

Probabilistic  
Graphical  
Models



Inference

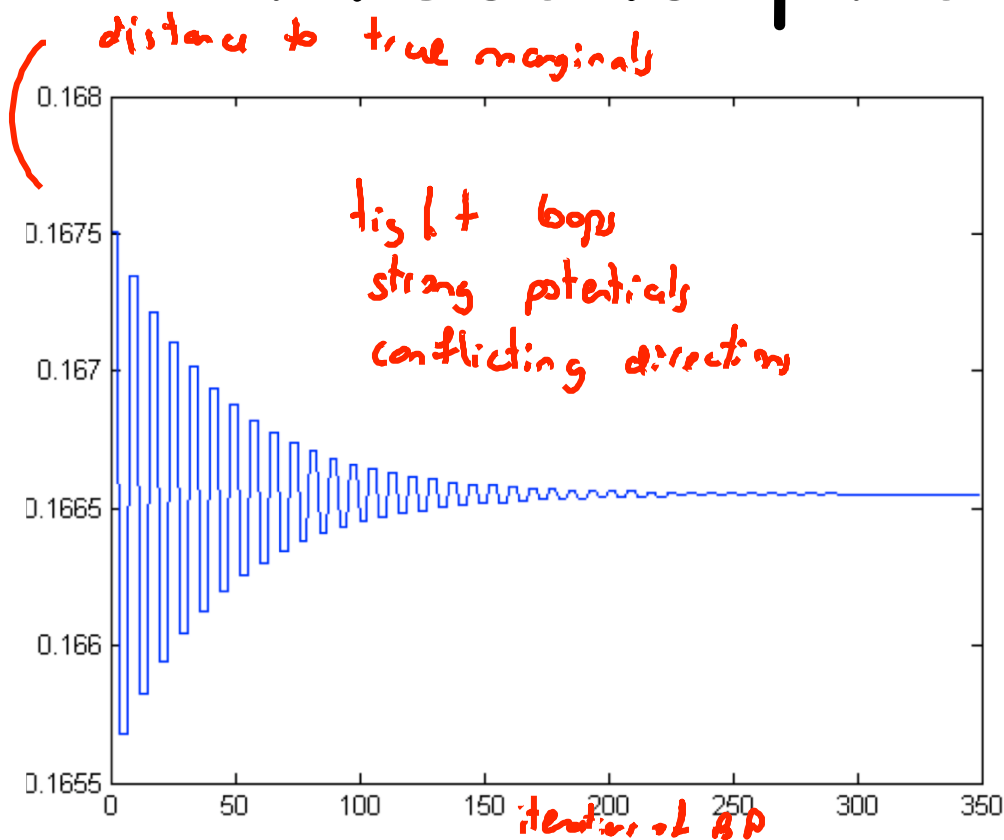
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Message Passing

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# BP In Practice

# Misconception Revisited

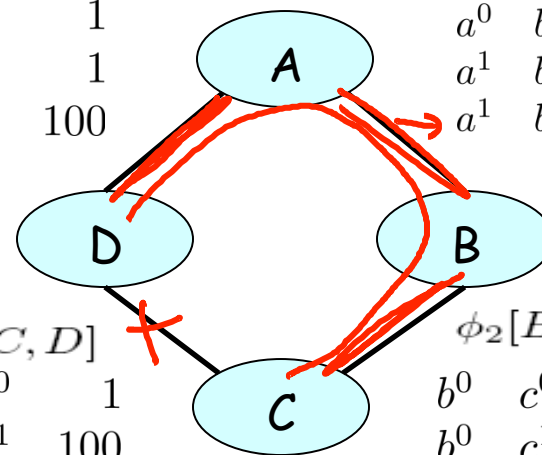


$$\phi_4[D, A]$$

|       |       |     |
|-------|-------|-----|
| $d^0$ | $a^0$ | 100 |
| $d^0$ | $a^1$ | 1   |
| $d^1$ | $a^0$ | 1   |
| $d^1$ | $a^1$ | 100 |

$$\phi_1[A, B]$$

|       |       |            |
|-------|-------|------------|
| $a^0$ | $b^0$ | <u>100</u> |
| $a^0$ | $b^1$ | 2          |
| $a^1$ | $b^0$ | 1          |
| $a^1$ | $b^1$ | <u>100</u> |



