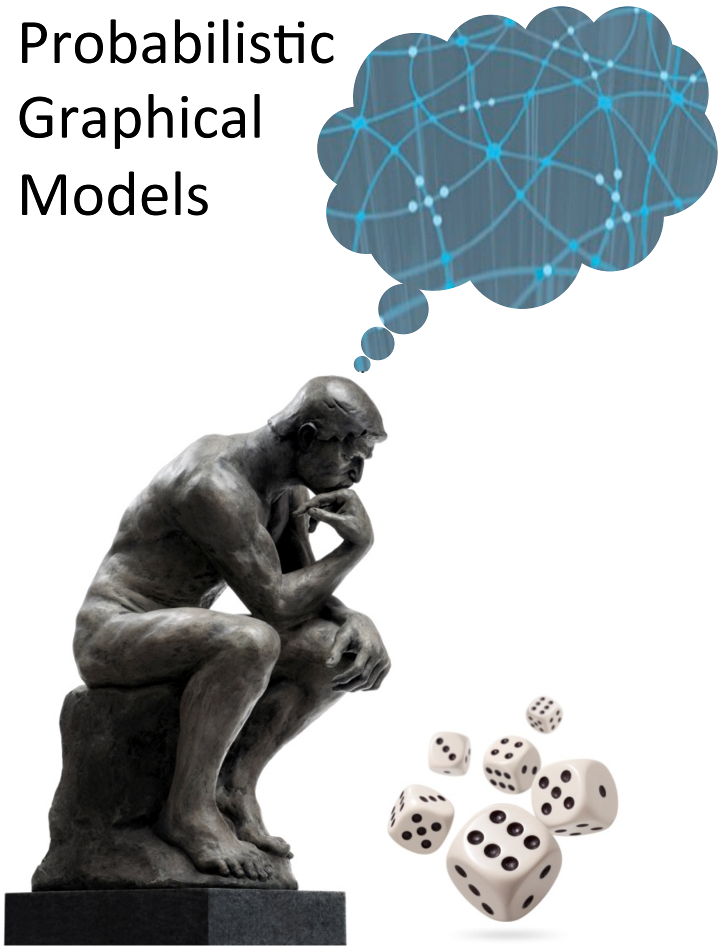
Probabilistic
Graphical
Models



Inference

Message Passing

Cluster Graph Properties

Cluster Graphs

- Undirected graph such that:
  - nodes are clusters $\boldsymbol{C}_i \subseteq \{X_1,\ldots,X_n\}$
  - edge between $\boldsymbol{C}_i$ and $\boldsymbol{C}_j$ associated with sepset $\mathbf{S}_{i,j} \subseteq \boldsymbol{C}_i \cap \boldsymbol{C}_j$

