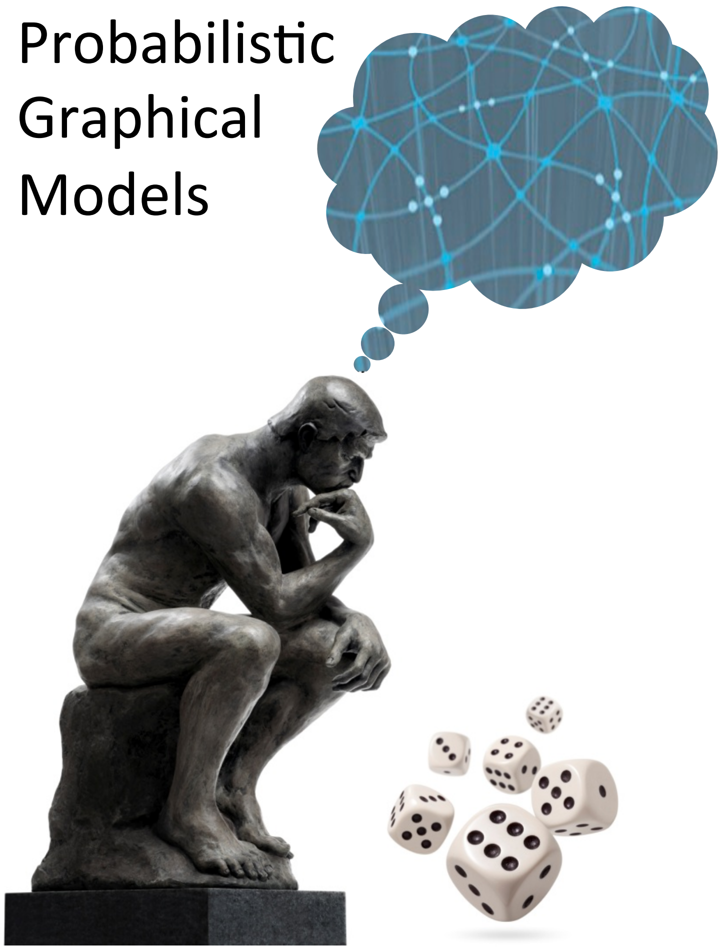
Probabilistic
Graphical
Models



Inference

Variable Elimination

Finding Elimination Orderings

Finding Elimination Orderings

- Theorem: For a graph H, determining whether there exists an elimination ordering for H with induced width ≤ K is NP-complete

- Note: This NP-hardness result is distinct from the NP-hardness result of inference
  - Even given the optimal ordering, inference may still be exponential

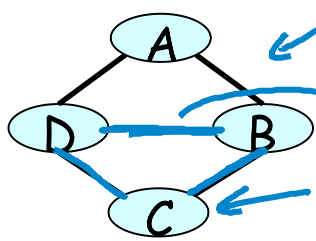
Finding Elimination Orderings

- Greedy search using heuristic cost function
  - At each point, eliminate node with smallest cost
- Possible cost functions:
  - min-neighbors: # neighbors in current graph
  - min-weight: weight (# values) of factor formed
  - min-fill: number of new fill edges
  - weighted min-fill: total weight of new fill edges (edge weight = product of weights of the 2 nodes)

smallest factor

Finding Elimination Orderings

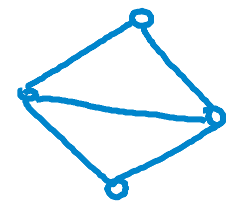
- Theorem: The induced graph is triangulated
  - No loops of length > 3 without a "bridge"



