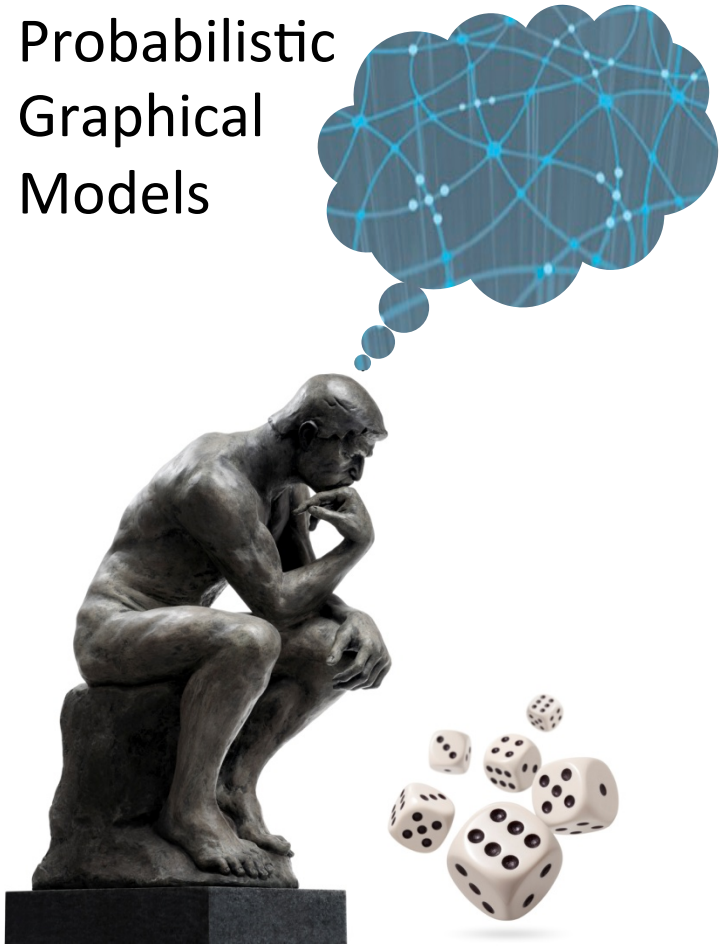
Probabilistic Graphical Models

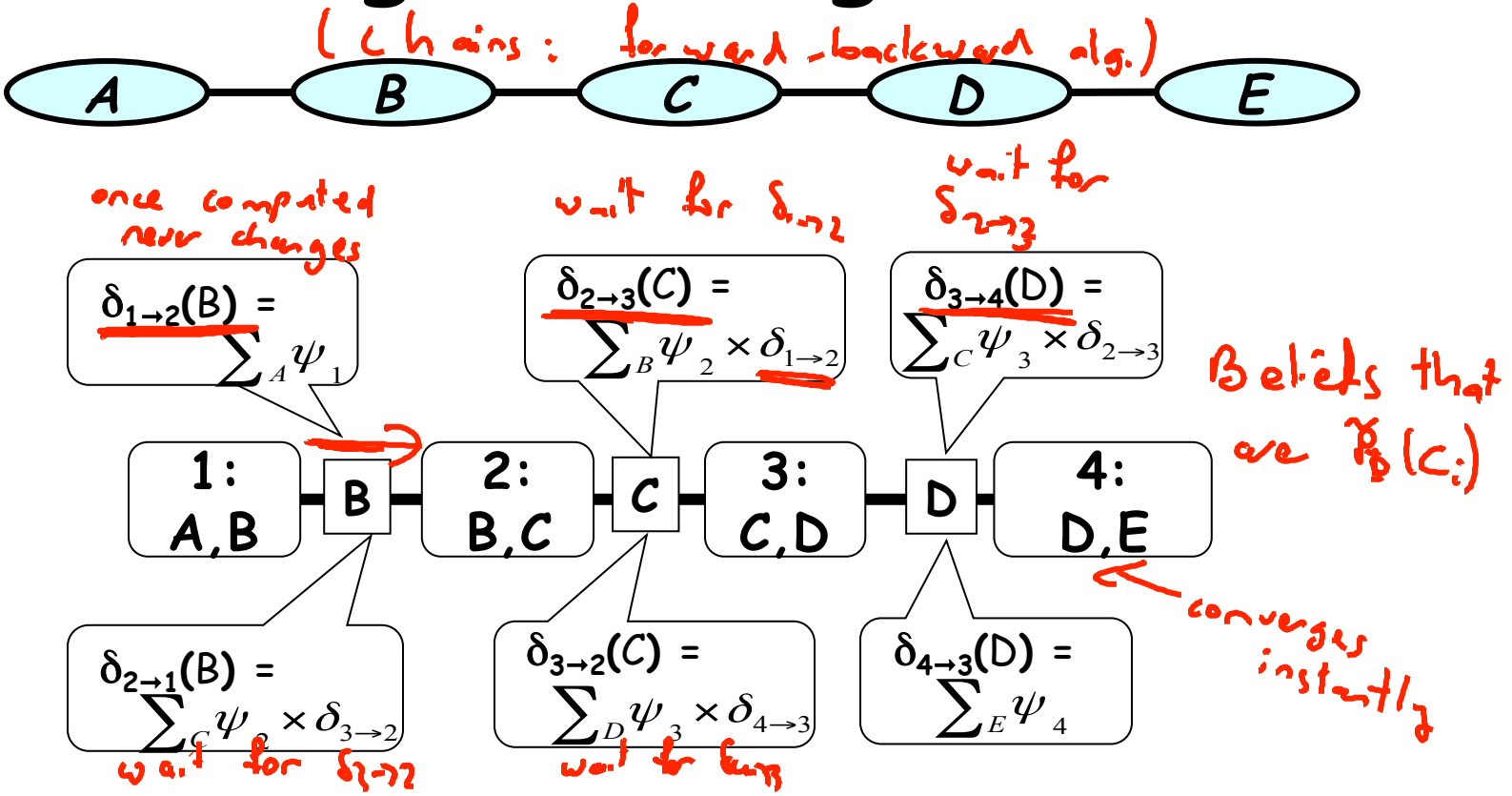


Inference

Message Passing

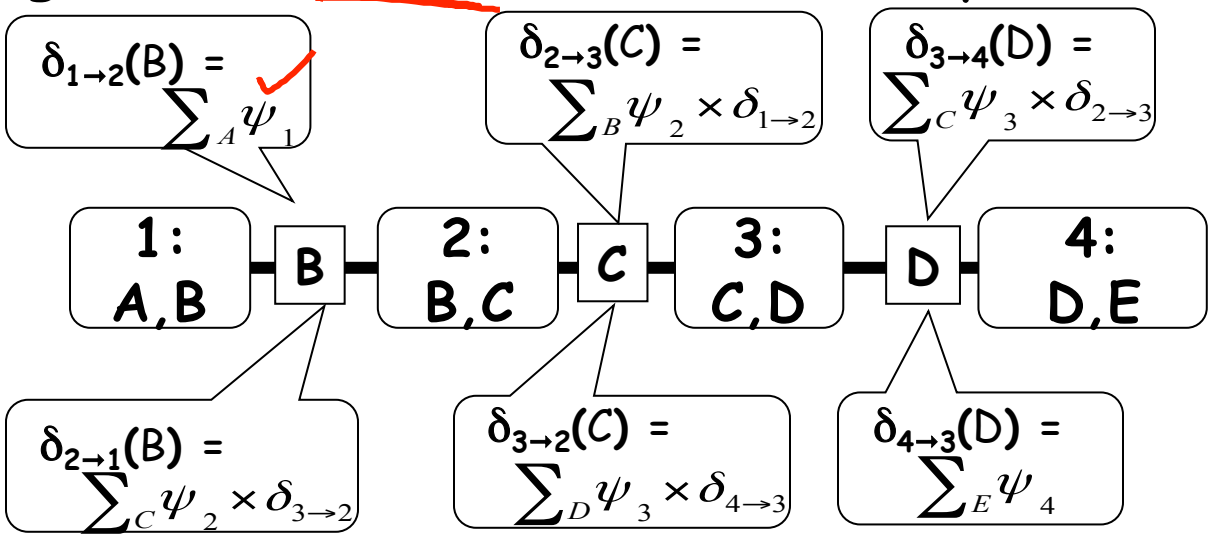
Clique Tree Algorithm: Computation