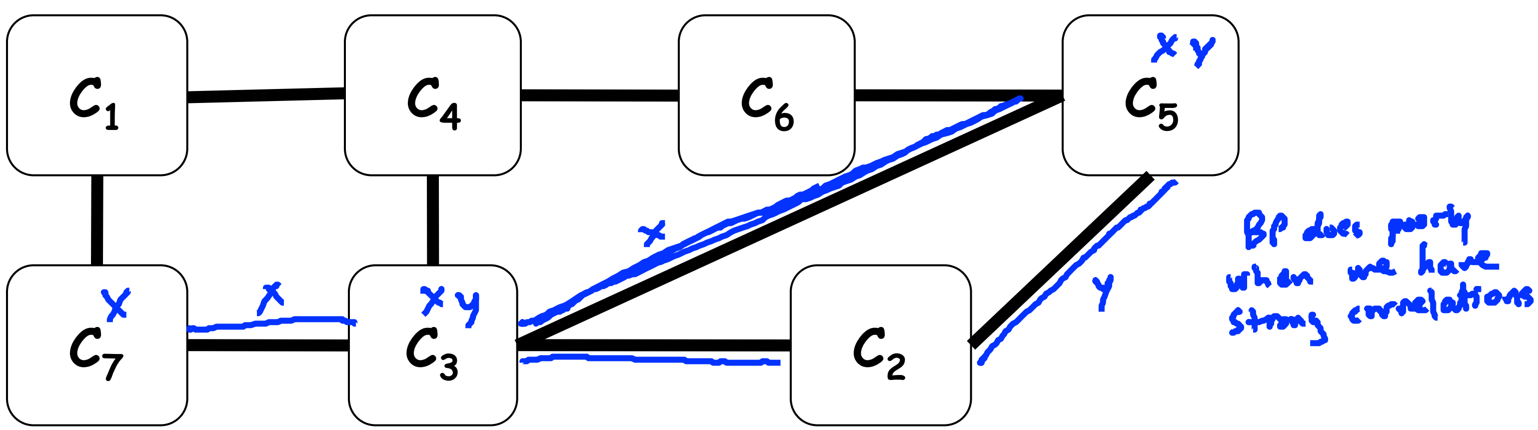
Family Preservation

- Given set of factors Φ, we assign each ϕ_k to a cluster $\boldsymbol{C}_{\alpha(k)}$ s.t. Scope$[\phi_k] \subseteq \boldsymbol{C}_{\alpha(k)}$

  cluster can accomodate ϕ_k
- For each factor $\phi_k \in \Phi$, there exists a cluster $\boldsymbol{C}_i$ s.t. Scope$[\phi_k] \subseteq \boldsymbol{C}_i$ ← accomodates ϕ_k

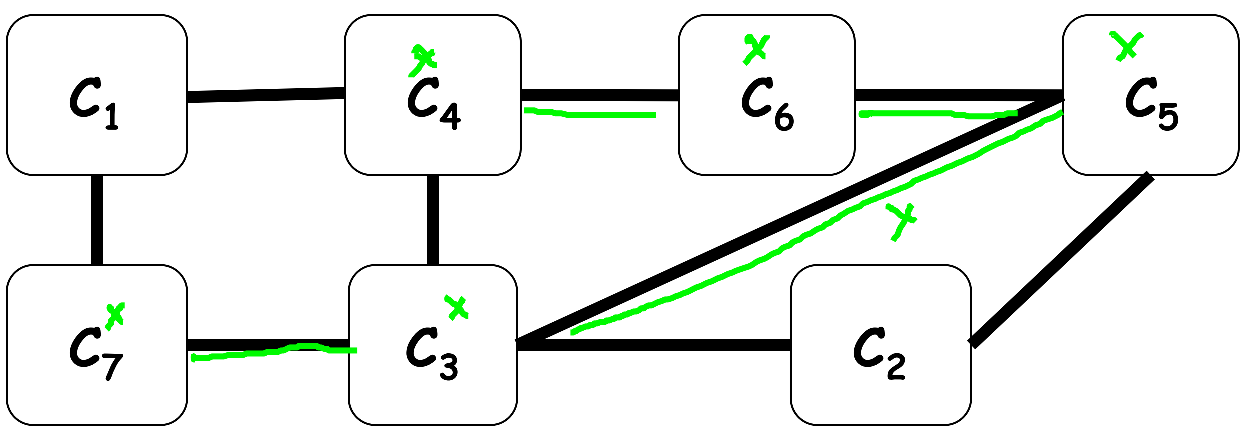
Running Intersection Property

- For each pair of clusters C_i, C_j and variable $X \in C_i \cap C_j$ there exists a unique path between C_i and C_j for which all clusters and sepsets contain X