Assume we have a loop of size > 3

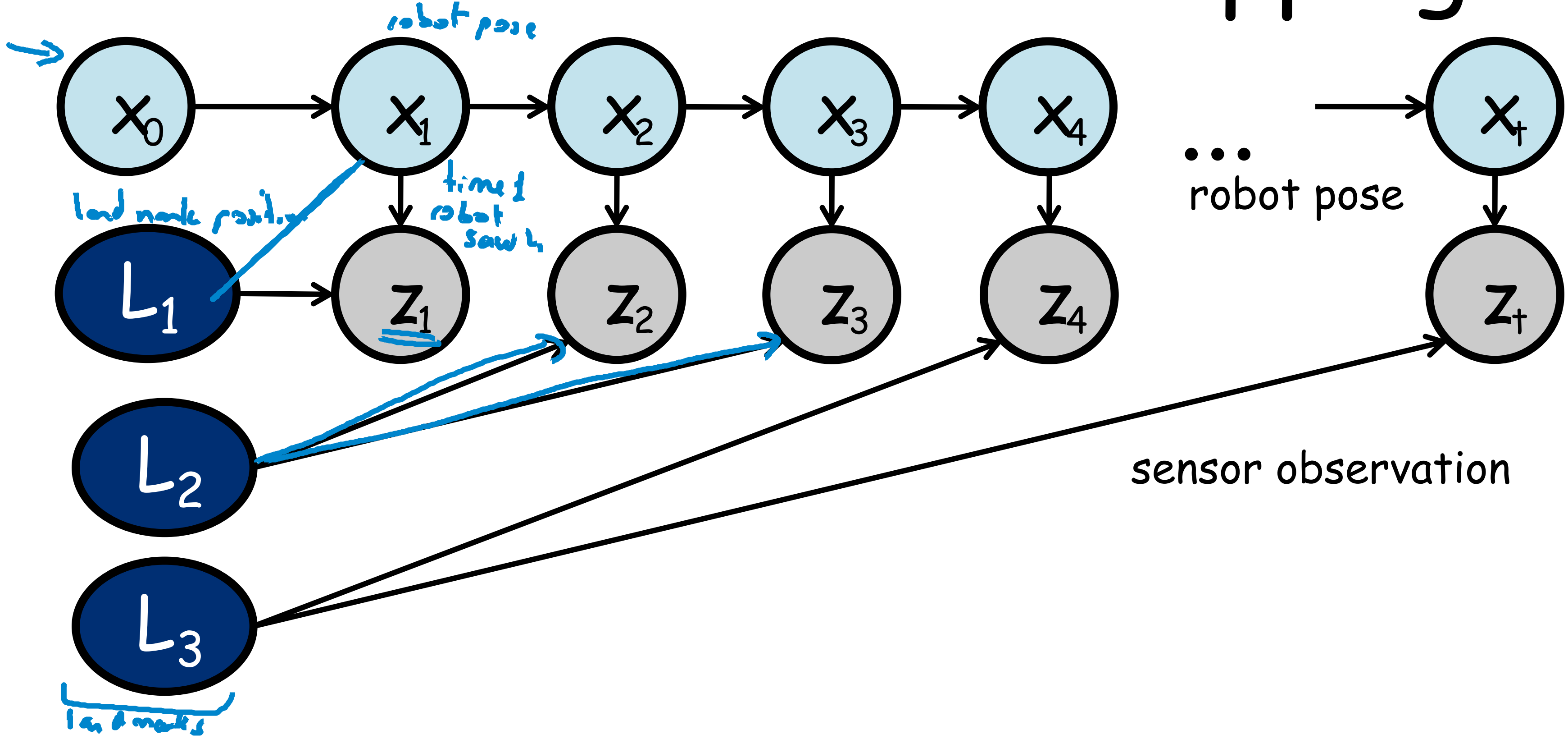
fill edge must be added

first to be eliminated



- Can find elimination ordering by finding a low-width triangulation of original graph H_Φ

