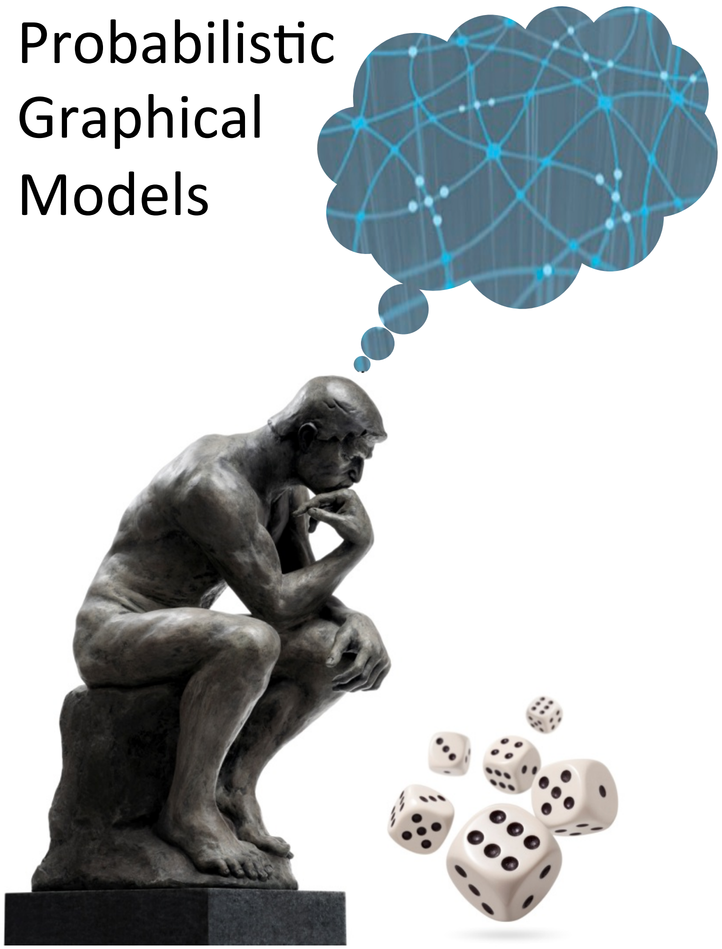
Probabilistic
Graphical
Models

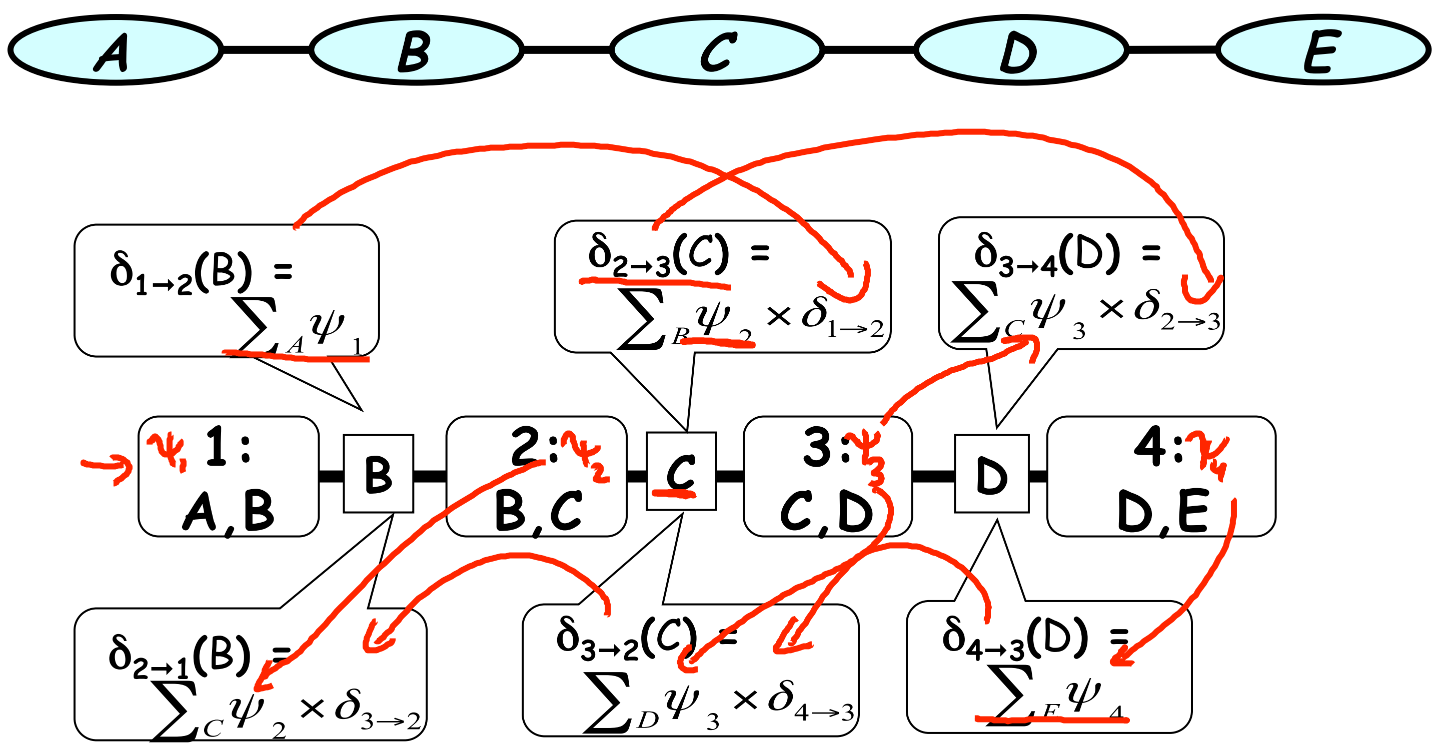


Inference

Message Passing

Clique Tree Algorithm & Correctness

Message Passing in Trees



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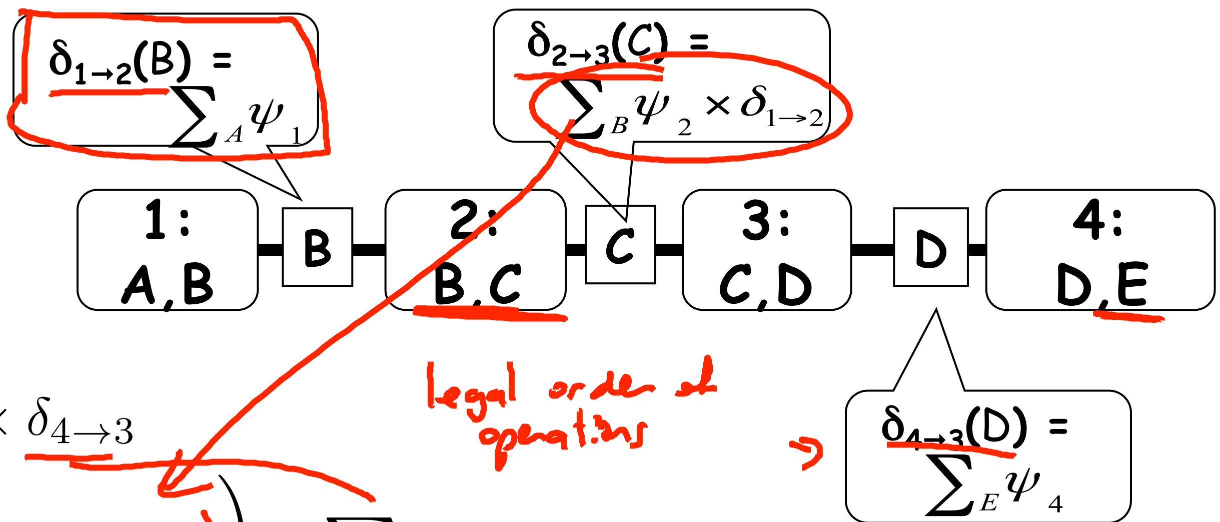
Correctness



legal order of operations

$$\beta_3(C,D) = \psi_3 \times \delta_{2\to 3} \times \delta_{4\to 3}$$

$$= \psi_3 \times \left(\sum_B (\psi_2 \times \delta_{1\to 2}) \right) \times \sum_E \psi_4$$

$$= \psi_3 \times \left(\sum_B \psi_2 \times \sum_A \psi_1 \right) \times \sum_E \psi_4$$

product of factors marginalized out unnecessary variables

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Clique Tree

- Undirected tree such that:
  - nodes are clusters $\boldsymbol{C}_i \subseteq \{X_1,\ldots,X_n\}$
  - edge between $\boldsymbol{C}_i$ and $\boldsymbol{C}_j$ associated with sepset $\boldsymbol{S}_{i,j} = \boldsymbol{C}_i \cap \boldsymbol{C}_j$

