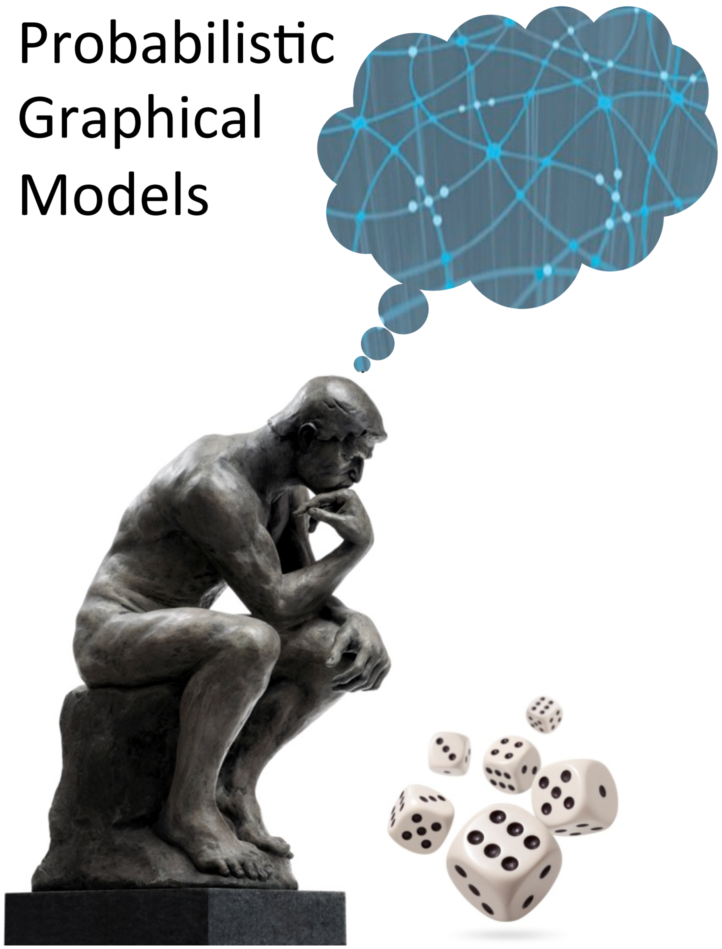
Probabilistic
Graphical
Models

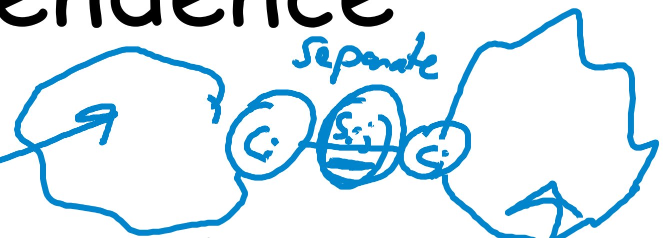


Inference

Message Passing

Clique Tree & Independence

RIP and Independence



- For an edge (i,j) in T, let:
  - $\mathbf{W}_{<(i,j)}$ = all variables that appear only on C_i side of T
  - $\mathbf{W}_{<(j,i)}$ = all variables that appear only on C_j side
  - Variables on both sides are in the sepset $S_{i,j}$

- Theorem: T satisfies RIP if and only if, for every (i,j) $P_\Phi \models (\boldsymbol{W}_{<(i,j)} \perp \boldsymbol{W}_{<(j,i)} \mid \boldsymbol{S}_{i,j})$

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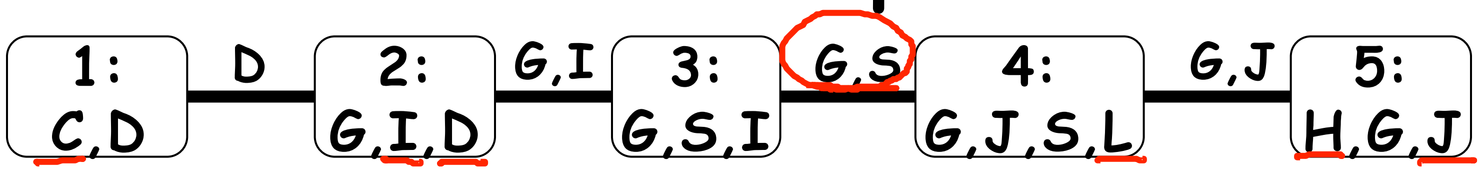
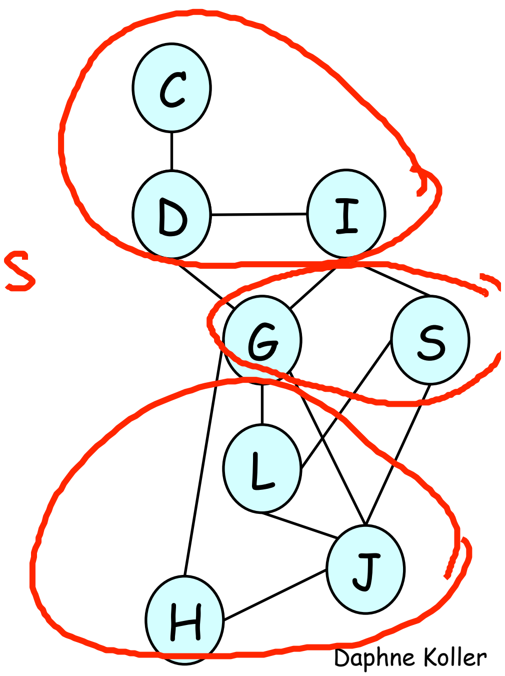
RIP and Independence



