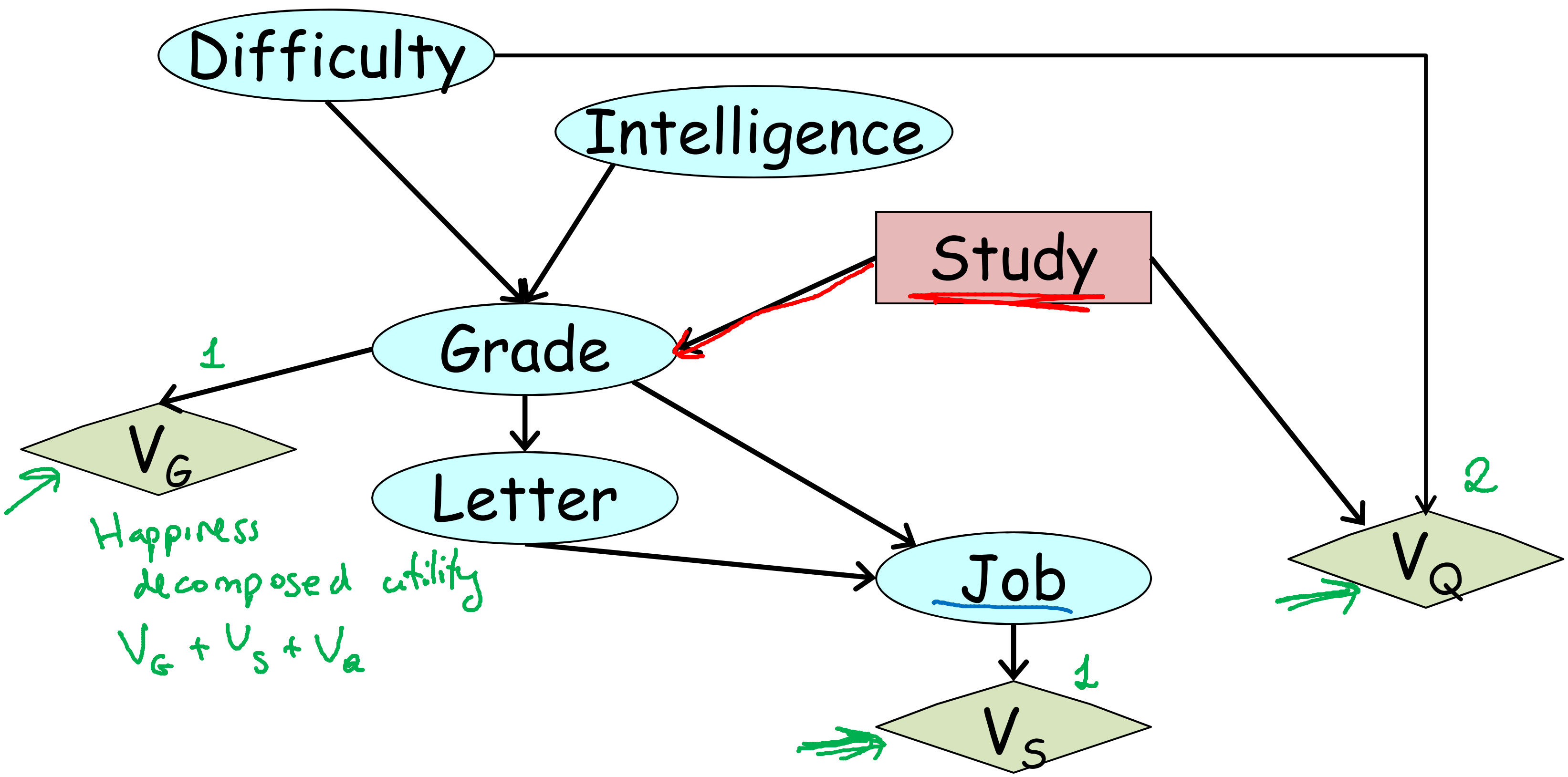
More Complex Influence Diagram



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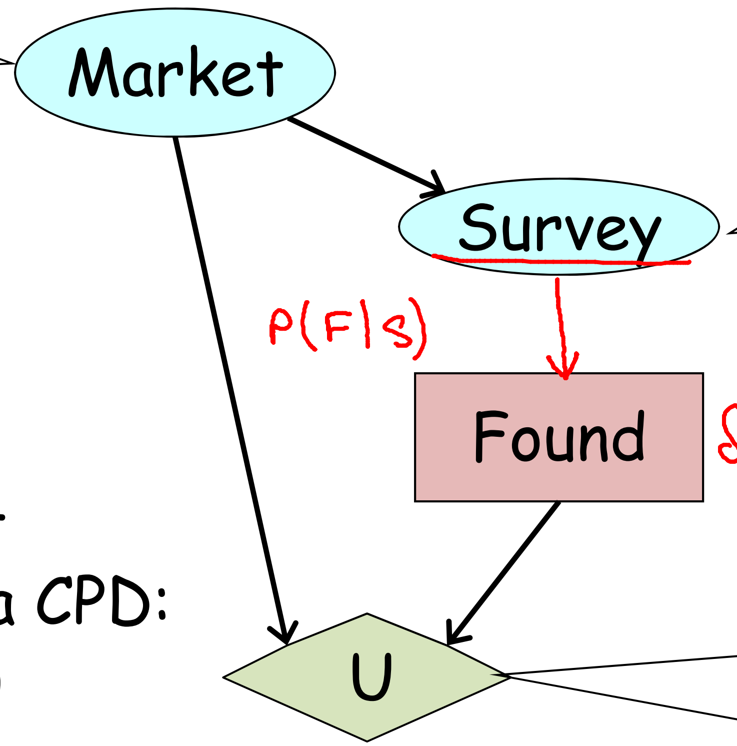
Information Edges

m^0	m^1	m^2
0.5	0.3	0.2



CPD P(S|M)

	s^0	s^1	s^2
m^0	0.6	0.3	0.1
m^1	0.3	0.4	0.3
m^2	0.1	0.4	0.5

	f^0	f^1
m^0	0	-7
m^1	0	5
m^2	0	20

Decision rule δ at action node A is a CPD: P(A | Parents(A))

Expected Utility with Information

joint prob. dist over $\bar{X} \cup \{A\}$

$$\mathrm{EU}[\mathcal{D}[\boxed{\delta_A}]] = \sum_{\boldsymbol{x},a} \underline{P_{\delta_A}(\boldsymbol{x},a)}\,\underline{U(\boldsymbol{x},a)}$$