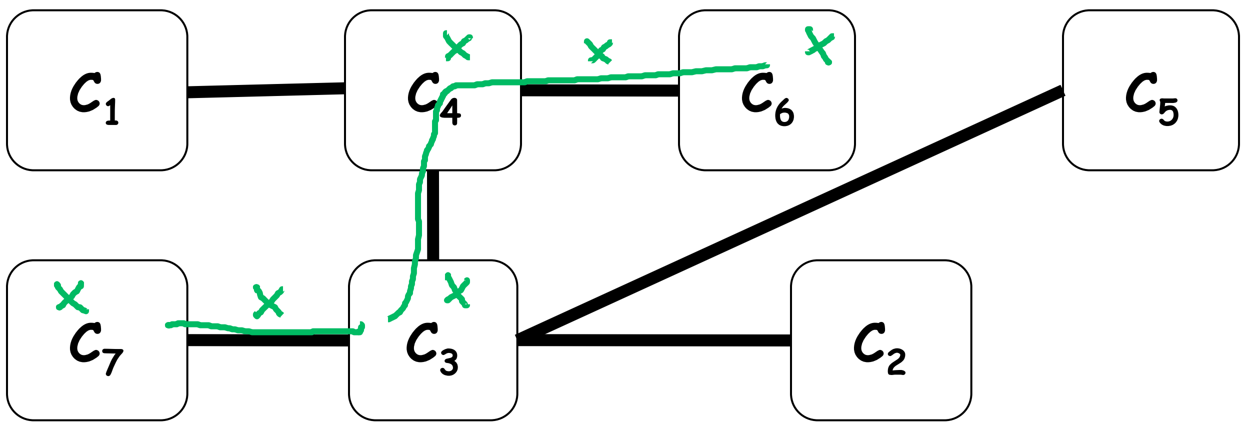
Family Preservation

- Given set of factors Φ, we assign each ϕ_k $\in \Phi$ to a cluster $\boldsymbol{C}_{\alpha(k)}$ s.t. $\text{Scope}[\phi_k] \subseteq \boldsymbol{C}_{\alpha(k)}$
- For each factor $\phi_k \in \Phi$, there exists a cluster $\boldsymbol{C}_i$ s.t. $\text{Scope}[\phi_k] \subseteq \boldsymbol{C}_i$

Running Intersection Property

Cluster graph variant

- For each pair of clusters $\mathbf{C}_i$, $\mathbf{C}_j$ and variable $X \in \mathbf{C}_i \cap \mathbf{C}_j$ there exists a unique path between $\mathbf{C}_i$ and $\mathbf{C}_j$ for which all clusters and sepsets contain X