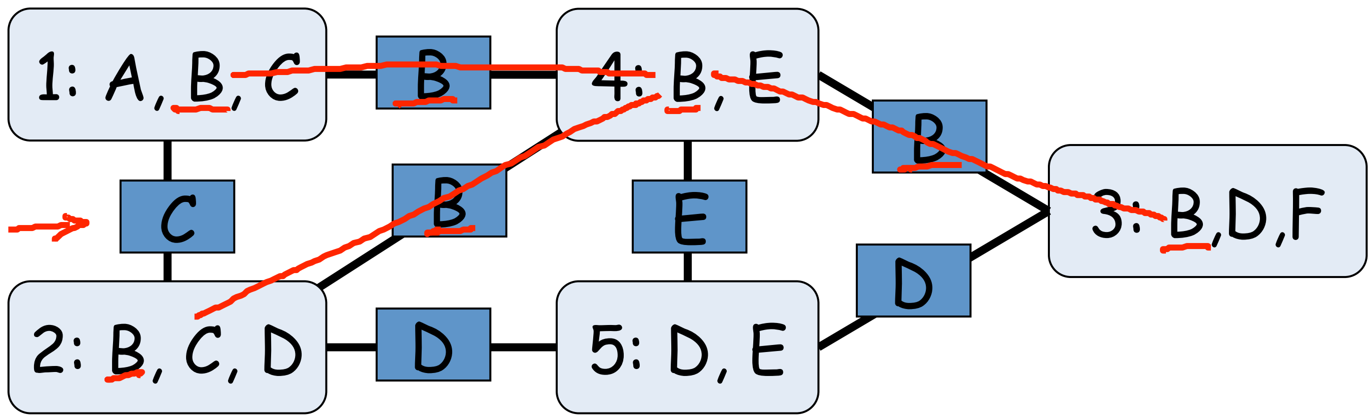


Running Intersection Property

- Equivalently: For any X, the set of clusters and sepsets containing X forms a tree