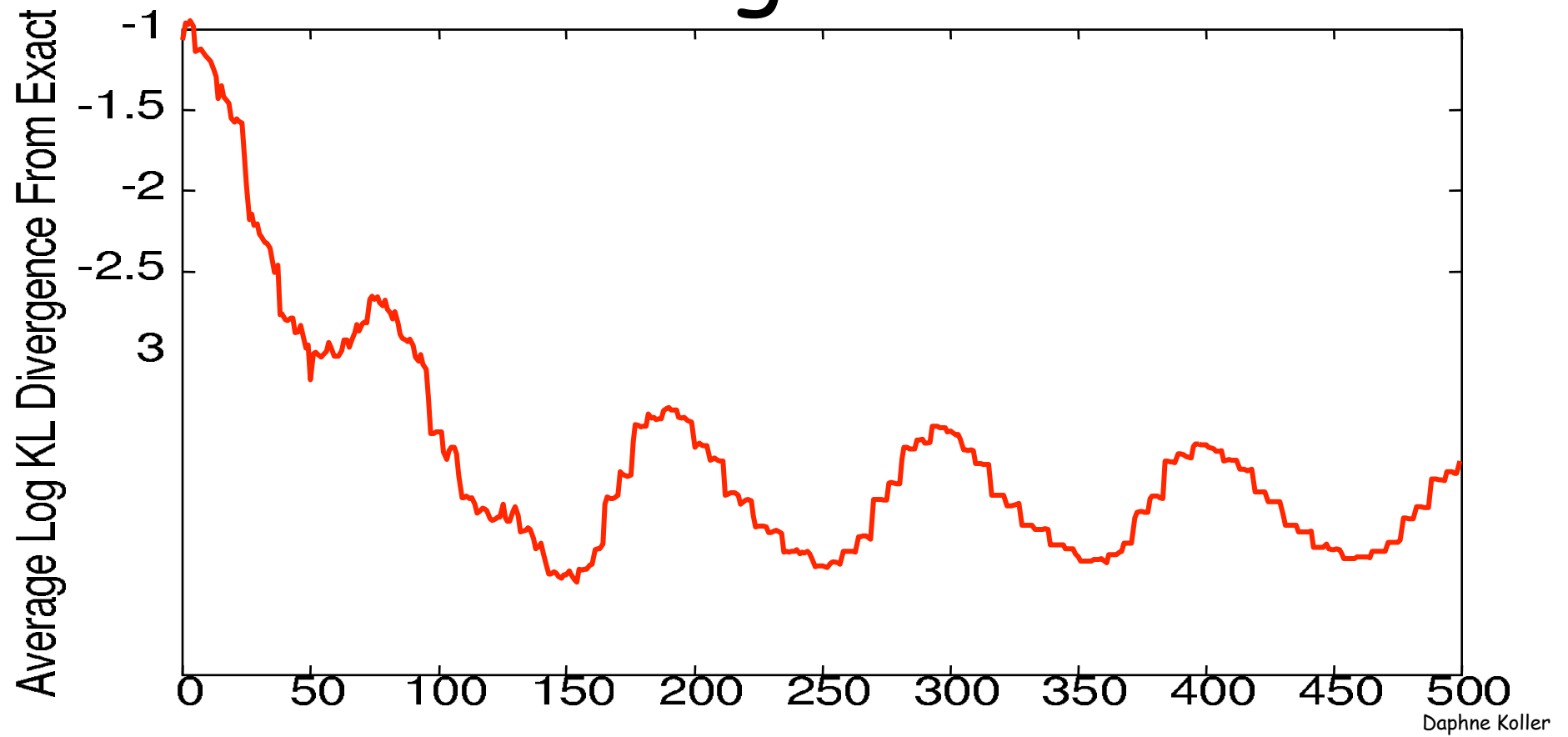
$$\phi_3[C, D]$$

|       |       |     |
|-------|-------|-----|
| $c^0$ | $d^0$ | 1   |
| $c^0$ | $d^1$ | 100 |
| $c^1$ | $d^0$ | 100 |
| $c^1$ | $d^1$ | 1   |

$$\phi_2[B, C]$$

|       |       |     |
