Case study: election fraud

INFERENCE FOR CATEGORICAL DATA IN R

Andrew Bray

Assistant Professor of Statistics at Reed College

Election fraud

- Vote buying
- Voting twice
- Altering vote totals

precinct	candidate A	candidate B
1	53	37
2	36	31
3	68	55
4	17	19
5	24	27

[1] The phrase election fraud can mean many things including vote buying, casting two ballots in different locations, and stuffing ballot boxes with fake ballots. We're going to focus on a version of the third, when the vote

Election fraud

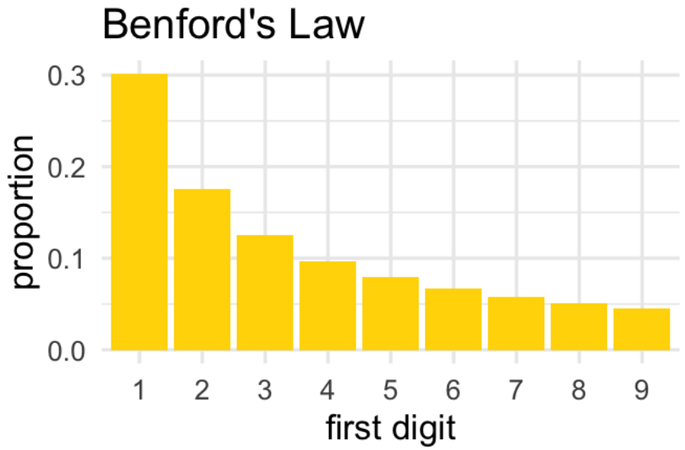
- Vote buying
- Voting twice
- Altering vote totals

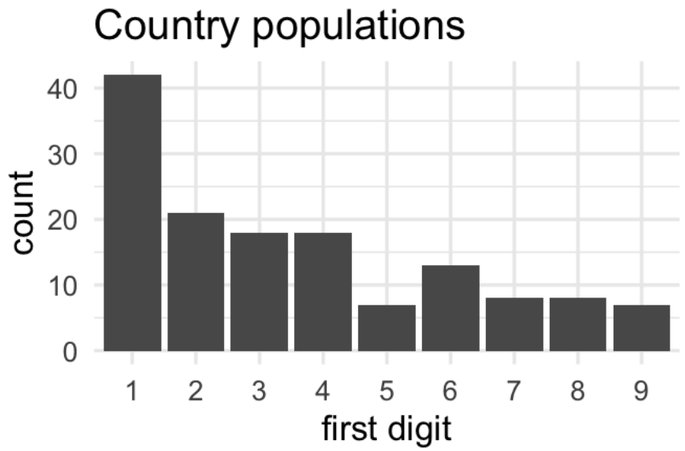
precinct	candidate A	candidate B
1	53	77 ~~37~~
2	36	31
3	68	85 ~~55~~
4	17	19
5	24	27

Benford's Law A.K.A. "the first digit law"

```
library(gapminder)
gapminder %>%
  filter(year == 2007) %>%
  select(country, pop)
```

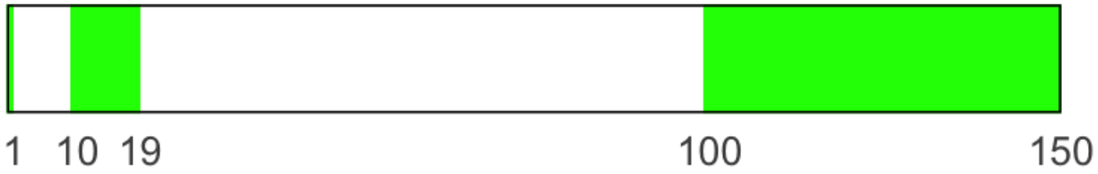
```
# A tibble: 142 x 2
   country           pop
   <fct>           <int>
 1 Afghanistan  31889923
 2 Albania       3600523
 3 Algeria      33333216
 4 Angola       12420476
 5 Argentina    40301927
 6 Australia    20434176
 7 Austria       8199783
 8 Bahrain        708573
 9 Bangladesh  150448339
10 Belgium      10392226
# … with 132 more rows
```

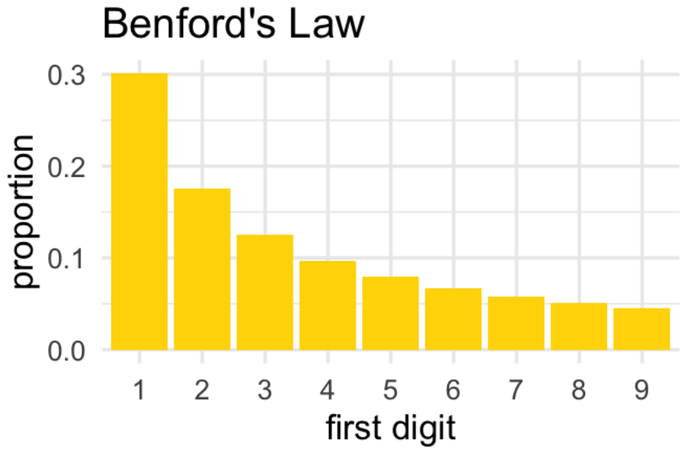




Benford's Law A.K.A. "the first digit law"

- If the election was fair then vote counts should follow Benford's Law.
- If the election was fraudulent then vote counts should not follow Benford's Law.