$$P_\Phi \models (\{C, I, D\} \perp \{J, L, H\} \mid \{G, S\})$$

C, I, D separated from H, L, J given G, S



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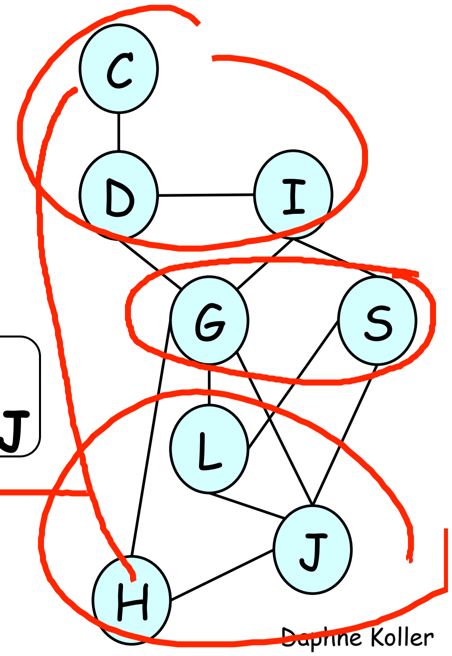
RIP and Independence

- Theorem: T satisfies RIP if and only if, for every edge (i,j) $P_\Phi \models (\boldsymbol{W}_{<(i,j)} \perp \boldsymbol{W}_{<(j,i)} \mid \boldsymbol{S}_{i,j})$

Assume otherwise ⇒ ∃ path in induced Markov network between $W_{<(i,j)}$ $W_{<(j,i)}$ that doesn't go through $S_{i,j}$

Factor $\phi(C, H)$



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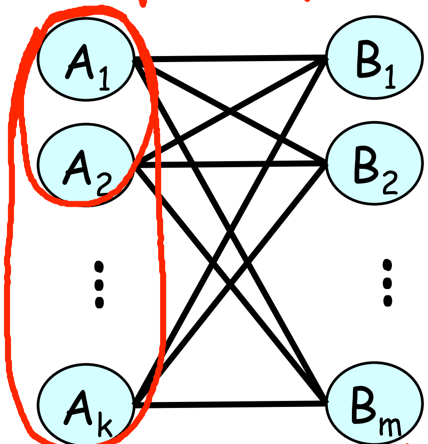
Implications

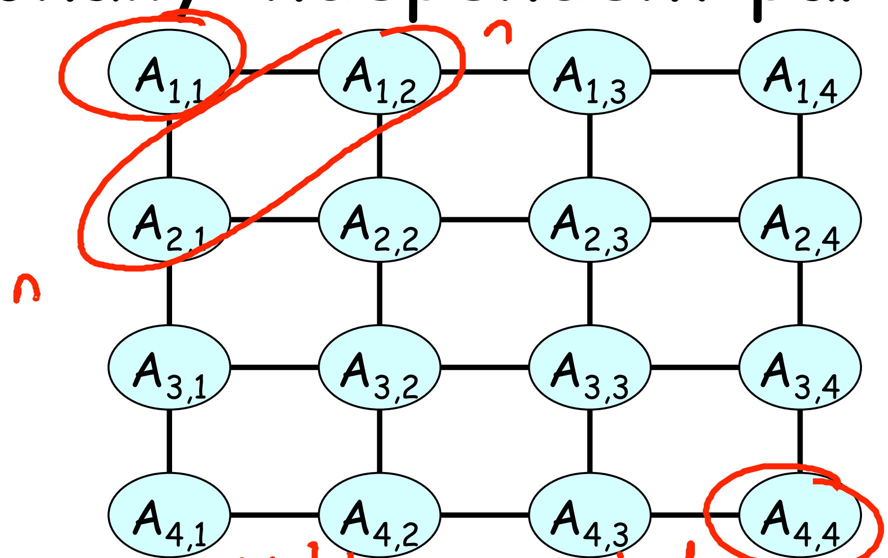
- Each sepset needs to separate graph into two conditionally independent parts

complete bipartite graph



Difficulty Intelligence

smallest sepset size $\geq \min(k, m)$



must have a sepset of size $\geq n$

Summary

- Correctness of clique tree inference relies on running intersection property
- Running intersection property implies separation in original distribution
- Implies minimal complexity incurred by any clique tree: sepsets
  - cliques
  - Related to minimal induced width of graph