

Probabilistic
Graphical
Models



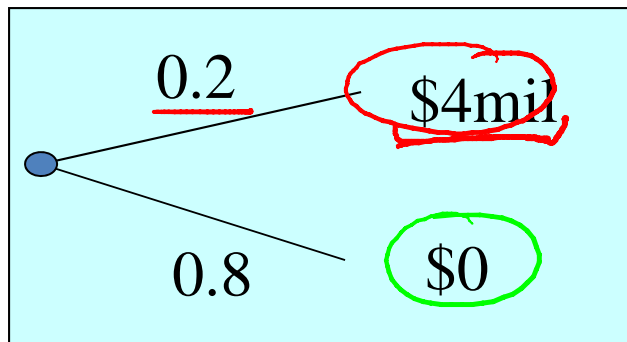
Acting

Decision Making

Utility
Functions

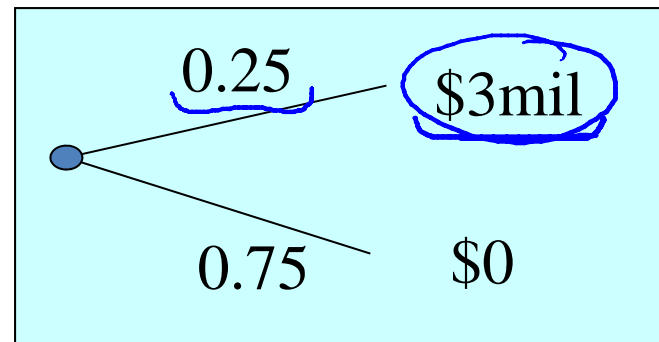
Utilities and Preferences

Lotteries



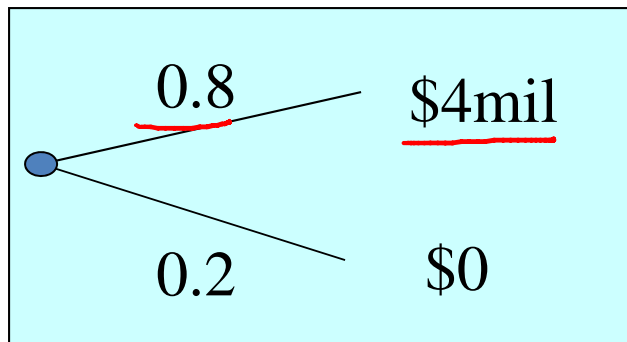
$$0.2 \times u(4) + 0.8 u(0)$$

\approx



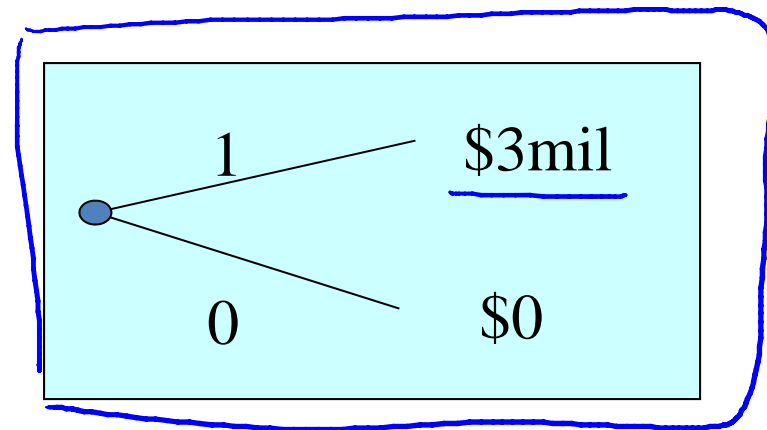
$$0.25 u(3) + 0.75 u(0)$$

Utility = Payoff?



$$\begin{aligned} & \$4\text{mil} \times 0.8 = \\ & \underline{\underline{\$3.2\text{mil}}} \end{aligned}$$