Iran election 2009

```
iran %>%
  select(city, ahmadinejad, mousavi, total_votes_cast)
```

```
# A tibble: 366 x 4
   city          ahmadinejad mousavi total_votes_cast
   <chr>               <dbl>   <dbl>            <dbl>
 1 Azar Shahr          37203   18312            56712
 2 Asko                32510   18799            52643
 3 Ahar                47938   26220            75500
 4 Bostan Abad         38610   12603            51911
 5 Bonab               36395   33695            71389
 6 Tabriz             435728  419983           876919
 7 Jalfa               20520   14340            35295
 8 Chahar o Imaq       12197    3975            16375
 9 Sarab               53196   17669            72152
10 Shabestar           37099   39182            77459
# … with 356 more rows
```

Ahmadinejad

Mousavi

Let's practice!

INFERENCE FOR CATEGORICAL DATA IN R

Goodness of fit

INFERENCE FOR CATEGORICAL DATA IN R

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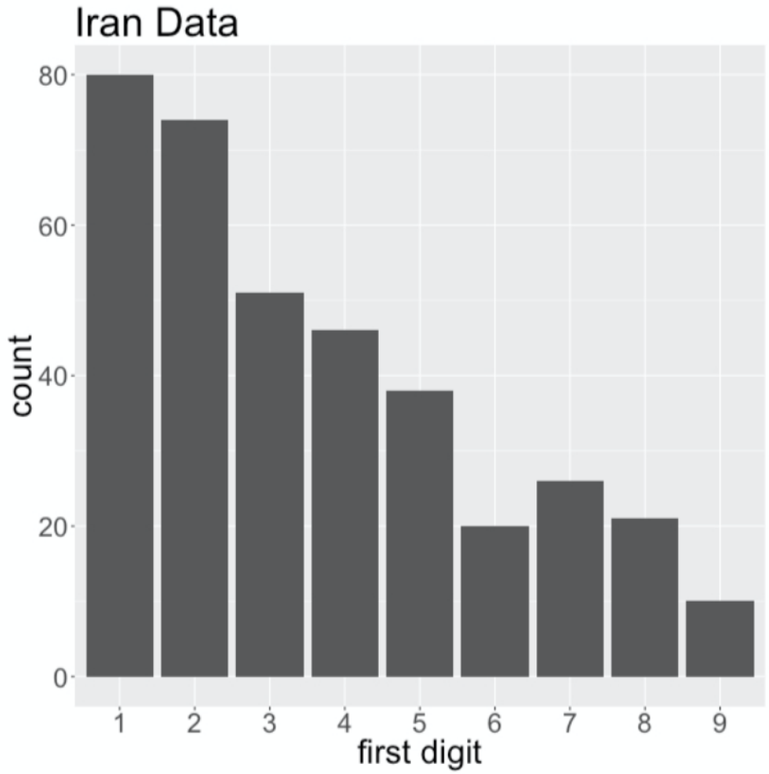
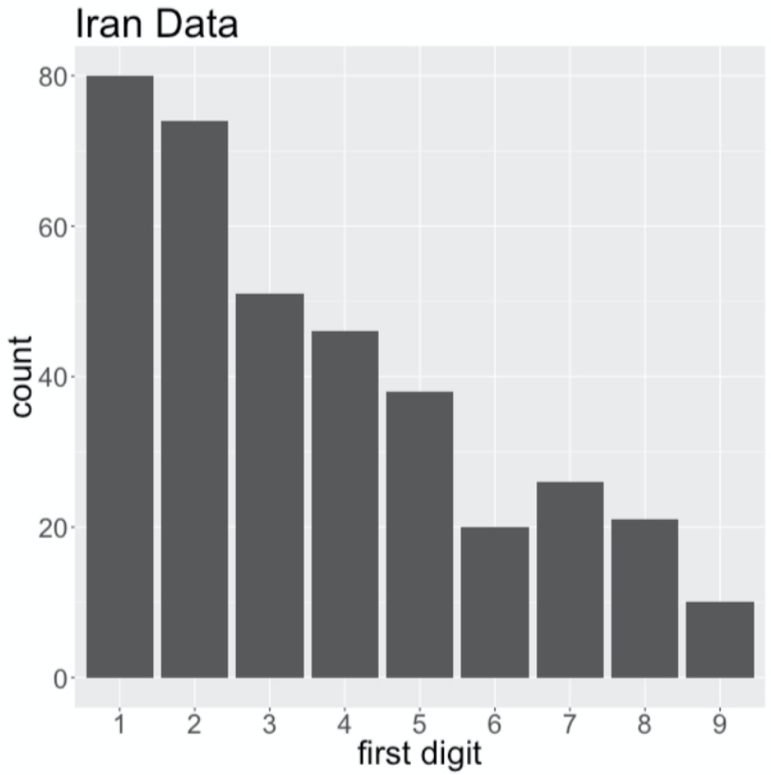
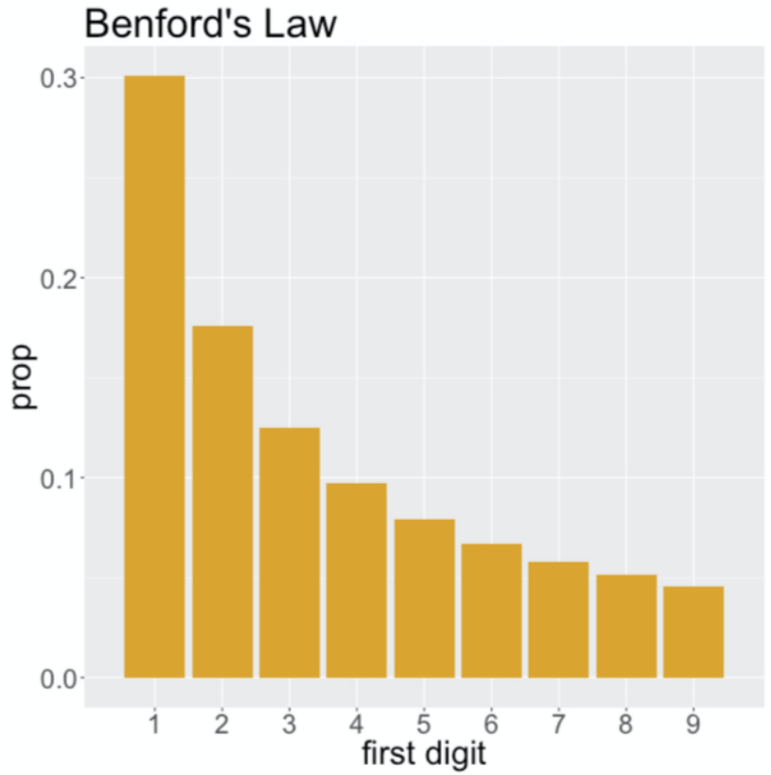
First Digit Distribution