Robot Localization & Mapping



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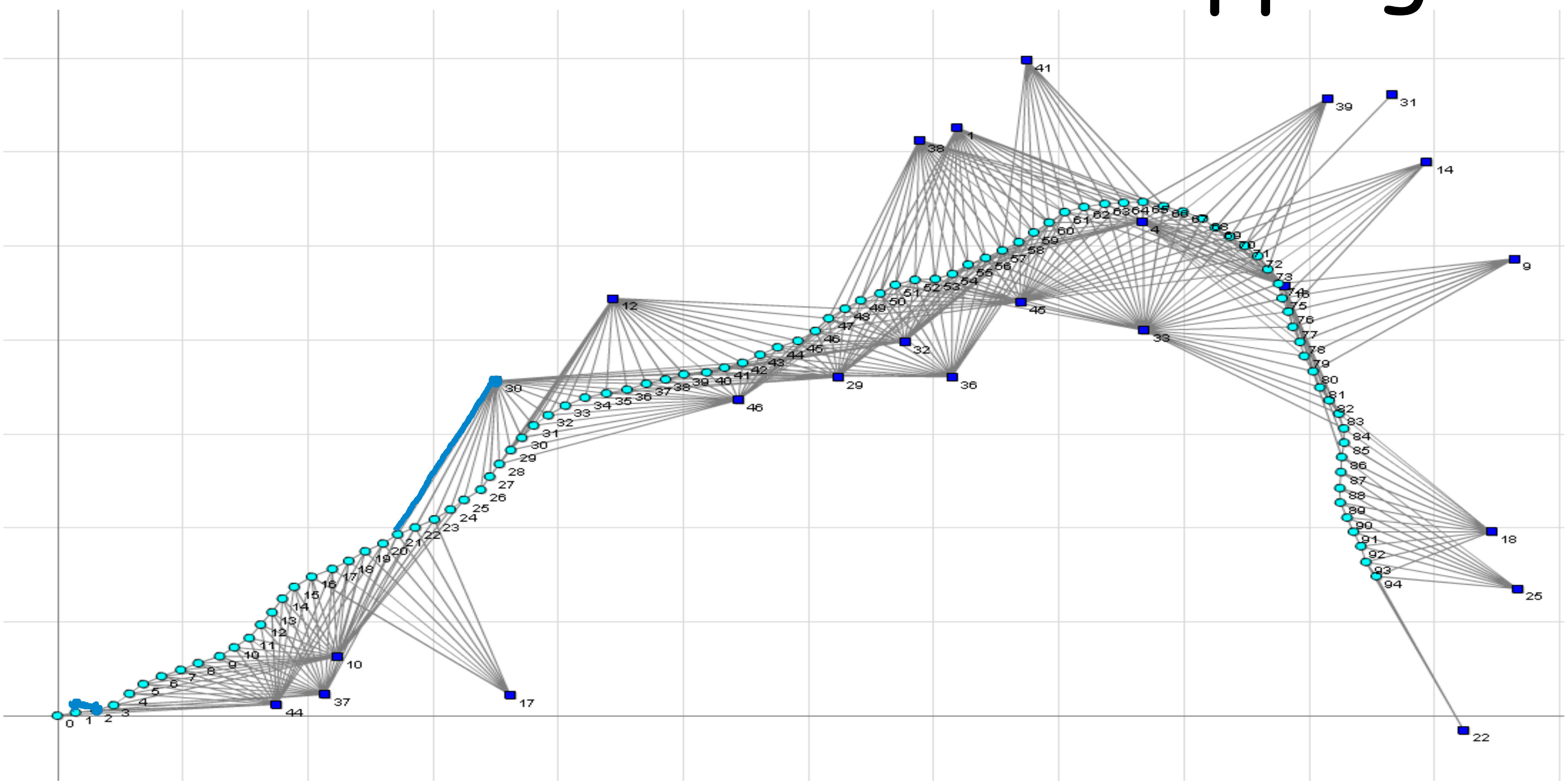