- Want to choose the decision rule $\boxed{\delta_A}$ that maximizes the expected utility

$$\mathrm{argmax}_{\delta_A} \mathrm{EU}[\mathcal{D}[\delta_A]]$$

$$\mathrm{MEU}(\mathcal{D}) = \max_{\delta_A} \mathrm{EU}[\mathcal{D}[\delta_A]]$$

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Finding MEU Decision Rules

optimize

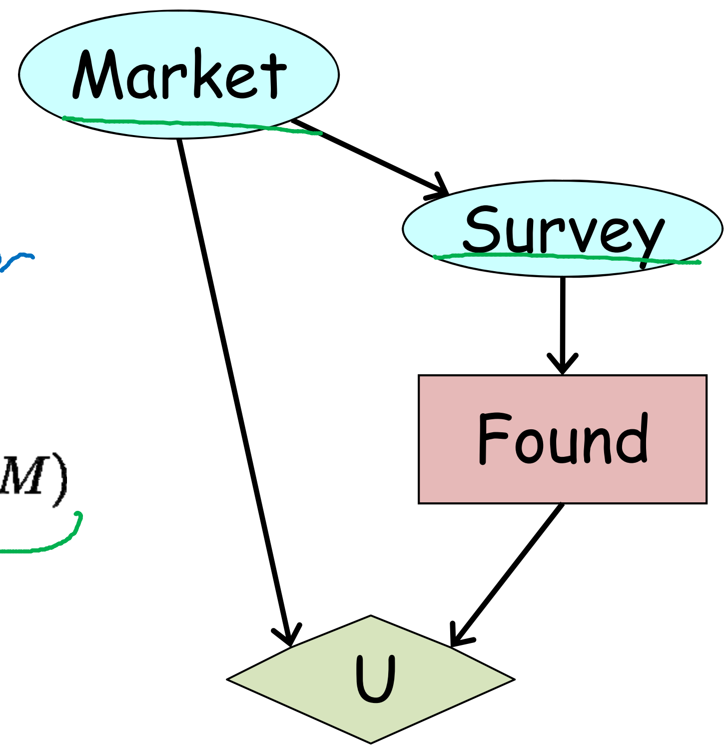
$$\mathrm{EU}[\mathcal{D}[\delta_A]] = \sum_{\boldsymbol{x},a} P_{\delta_A}(\boldsymbol{x},a)U(\boldsymbol{x},a)$$