|-------|-------|-----|
| $b^0$ | $c^0$ | 100 |
| $b^0$ | $c^1$ | 1   |
| $b^1$ | $c^0$ | 1   |
| $b^1$ | $c^1$ | 100 |

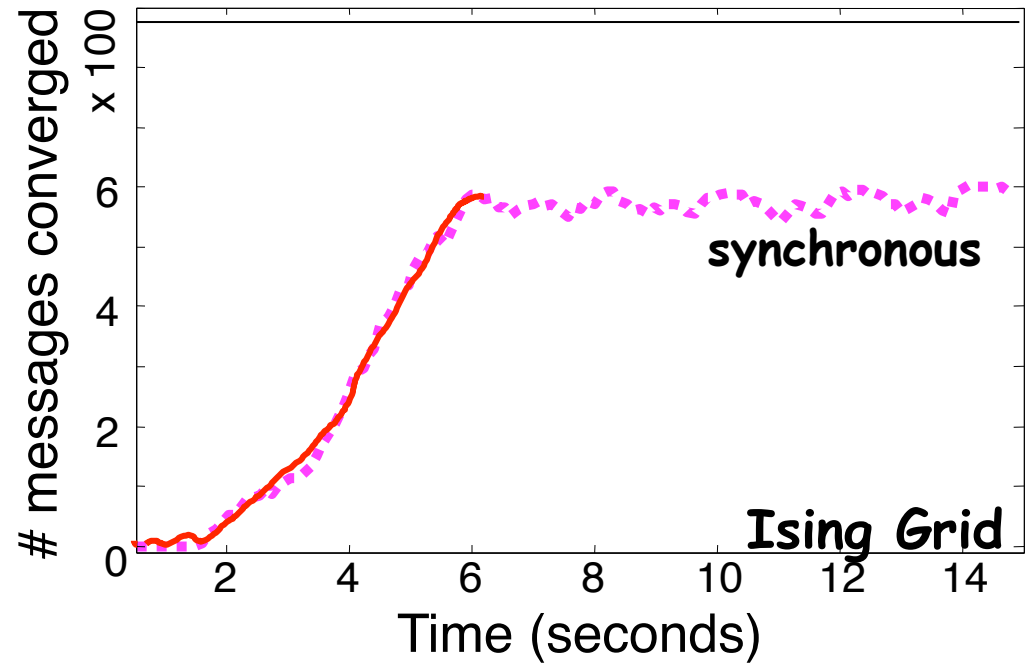
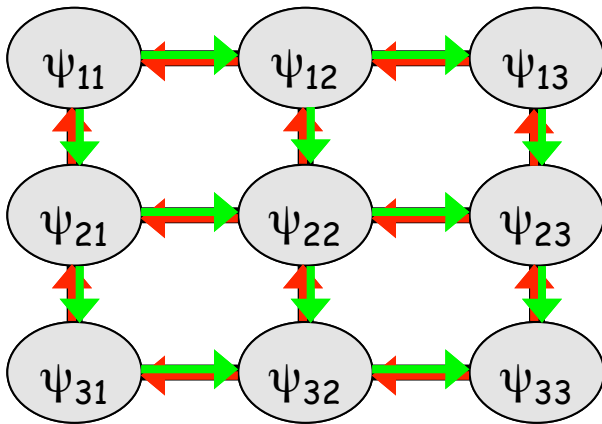
# Nonconvergent BP Run