Message Passing in Trees



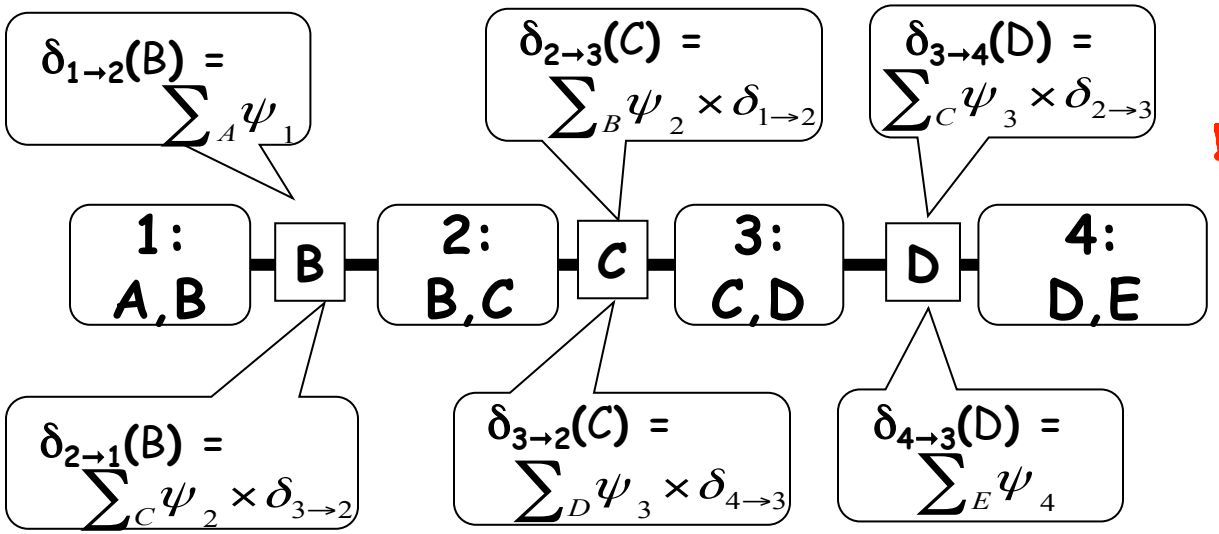
Convergence of Message Passing

- Once C_i receives a final message from all neighbors except C_j, then $\delta_{i \to j}$ is also final (will never change)