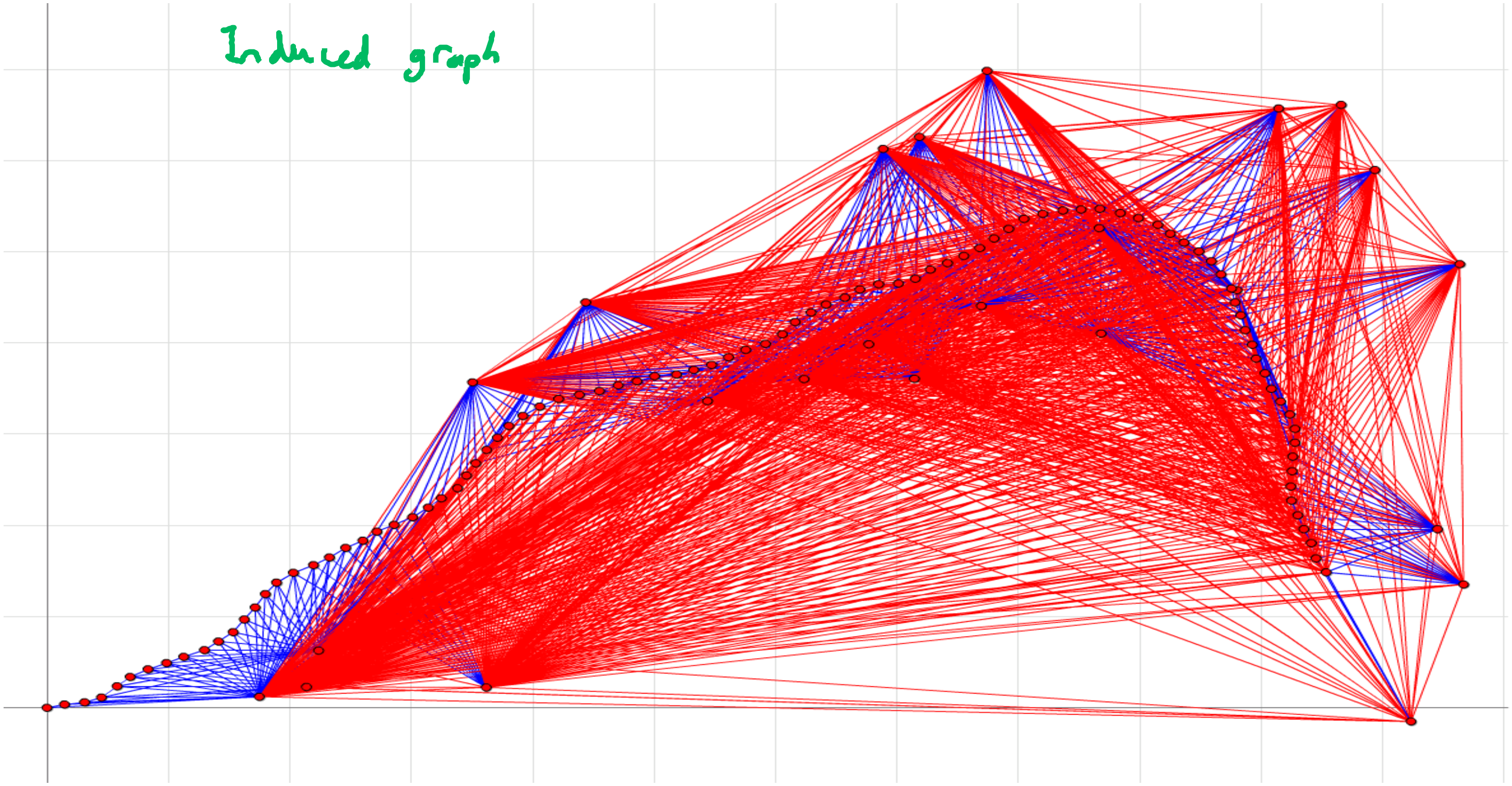
Square Root SAM, F. Dellaert and M. Kaess, IJRR, 2006