# Different Variants of BP

## Synchronous BP:

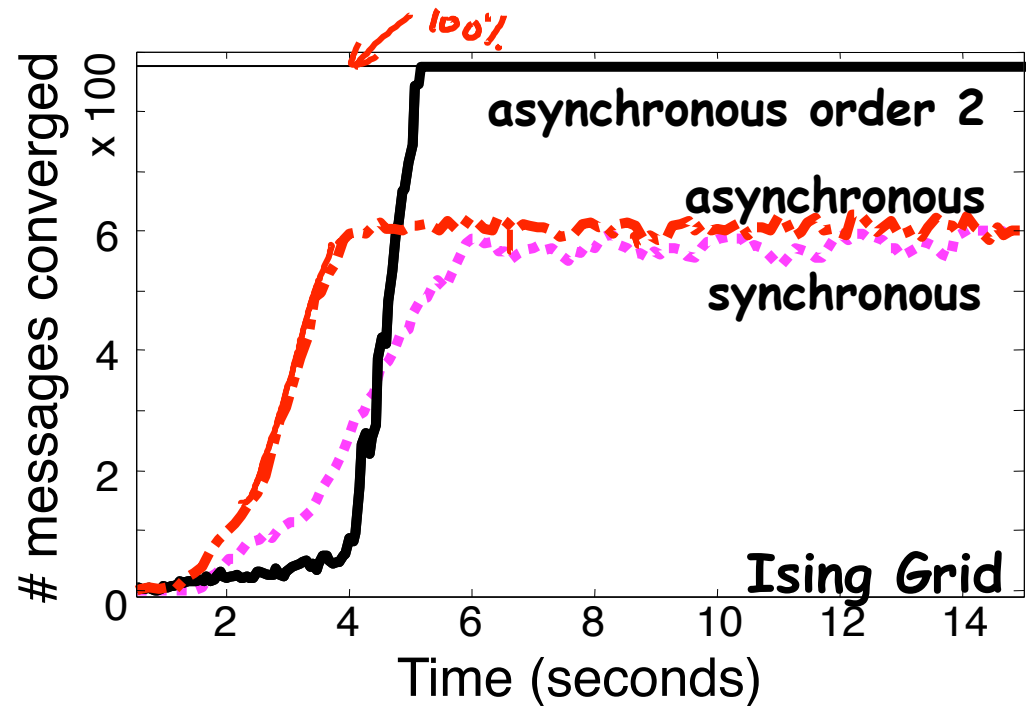
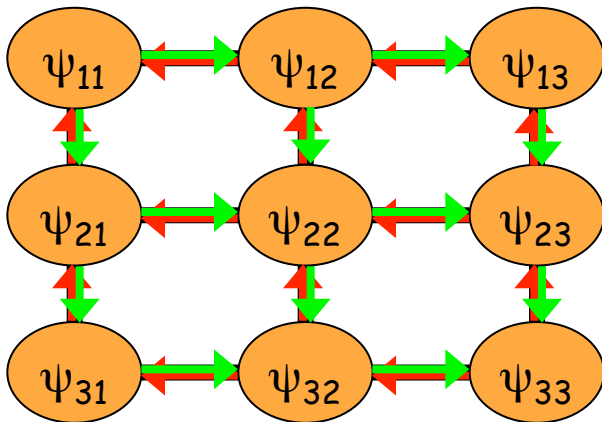
all messages are updated in parallel