- Messages from leaves are immediately final



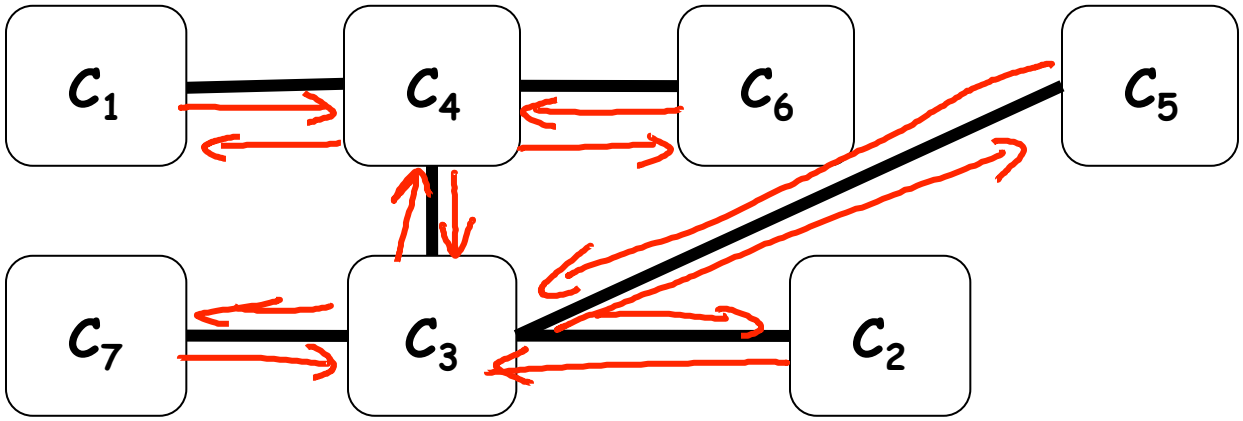
Convergence of Message Passing

- Can pass messages from leaves inward
- If messages are passed in the right order, only need to pass 2(K-1) messages

