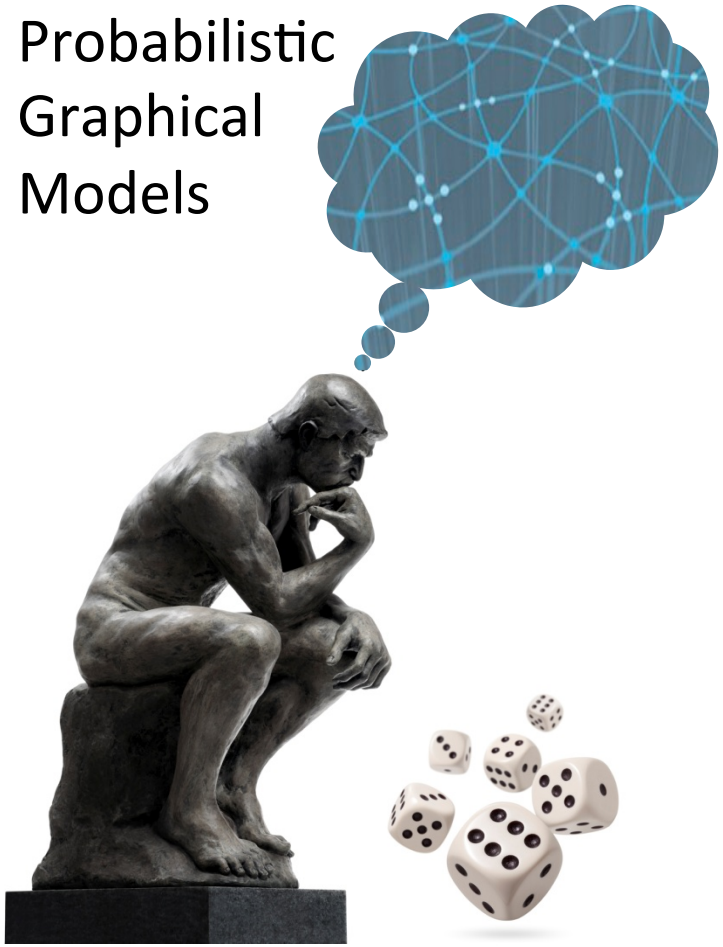
Probabilistic Graphical Models



Acting

Decision Making

Value of Perfect Information

Value of Information

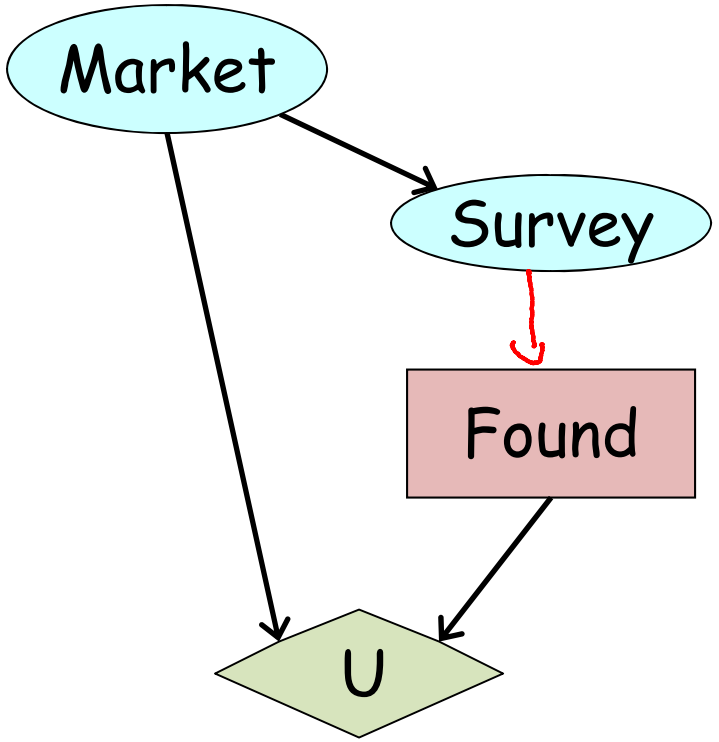
- VPI($A \mid X$) is the value of observing X before choosing an action at A
- $\mathcal{D}$ = original influence diagram
- $\mathcal{D}_{X \rightarrow A}$ = influence diagram with edge $X \rightarrow A$

$$\mathrm{VPI}(A \mid X) := \mathrm{MEU}(\mathcal{D}_{X \rightarrow A}) - \mathrm{MEU}(\mathcal{D})$$

Finding MEU Decision Rules

$MEU(D_{S \to F}) - MEU(D)$

$3.25 \qquad 2 \qquad = 1.25$



Value of Information

$$\mathrm{VPI}(A \mid X) := \mathrm{MEU}(\mathcal{D}_{X \to A}) - \mathrm{MEU}(\mathcal{D})$$

optimizing $\delta(A \mid \bar{z}, x)$ — optimizing $\delta(A \mid \bar{z})$

- Theorem:
  - VPI(A | X) ≥ 0
  - VPI(A | X) = 0 if and only if the optimal decision rule for $\mathcal{D}$ is still optimal for $\mathcal{D}_{X \to A}$