First Digit Distribution





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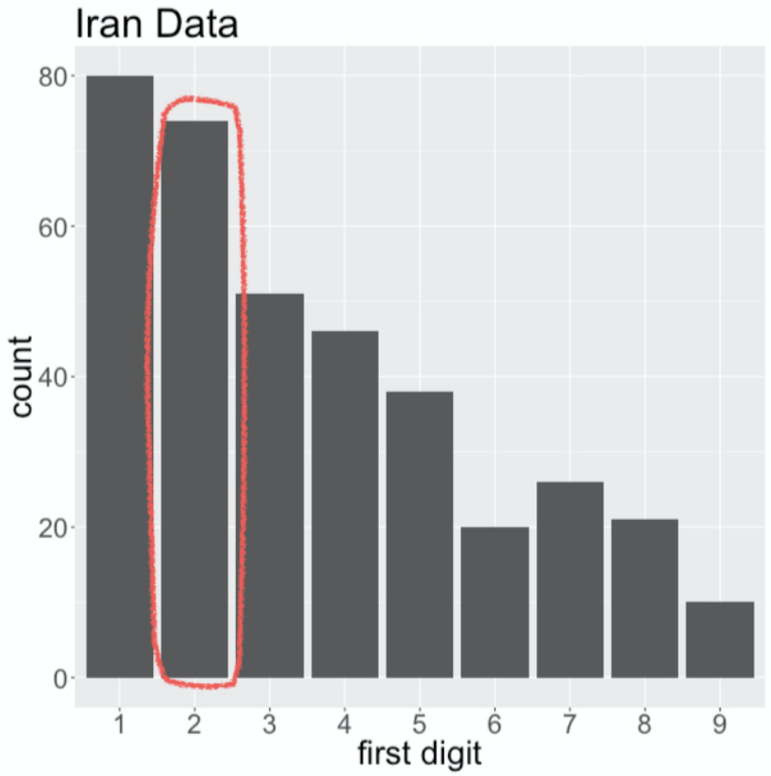