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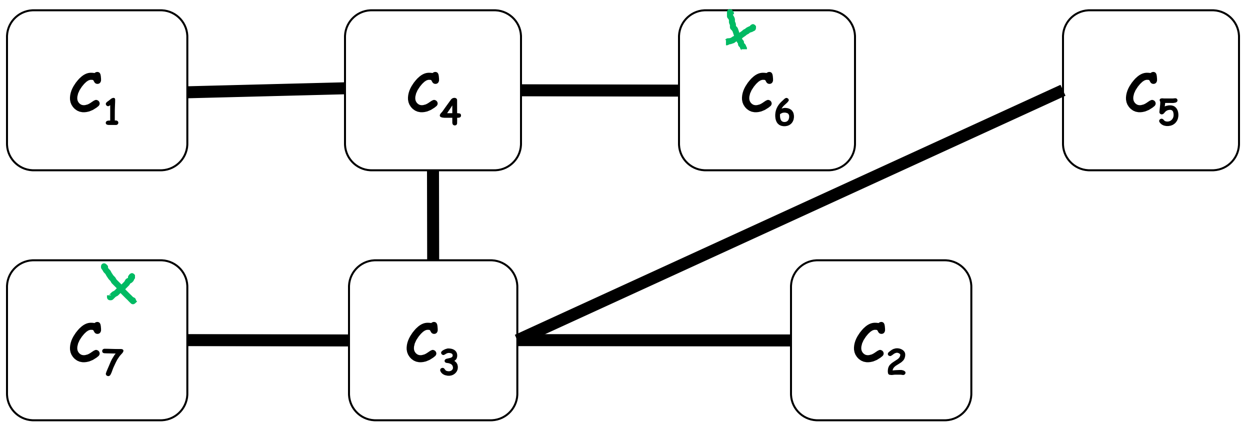
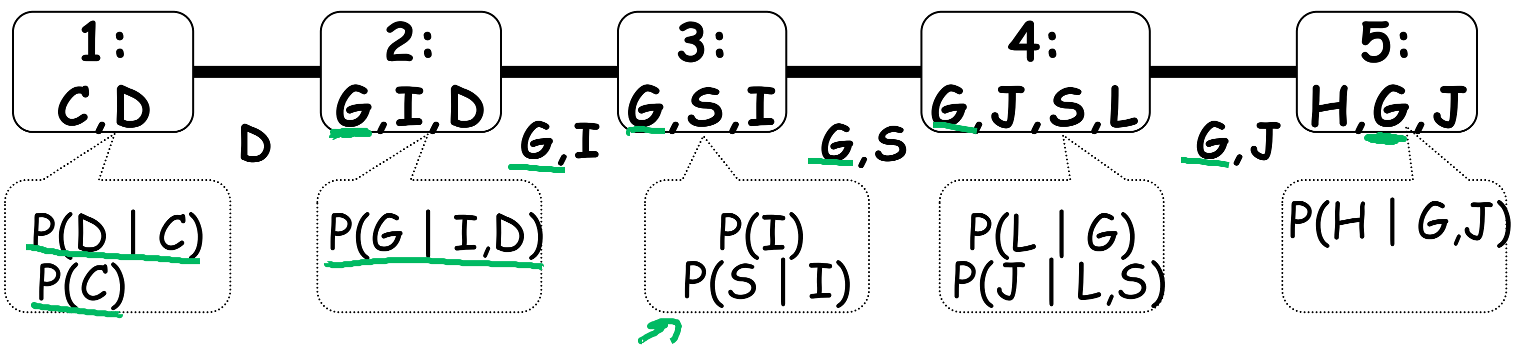
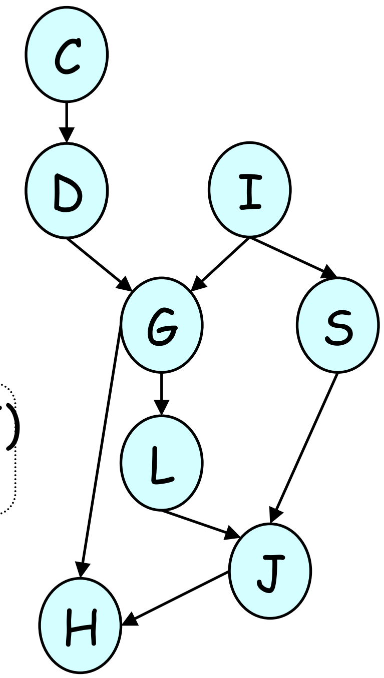
Running Intersection Property

- For each pair of clusters $\boldsymbol{C}_i$, $\boldsymbol{C}_j$ and variable $X \in \boldsymbol{C}_i \cap \boldsymbol{C}_j$, in the unique path between $\boldsymbol{C}_i$ and $\boldsymbol{C}_j$, all clusters and sepsets contain X