Example Cluster Graph



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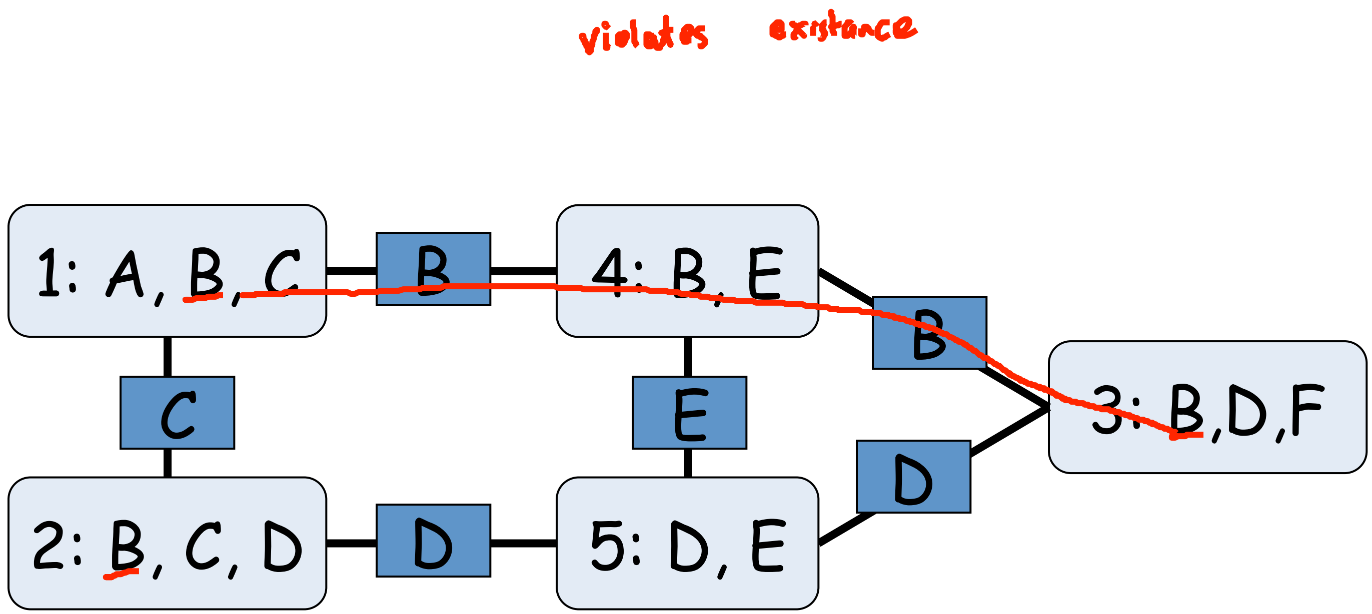
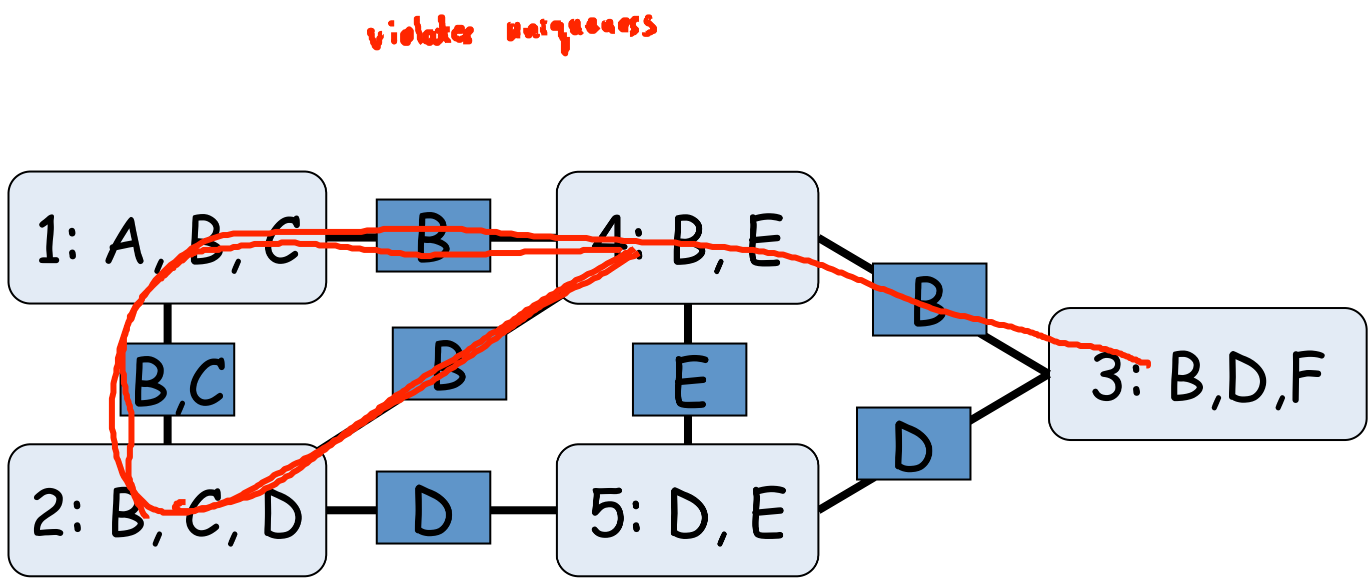
Illegal Cluster Graph I