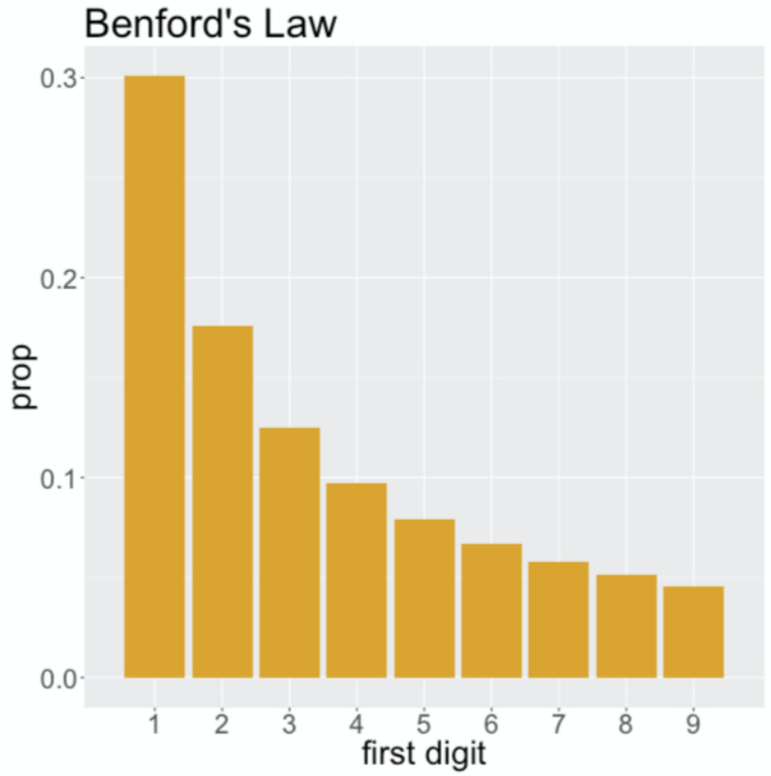
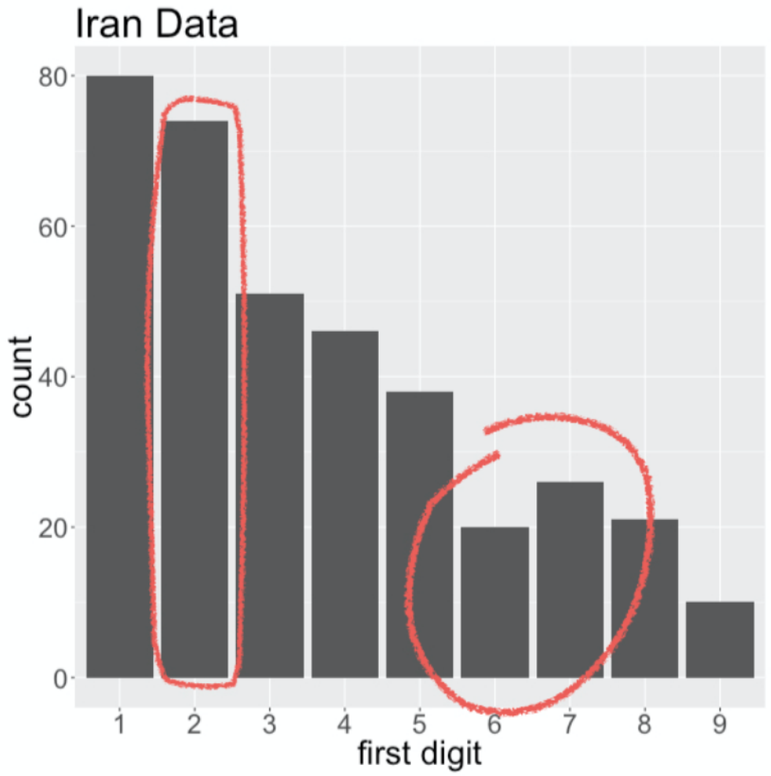
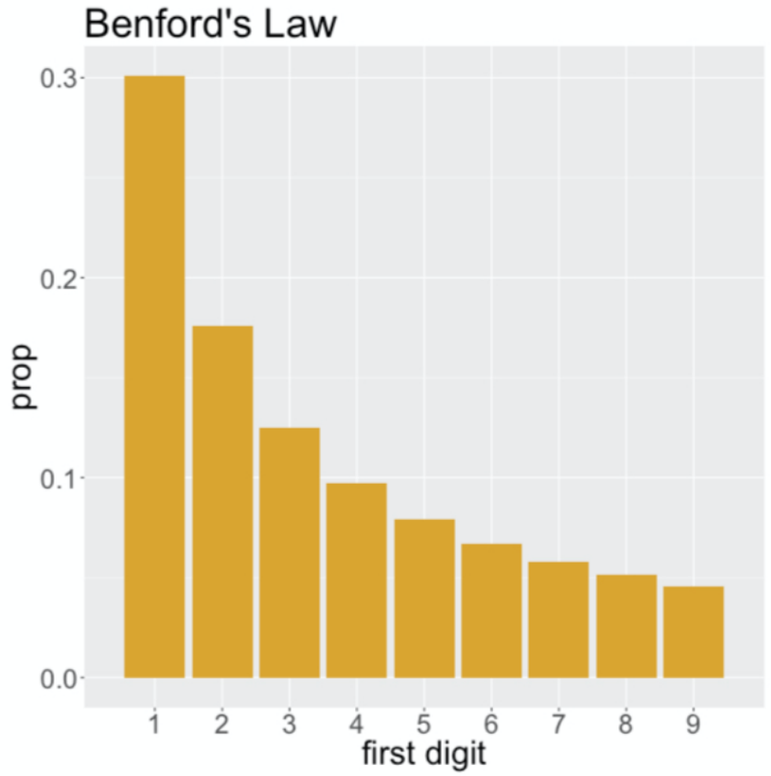