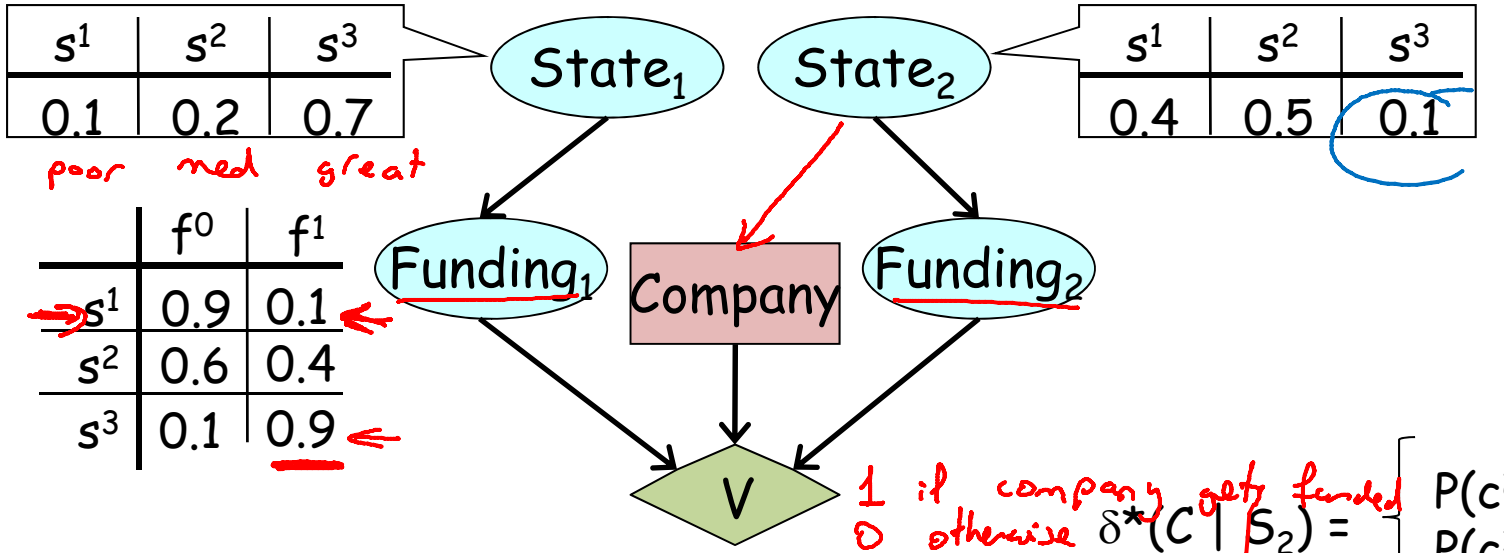
Any CPD $\delta(A \mid \bar{z})$ is also a CPD $\delta(A \mid \bar{z}, x)$