Robot Localization & Mapping



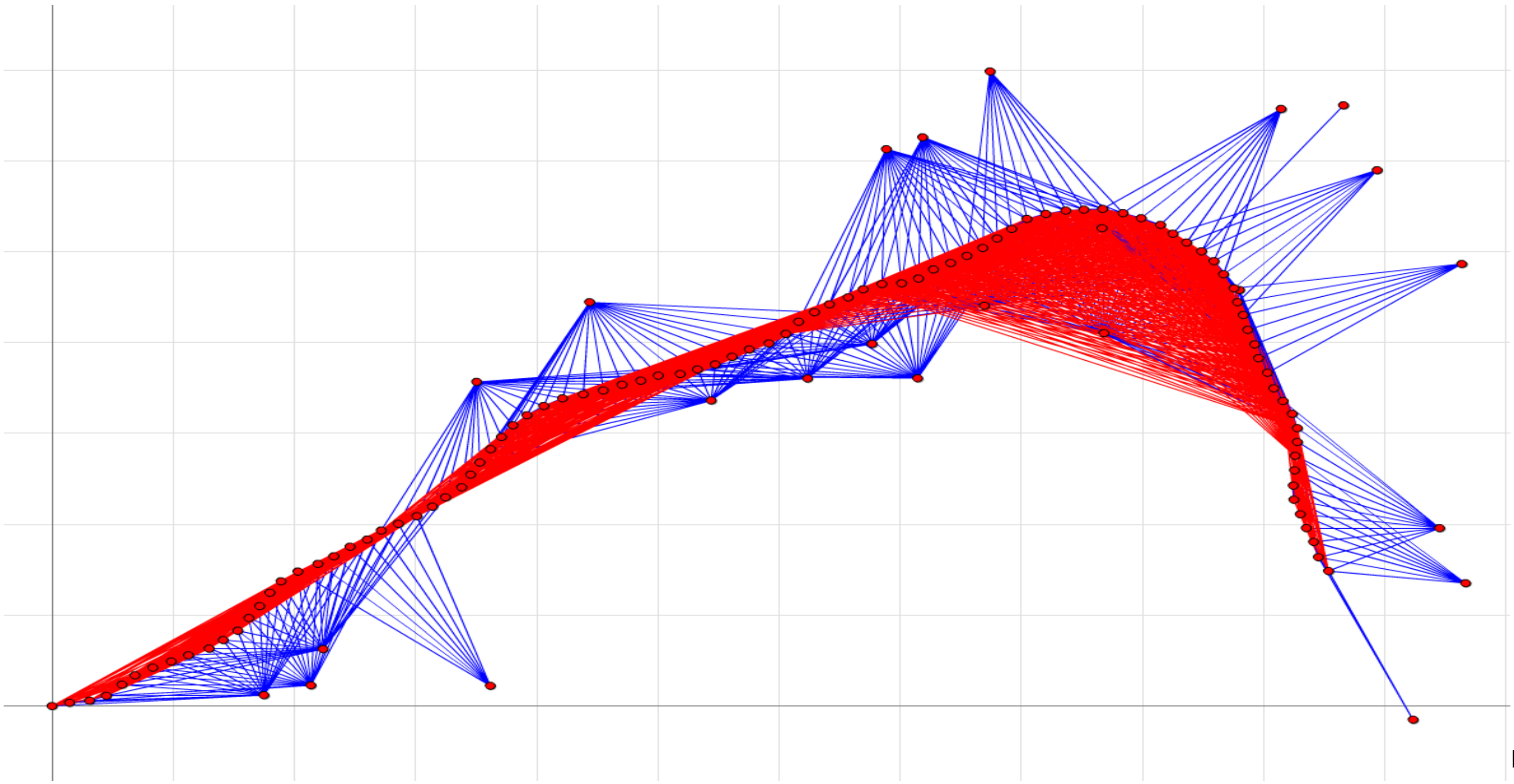
Square Root SAM, F. Dellaert and M. Kaess, IJRR, 2006

Eliminate Poses then Landmarks



Square Root SAM, F. Dellaert and M. Kaess, IJRR, 2006

Eliminate Landmarks then Poses



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Summary

- Finding the optimal elimination ordering is NP-hard
- Simple heuristics that try to keep induced graph small often provide reasonable performance