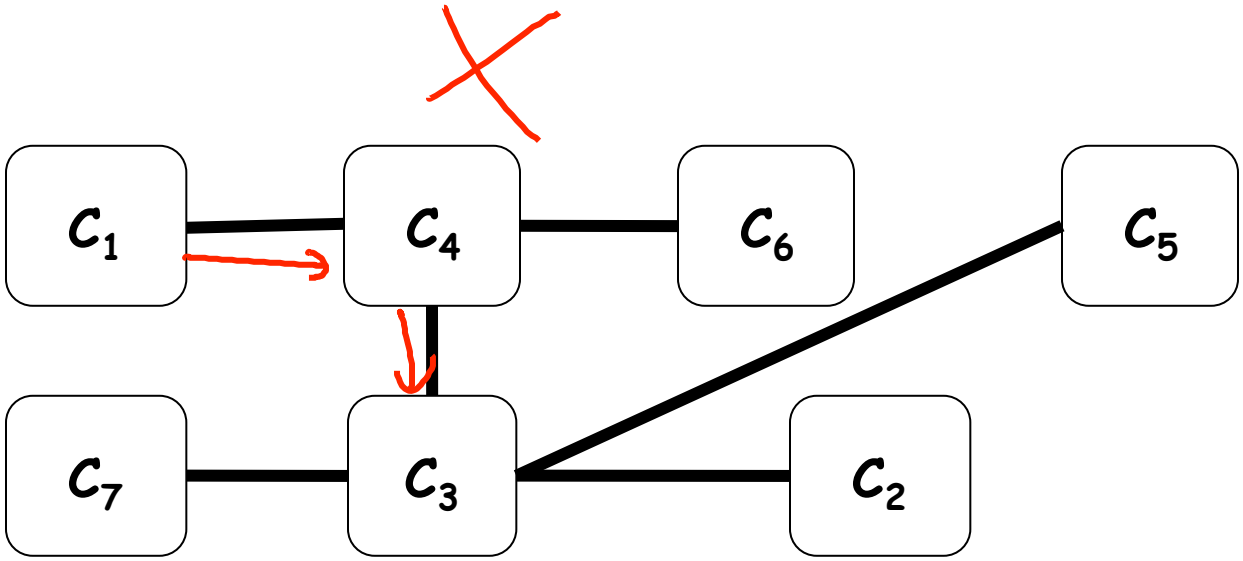


K total number of clusters.
K-1 edges
2(K-1) msgs

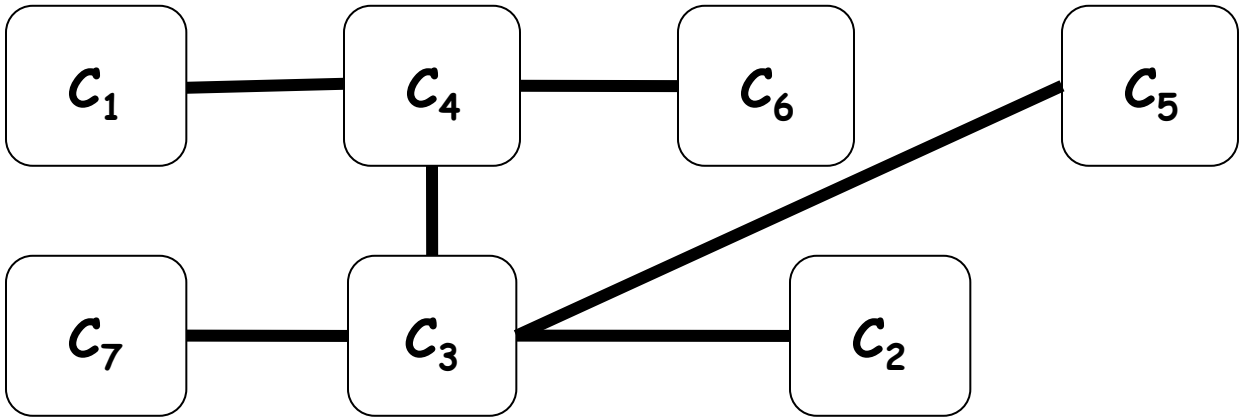
Message Passing Order I



Message Passing Order II



Message Passing Order III

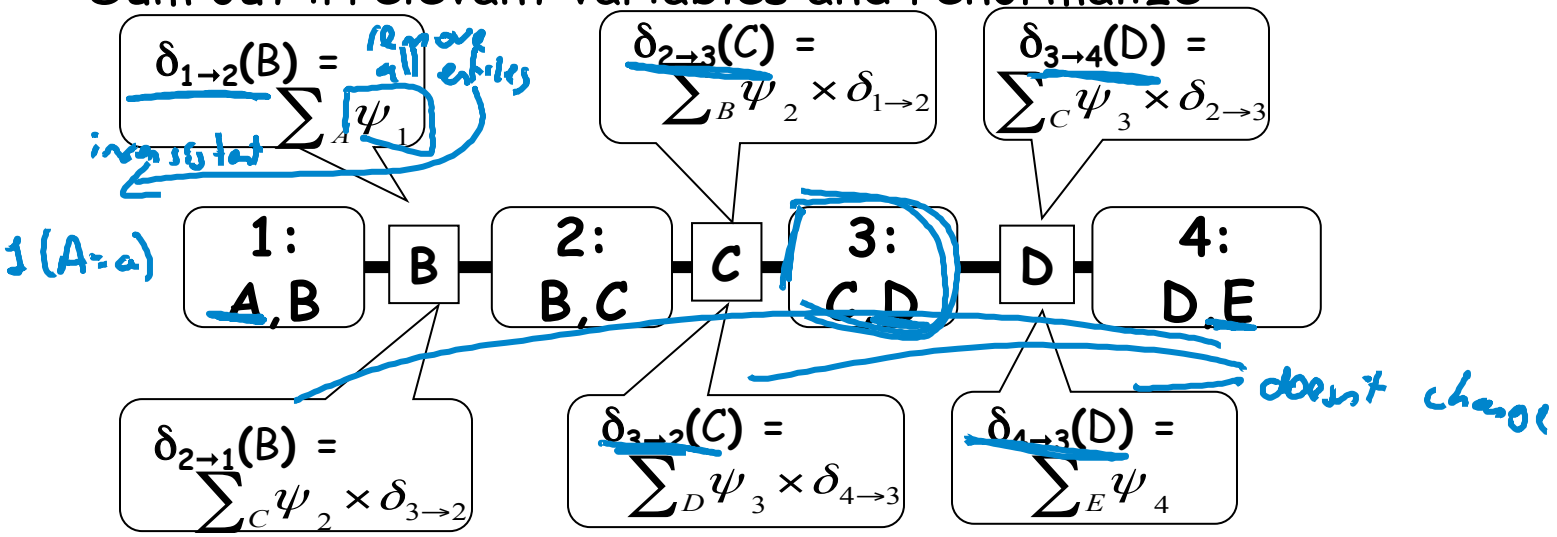


Answering Queries

- Posterior distribution queries on variables that appear together in clique
  - Sum out irrelevant variables from any clique containing those variables
- Introducing new evidence Z=z and querying X
  - If X appears in clique with Z
    - Multiply clique that contains X and Z with indicator function 1(Z=z)
    - Sum out irrelevant variables and renormalize

And More Queries

- Introducing new evidence Z=z and querying X if X does not share a clique with Z
  - Multiply **1**(Z=z) into some clique containing Z
  - Propagate messages along path to clique containing X
  - Sum out irrelevant variables and renormalize



Summary

- In clique tree with K cliques, if messages are passed starting at leaves, 2(K-1) messages suffice to compute all beliefs
- Can compute marginals over all variables at only twice the cost of variable elimination
- By storing messages, inference can be reused in incremental queries