First Digit Distribution





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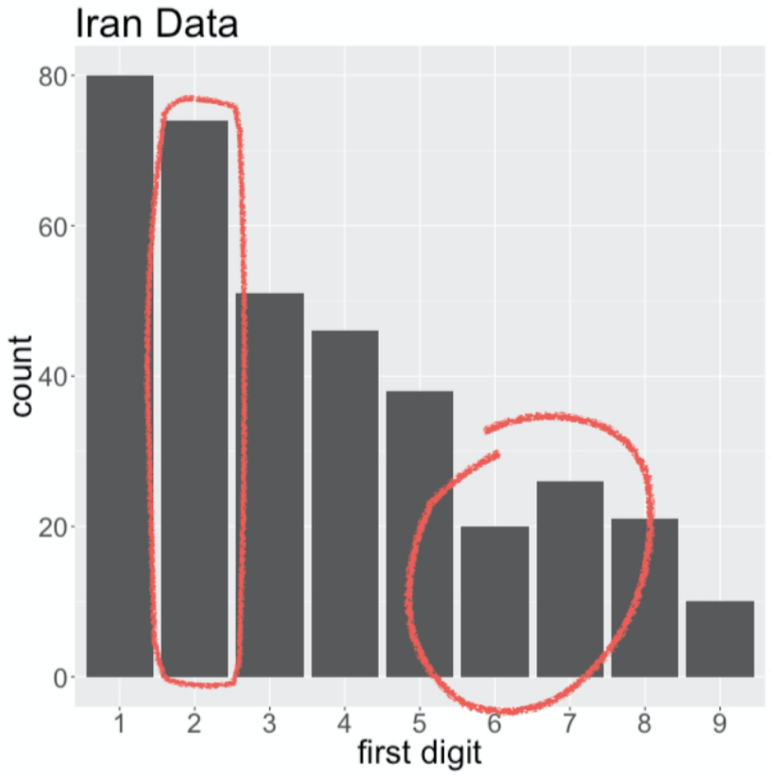
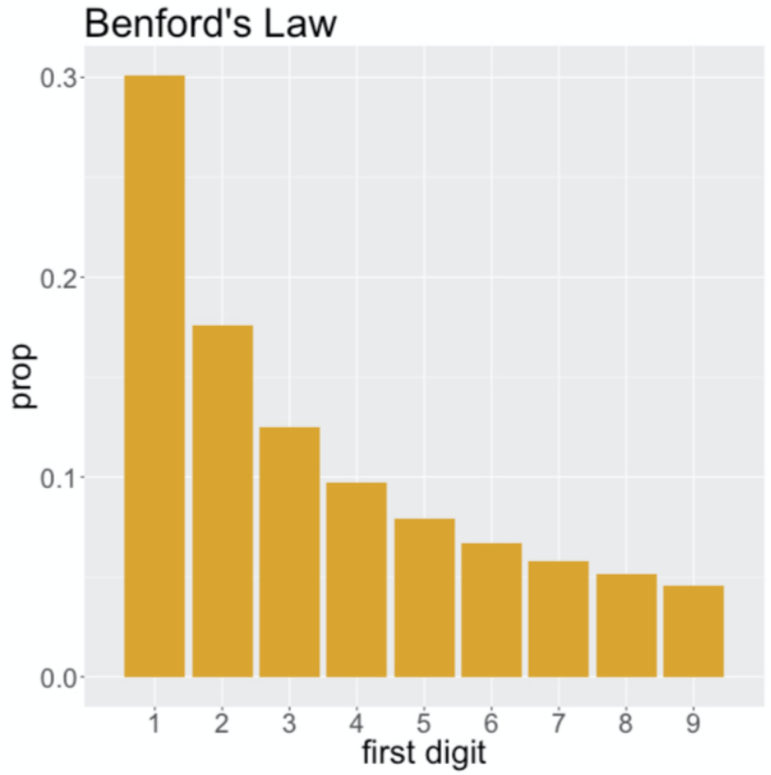
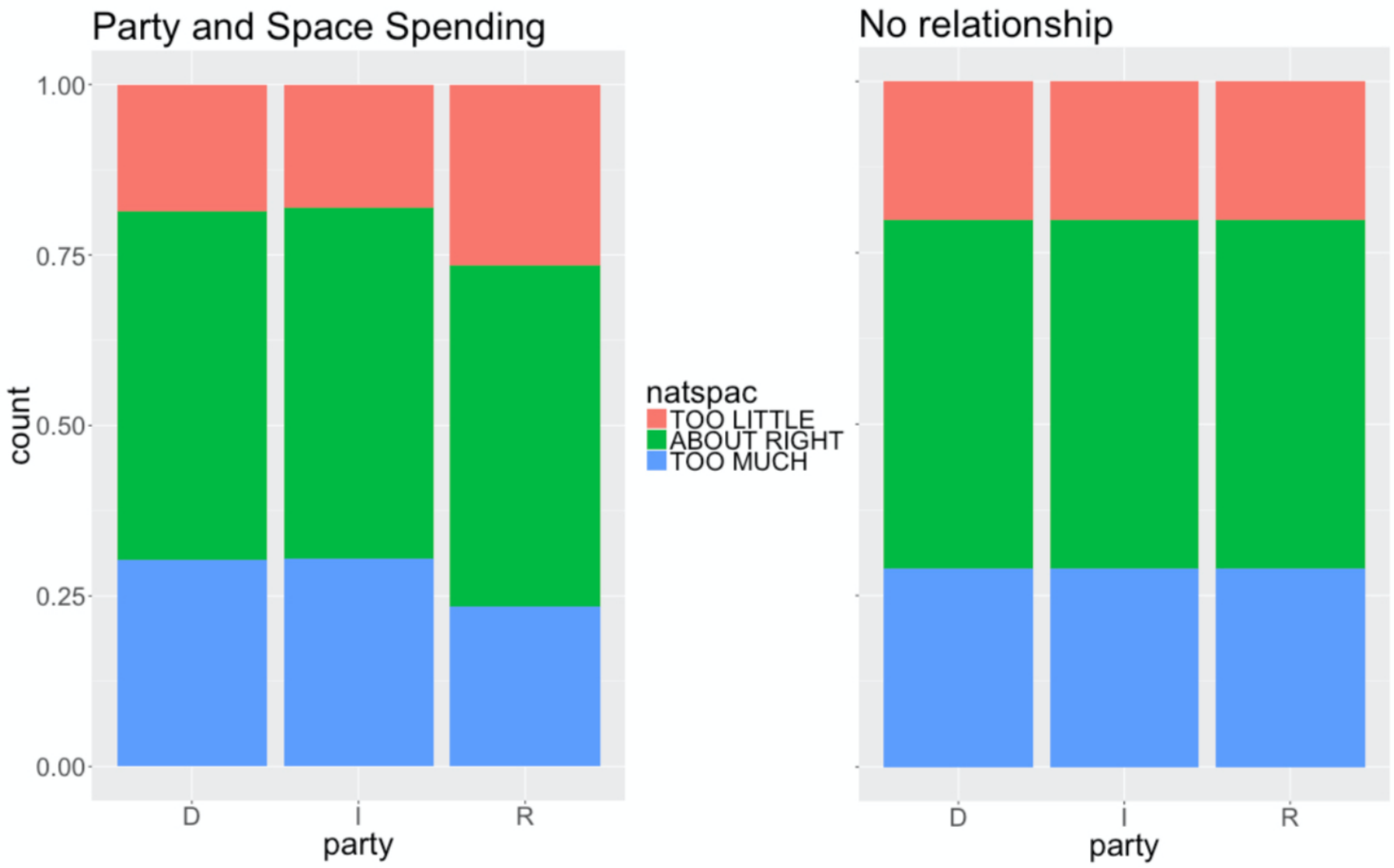