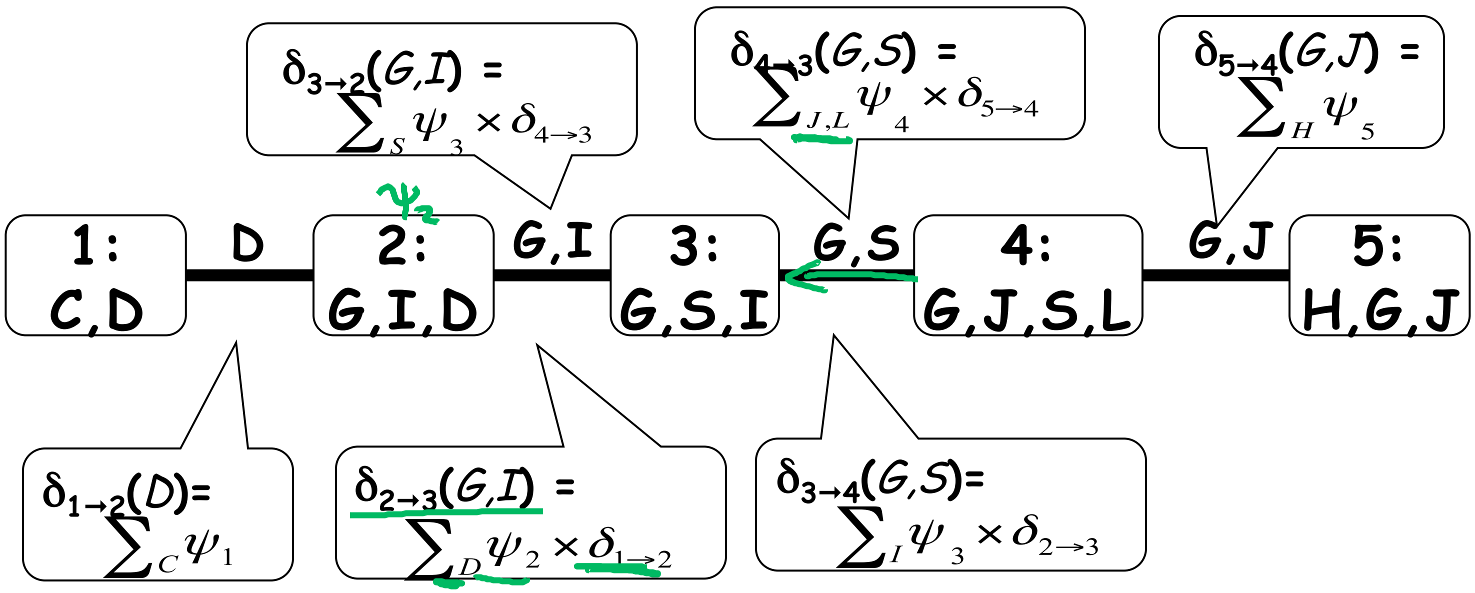


More Complex Clique Tree





Clique Tree Message Passing



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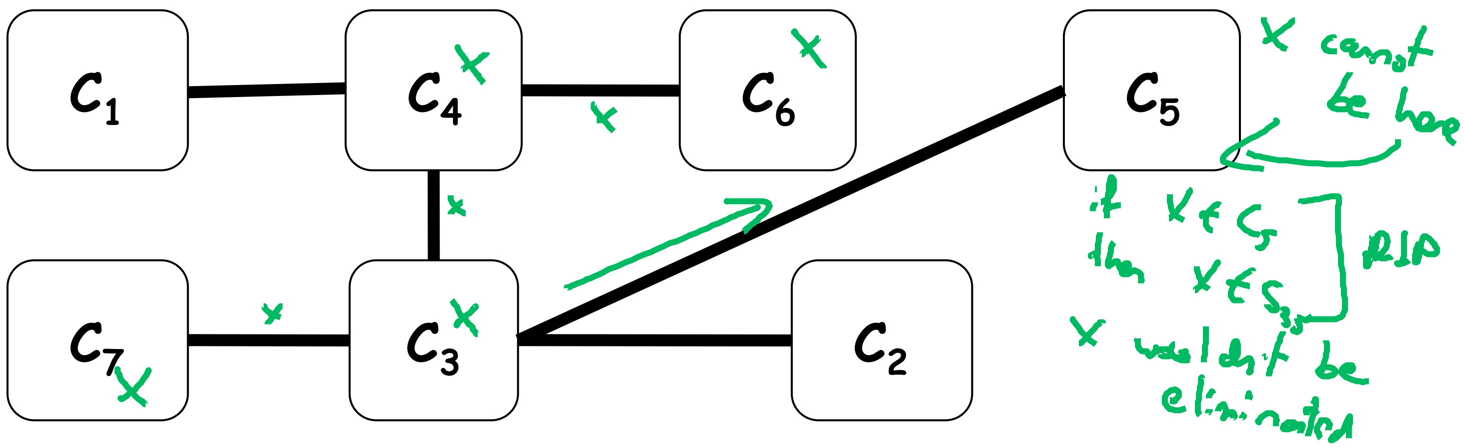
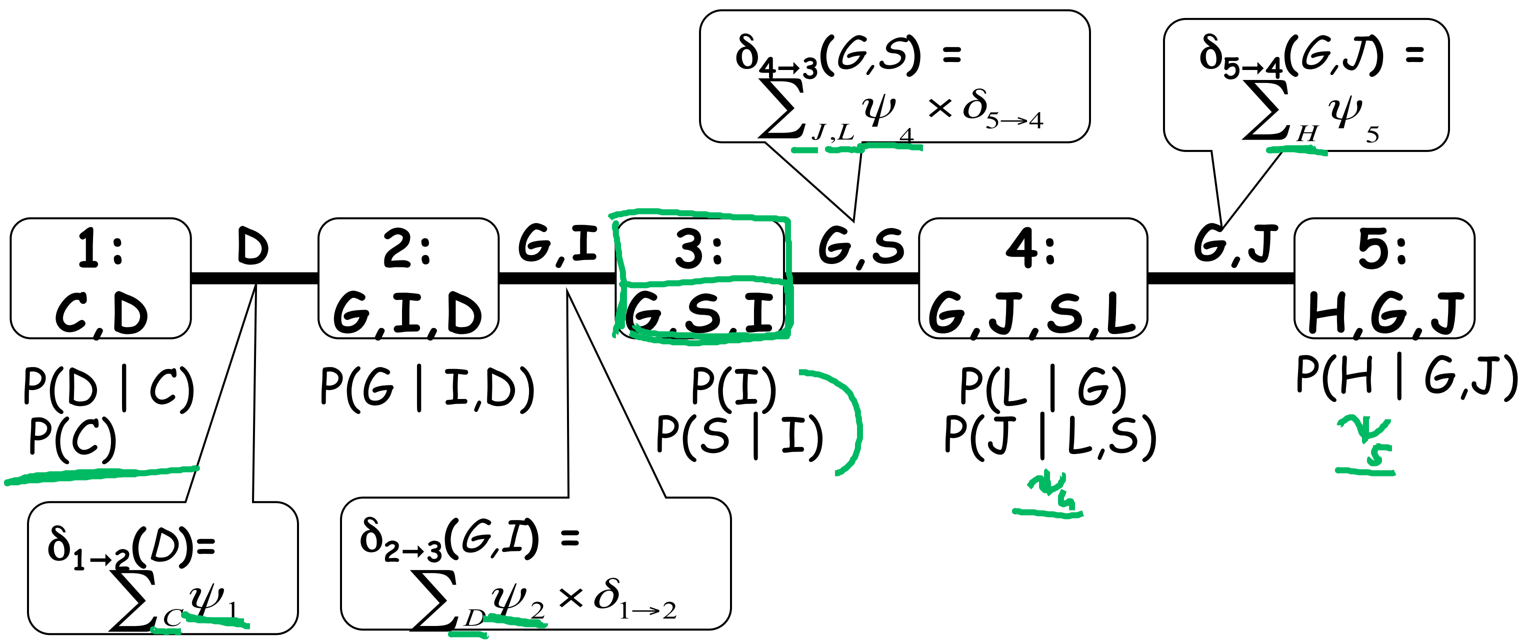
RIP ⇨ Clique Tree Correctness

- If X is eliminated when we pass the message $C_i \rightarrow C_j$
- Then X does not appear in the C_j side of the tree



Clique Tree Correctness



Summary

- Belief propagation can be run over a tree-structured cluster graph
- In this case, computation is a variant of variable elimination
- Resulting beliefs are guaranteed to be correct marginals of $\tilde{P}_{\Phi}$