Illegal Cluster Graph II



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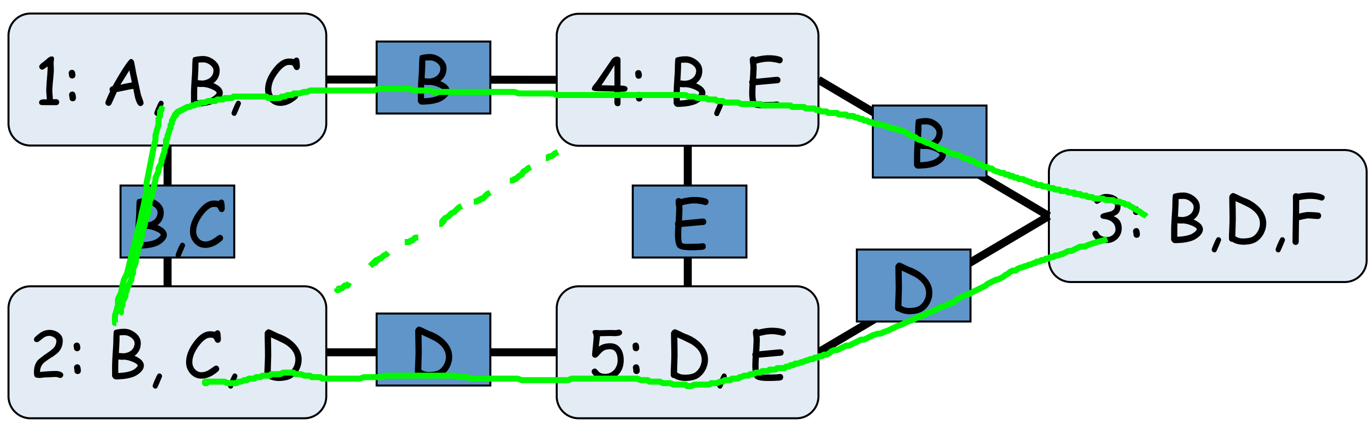
Alternative Legal Cluster Graph



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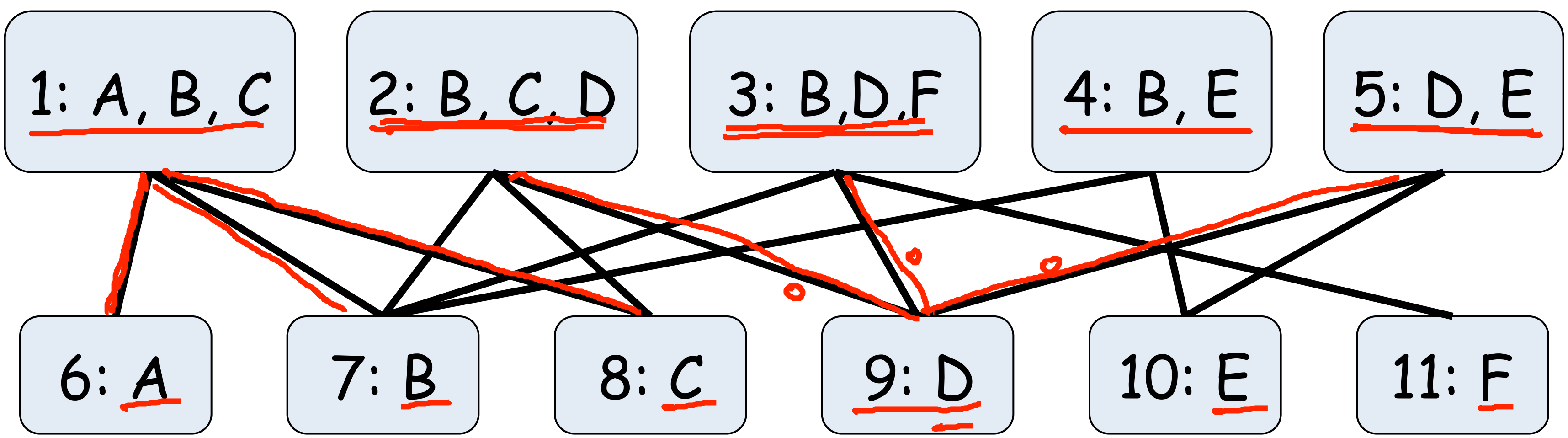
Bethe Cluster Graph

big clusters = factor in Φ

- For each $\phi_k \in \Phi$, a factor cluster $\boldsymbol{C}_k$ =Scope$[\phi_k]$
- For each X_i a singleton cluster $\{X_i\}$
- Edge $\boldsymbol{C}_k$ — X_i if $X_i \in \boldsymbol{C}_k$



Summary

- Cluster graph must satisfy two properties
  - family preservation: allows Φ to be encoded
  - running intersection: connects all information about any variable, but without feedback loops
- Bethe cluster graph is often first default
- Richer cluster graph structures can offer different tradeoffs wrt computational cost and preservation of dependencies