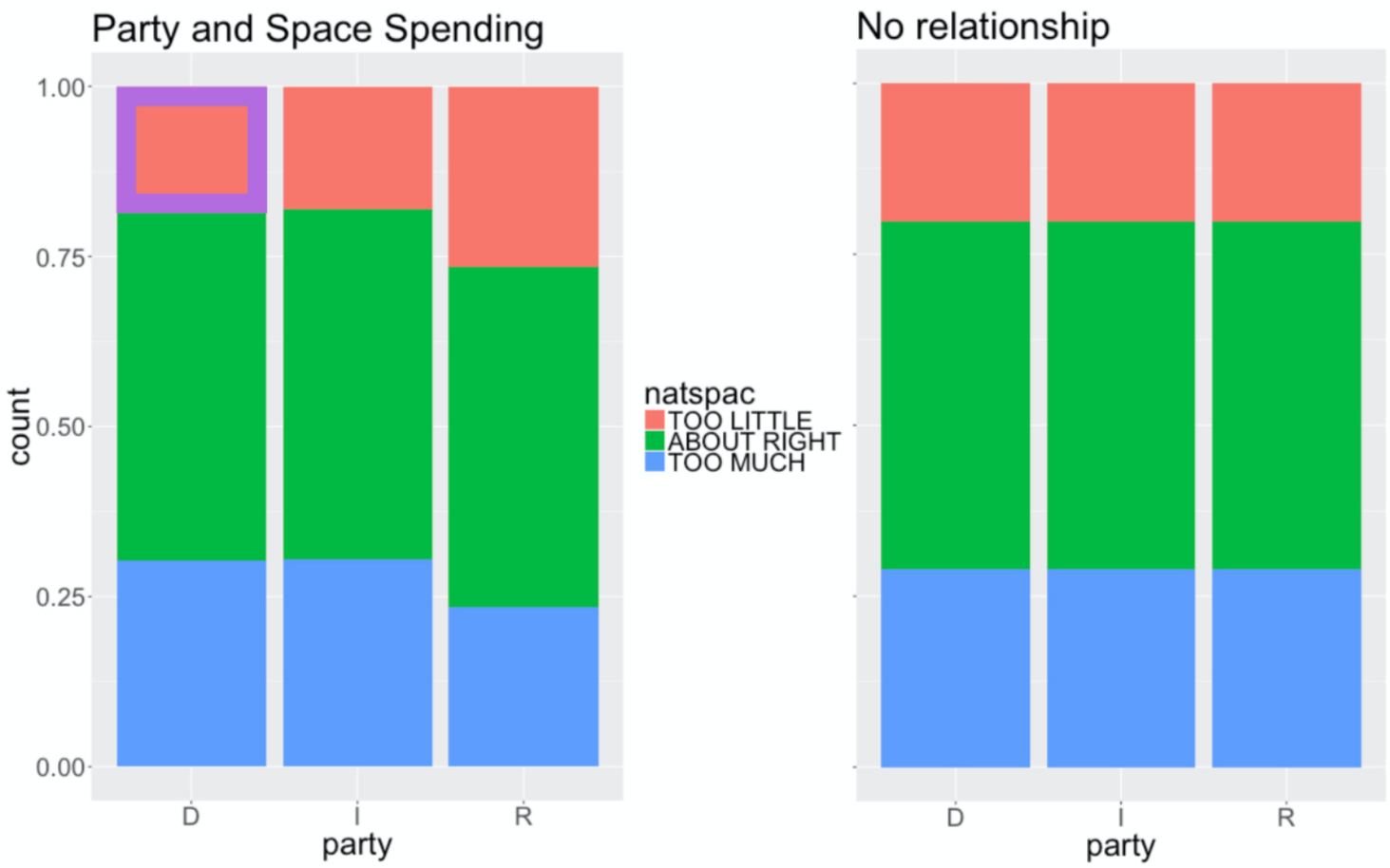


How far apart are these distributions?