factor

$$\sum_{M,S,F} P(M)P(S \mid M)\delta_F(F \mid S)U(F,M) =$$

$$= \sum_{S,F} \delta_F(F \mid S) \sum_{M} P(M)P(S \mid M)U(F,M)$$

$$= \sum_{S,F} \delta_F(F \mid S)\mu(F,S)$$

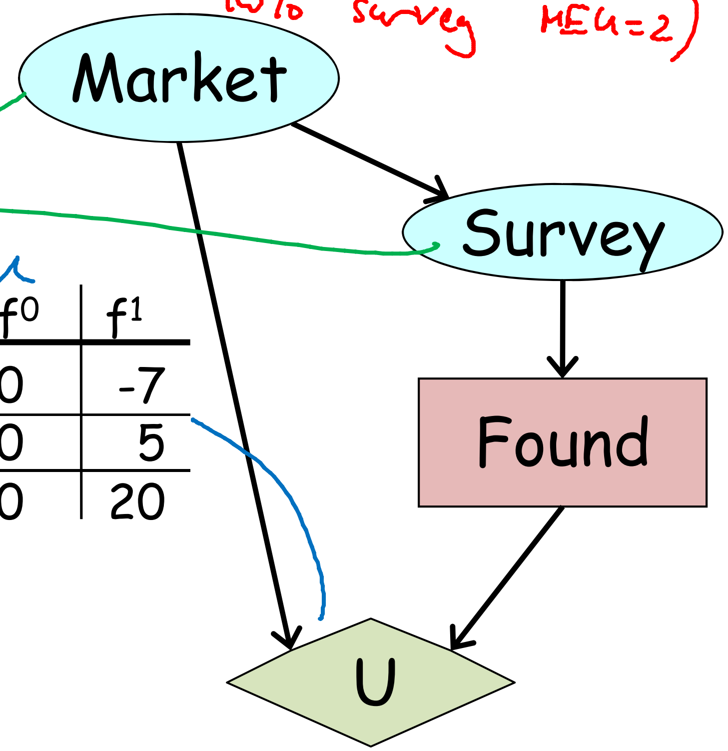


Finding MEU Decision Rules

(w/o survey MEU=2)

$$\sum_{S,F} \delta_F(F \mid S) \boxed{\sum_M P(M)P(S \mid M)U(F,M)}$$

$$= \boxed{\sum_{S,F} \delta_F(F \mid S)\mu(F,S)}$$



m^0	m^1	m^2
0.5	0.3	0.2

	s^0	s^1	s^2
m^0	0.6	0.3	0.1
m^1	0.3	0.4	0.3
m^2	0.1	0.4	0.5

u

	f^0	f^1
m^0	0	-7
m^1	0	5
m^2	0	20

	f^0	f^1
s^0	0	-1.25
s^1	0	1.15
s^2	0	2.1

$s_0 \rightarrow f_0$

$s_1 \rightarrow f_1$

$s_2 \rightarrow f_1$

$$\begin{array}{r} 0 \\ +\ 1.15 \\ +\ 2.1 \\ \hline 3.25 \end{array}$$

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More Generally

$$\mathrm{EU}[\mathcal{D}[\delta_A]] = \sum_{\boldsymbol{x},a} \overbrace{P_{\delta_A}(\boldsymbol{x},a)}^{\text{joint dist.}} U(\boldsymbol{x},a)$$

$$\boldsymbol{Z} = \mathbf{Pa}_A$$ observations prior to A

$$\boldsymbol{W} = \{X_1,\ldots,X_n\} - \boldsymbol{Z}$$

$$= \sum_{X_1,\ldots,X_n,A} \left(\left(\prod_i P(X_i \mid \mathbf{Pa}_{X_i}) \right) U(\mathbf{Pa}_U) \delta_A(A \mid \boldsymbol{Z}) \right)$$

$$= \sum_{\boldsymbol{Z},A} \delta_A(A \mid \boldsymbol{Z}) \underbrace{\sum_{\boldsymbol{W}} \left(\left(\prod_i P(X_i \mid \mathbf{Pa}_{X_i}) \right) U(\mathbf{Pa}_U) \right)}_{\text{prob}}$$

$$= \sum_{\boldsymbol{Z},A} \delta_A(A \mid \boldsymbol{Z}) \mu(A,\boldsymbol{Z})$$

$$\delta_A^*(a \mid \boldsymbol{z}) = \begin{cases} 1 & a = \mathrm{argmax}_A \mu(A,\boldsymbol{z}) \\ 0 & \text{otherwise} \end{cases}$$

MEU Algorithm Summary

- To compute MEU & optimize decision at A:
  - Treat A as random variable with arbitrary CPD
  - Introduce utility factor with scope Pa_U
  - VE Eliminate all variables except A, $\mathbf{Z}$ (A's parents) to produce factor $\mu(A, \mathbf{Z})$
  - For each $\mathbf{z}$, set:

$$\delta_A^*(a \mid \boldsymbol{z}) = \begin{cases} 1 & a = \text{argmax}_A \mu(A, \boldsymbol{z}) \\ 0 & \text{otherwise} \end{cases}$$

Decision Making under Uncertainty

- MEU principle provides rigorous foundation
- PGMs provide structured representation for probabilities, actions, and utilities
- PGM inference methods (VE) can be used for
  - Finding the optimal strategy
  - Determining overall value of the decision situation
- Efficient methods also exist for:
  - Multiple utility components
  - Multiple decisions