# Different Variants of BP

## Asynchronous BP:

Messages are updated one at a time



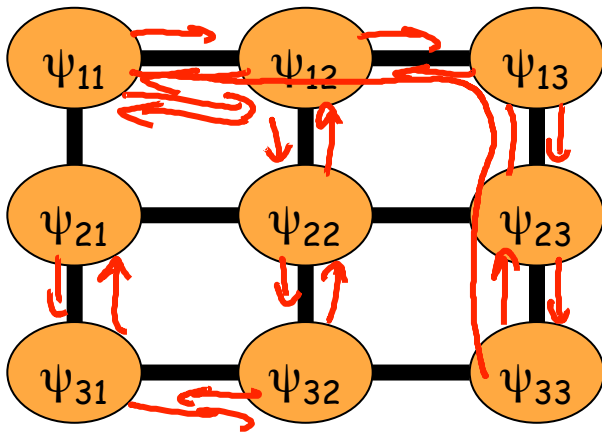
# Observations

- Convergence is a local property:
  - some messages converge soon
  - others may never converge
- Synchronous BP converges considerably worse than asynchronous
- Message passing order makes a difference to extent and rate of convergence

# Informed Message Scheduling

- Tree reparameterization (TRP)
  - Pick a tree and pass messages to calibrate

*trees must cover all edges  
improves performance if trees are larger (spanning trees)*



# Informed Message Scheduling

- Tree reparameterization (TRP)
  - Pick a tree and pass messages to calibrate
- Residual belief propagation (RBP)
  - Pass messages between two clusters whose beliefs over the sepset disagree the most  
priority over edges