Clear notion of when information useful

⇕

it changes my decision

Value of Information Example



s^1	s^2	s^3
0.1	0.2	0.7

poor med great

s^1	s^2	s^3
0.4	0.5	0.1

	f^0	f^1
s^1	0.9	0.1
s^2	0.6	0.4
s^3	0.1	0.9

1 if company gets funded
0 otherwise

$\delta^*(C \mid S_2)$ =

$P(c^2)=1$ if $S_2 = s^3$
$P(c^1)=1$ otherwise

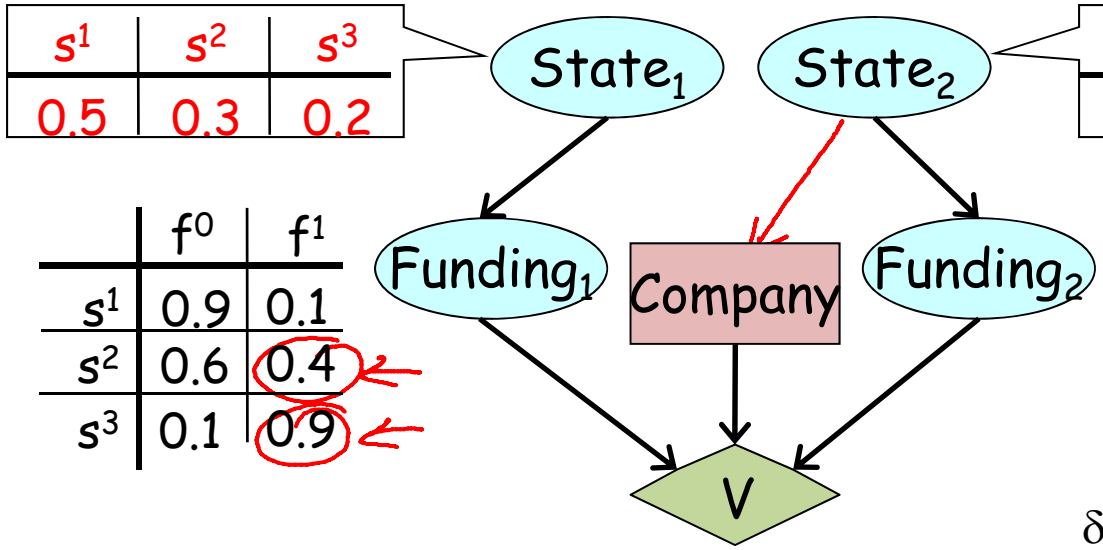
$EU(\mathcal{D}[c_1]) = 0.72$

$EU(\mathcal{D}[c_2]) = 0.33$

EU if $C=c_2$ $S_2=s^1$ 0.1 ⇒ Prefer c_1
EU if $C=c_2$ $S_2=s^2$ 0.4
EU if $C=c_2$ $S_2=s^3$ 0.9 ⇒ Prefer c_2

$MEU(\mathcal{D}_{S_1 \to C}) = 0.743$

Value of Information Example



	f^0	f^1
s^1	0.9	0.1
s^2	0.6	0.4
s^3	0.1	0.9

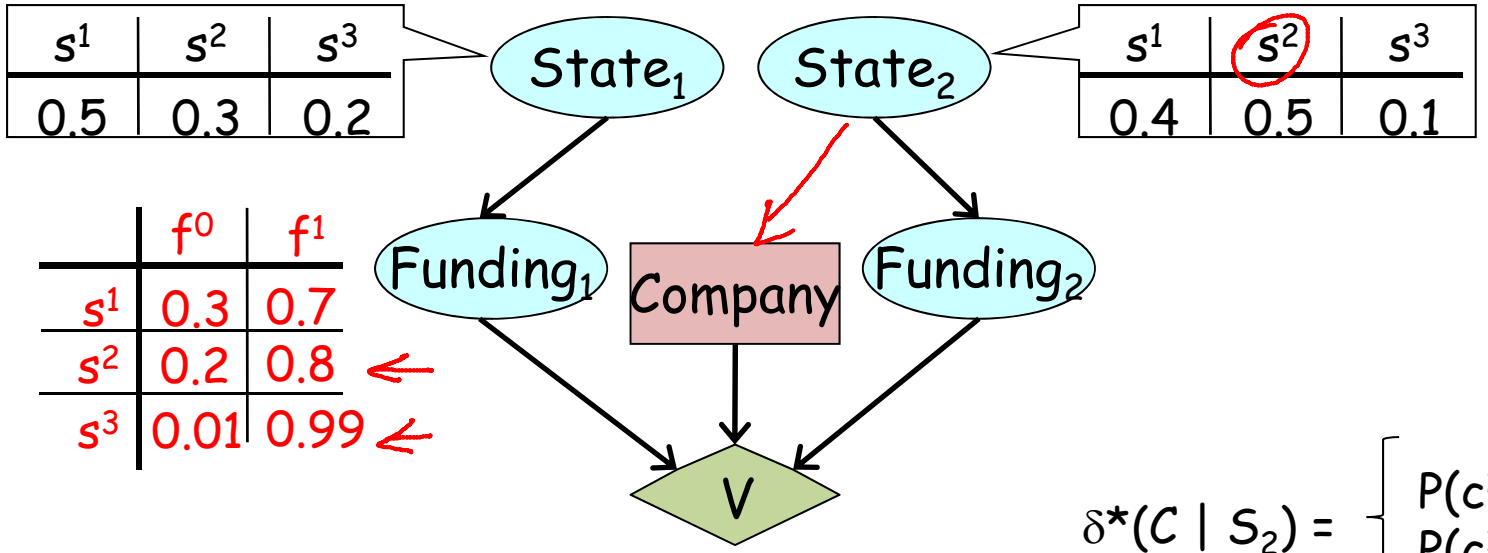
$$\delta^*(C \mid S_2) = \begin{cases} P(c^2)=1 \text{ if } S_2 = s^2, s^3 \\ P(c^1)=1 \text{ otherwise} \end{cases}$$

$EU(\mathcal{D}[c_1]) = 0.35$

$EU(\mathcal{D}[c_2]) = 0.33$

$MEU(\mathcal{D}_{S_2 \rightarrow C}) = 0.43$

Value of Information Example



$$\delta^*(C \mid S_2) = \begin{cases} P(c^2)=1 \text{ if } S_2 = s^2, s^3 \\ P(c^1)=1 \text{ otherwise} \end{cases}$$

$EU(\mathcal{D}[c_1]) = 0.788$

$EU(\mathcal{D}[c_2]) = 0.779$

$MEU(\mathcal{D}_{S_1 \rightarrow C}) = 0.8142$

Summary

- Influence diagrams provide clear and coherent semantics for the value of making an observation
  - Difference between values of two IDs
- Information is valuable if and only if it induces a change in action in at least one context