\approx



$$\$3\text{mil}$$

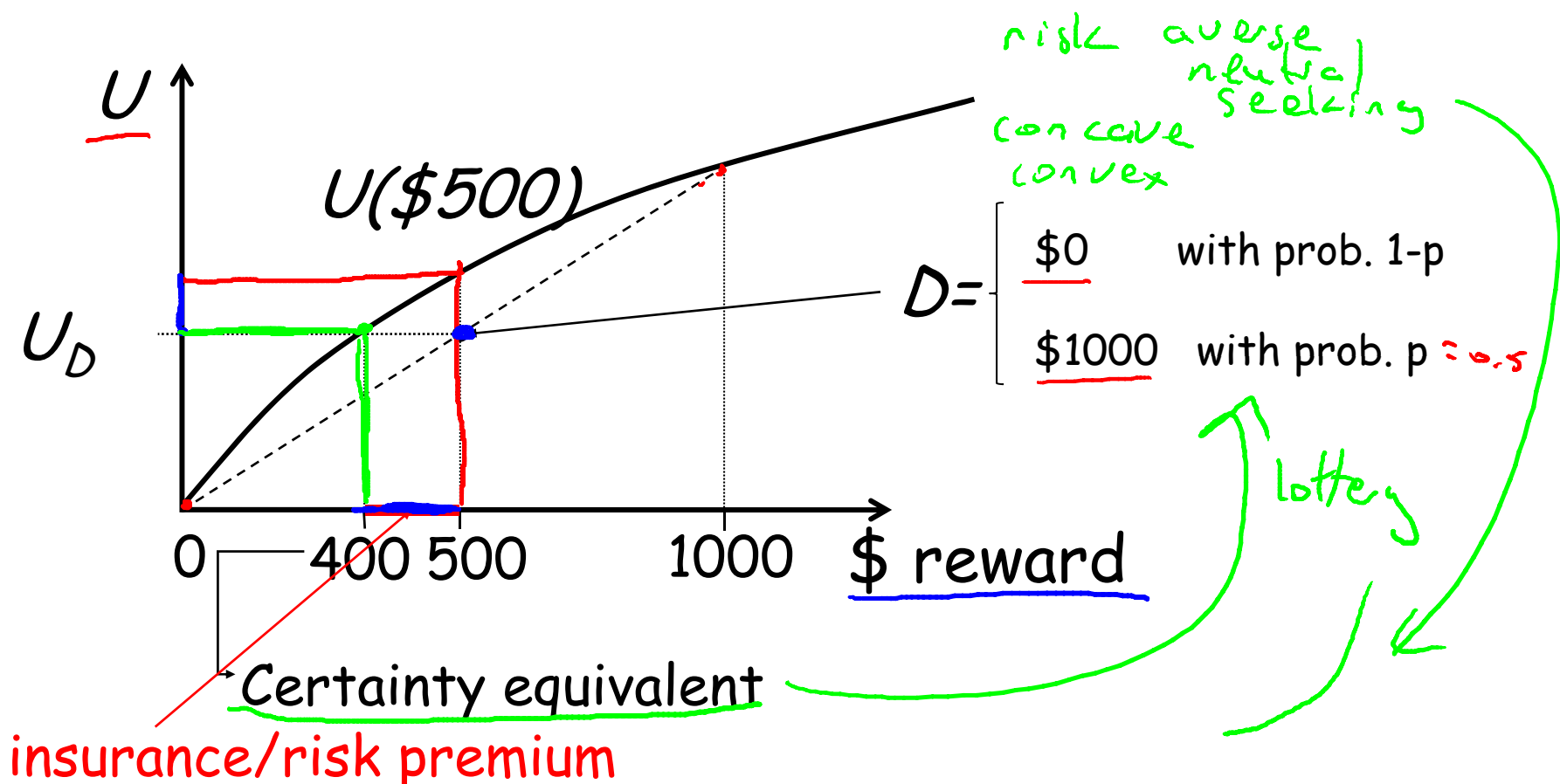
St. Petersburg Paradox



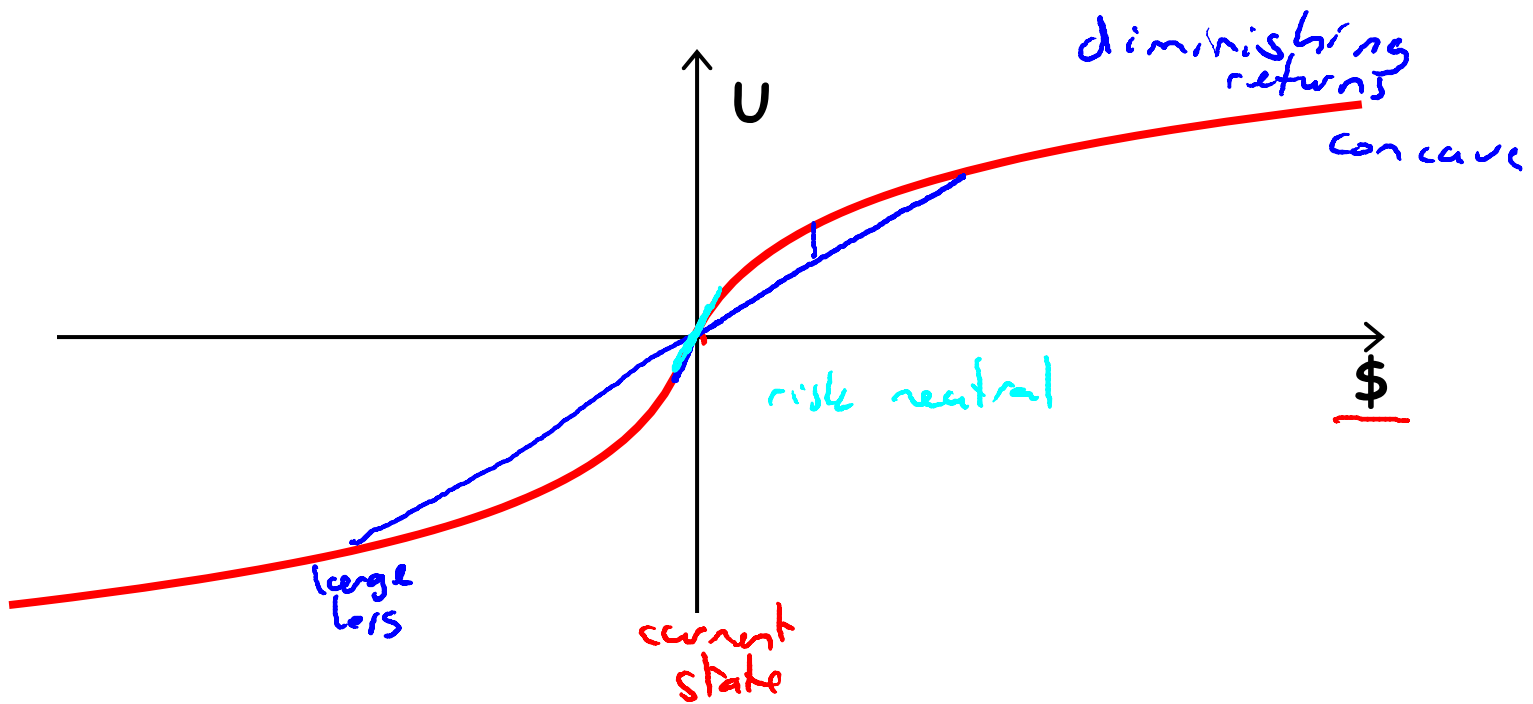
- Fair coin is tossed repeatedly until it comes up heads, say on the n^{th} toss
- Payoff = $\$2^n$

$$\frac{1}{2} \times 2 + \frac{1}{4} \times 4 + \frac{1}{8} \times 8 + \dots = \infty$$

most people value \approx \$2



Typical Utility Curve



Multi-Attribute Utilities

- All attributes affecting preferences must be integrated into one utility function

money, time, pleasure, ...

- Human life

- Micromorts *1/1000000 chance of death ≈ \$20 1930*
- QALY (quality-adjusted life year)

Example: Prenatal diagnosis

$$\underline{U_1(T)} + \underline{U_2(K)} + \underline{U_3(D,L)} + \underline{U_4(L,F)}$$

Testing

Knowledge

Down's
syndrome

Loss of
fetus

Future
pregnancy

Summary

- Our utility function determines our preferences about decisions that involve uncertainty
- Utility generally depends on multiple factors
 - Money, time, chances of death, ...
- Relationship is usually nonlinear
 - Shape of utility curve determines attitude to risk
- Multi-attribute utilities can help decompose high-dimensional function into tractable pieces