# Smoothing (Damping) Messages

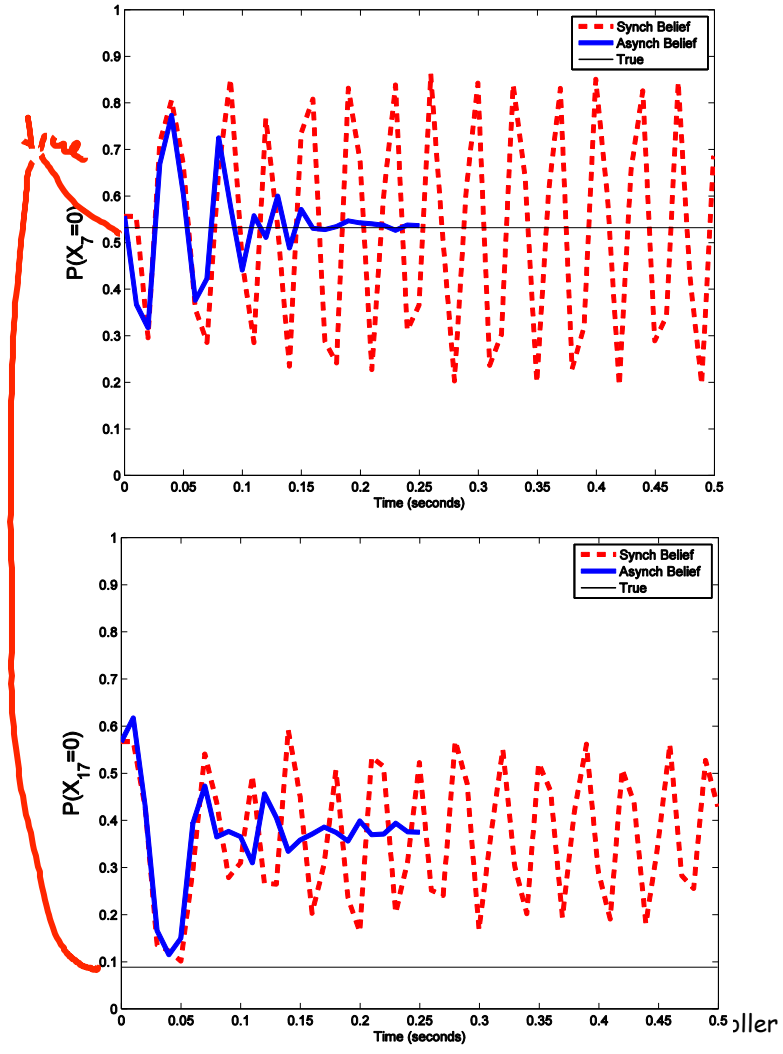
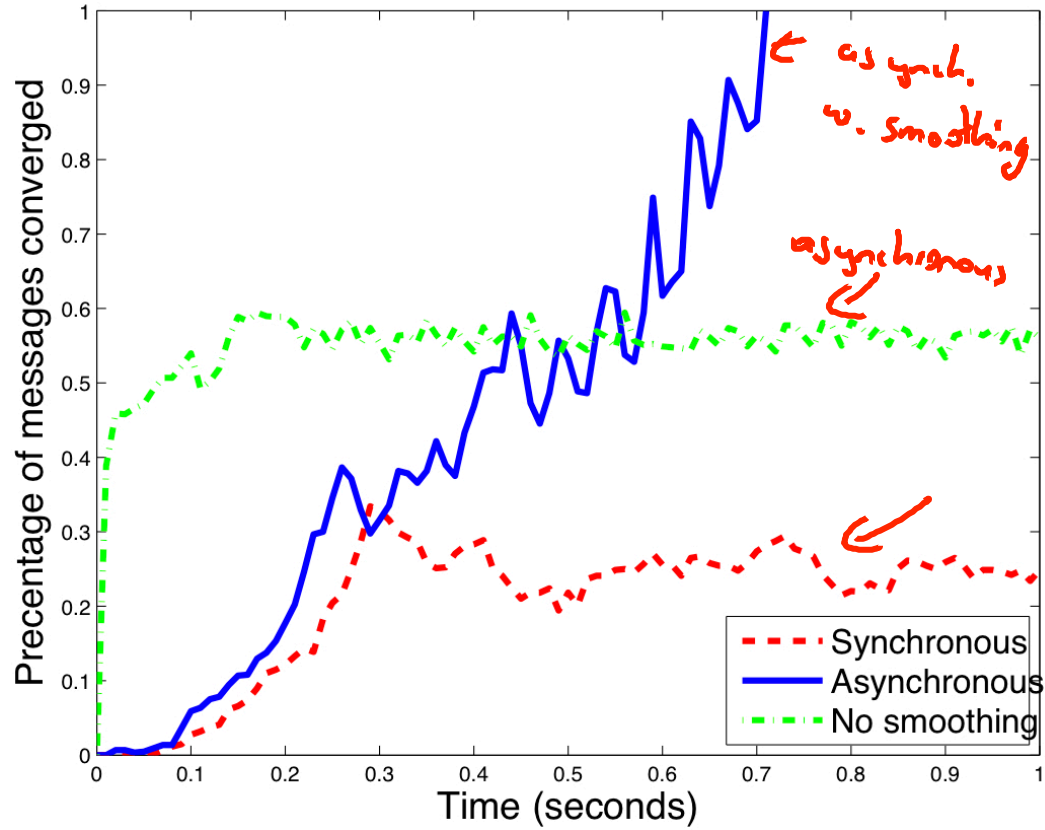
$$\delta_{i \rightarrow j} \leftarrow \sum_{C_i - S_{i,j}} \psi_i \prod_{k \neq j} \delta_{k \rightarrow i}$$
$$\delta_{i \rightarrow j} \leftarrow \lambda \left( \sum_{C_i - S_{i,j}} \psi_i \prod_{k \neq j} \delta_{k \rightarrow i} \right) + (1 - \lambda) \delta_{i \rightarrow j}^{\text{old}}$$

*new msg*

*old msg*

- Dampens oscillations in messages

Using grid



# Summary

- To achieve BP convergence, two main tricks
  - Damping
  - Intelligent message ordering
- Convergence doesn't guarantee correctness
- Bad cases for BP - both convergence & accuracy:
  - Strong potentials pulling in different directions
  - Tight loops
- Some new algorithms have better convergence:
  - Optimization-based view to inference