Chi-squared distance

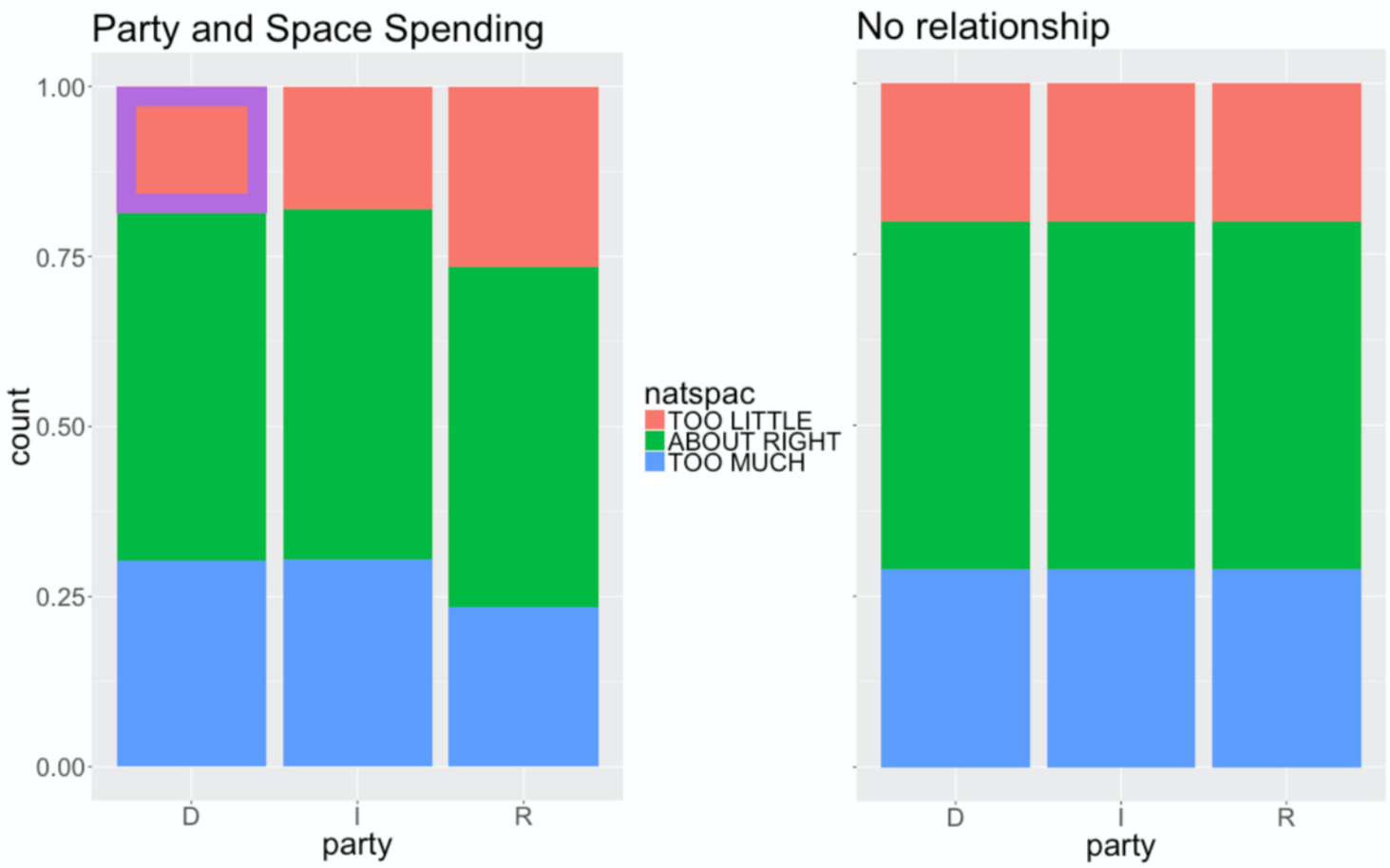


Chi-squared distance



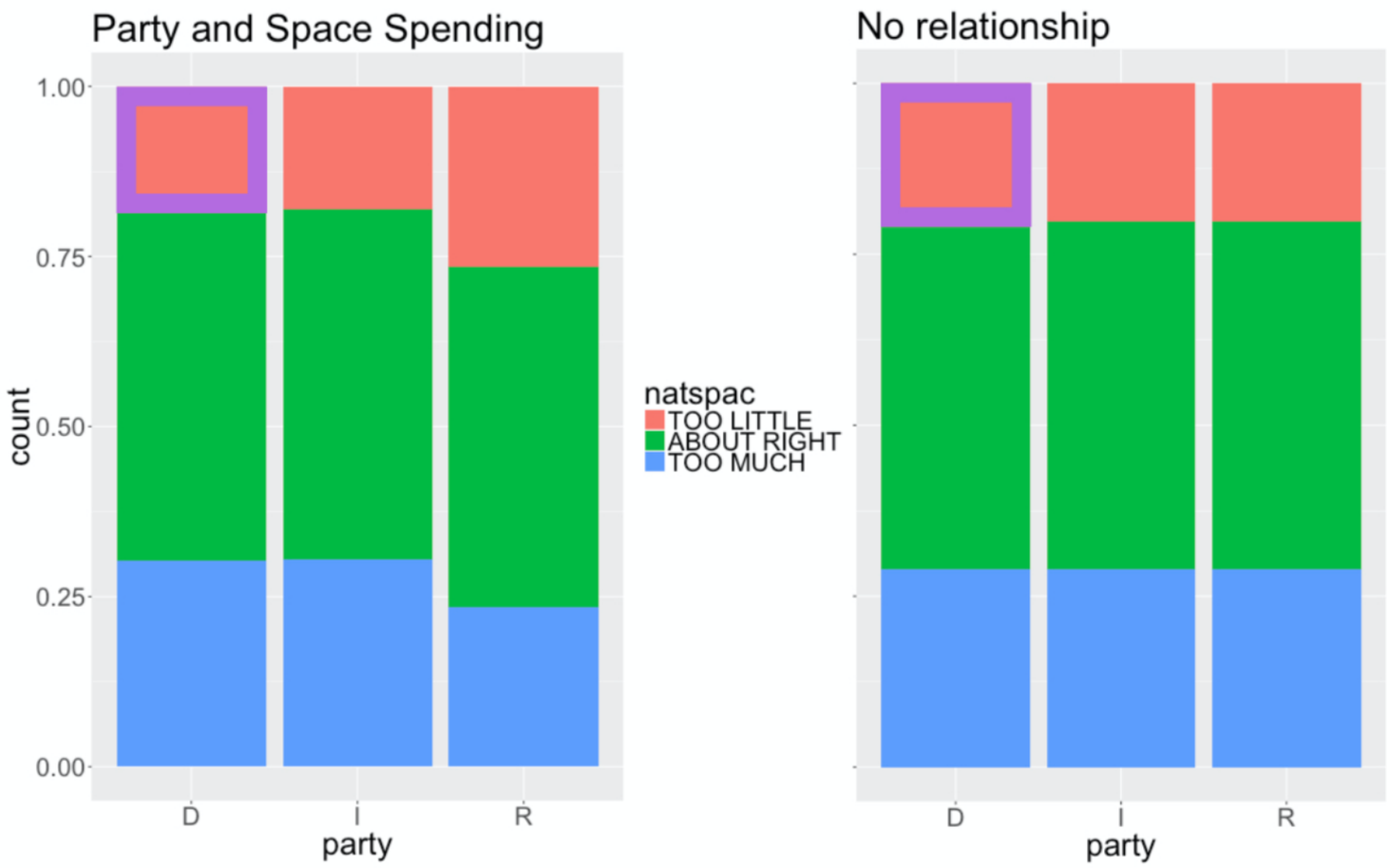
Cell 1:

Chi-squared distance



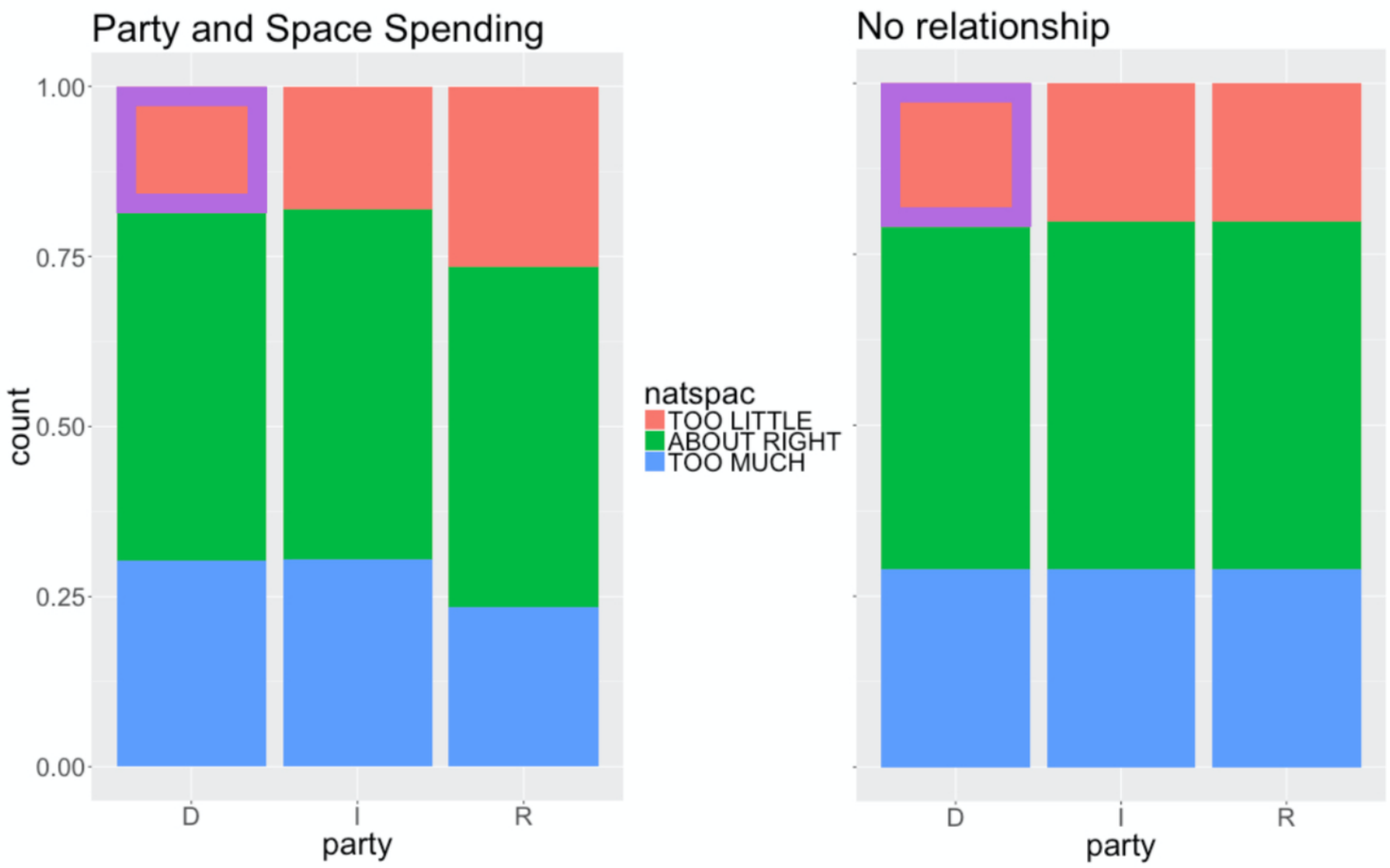
Cell 1: O

Chi-squared distance



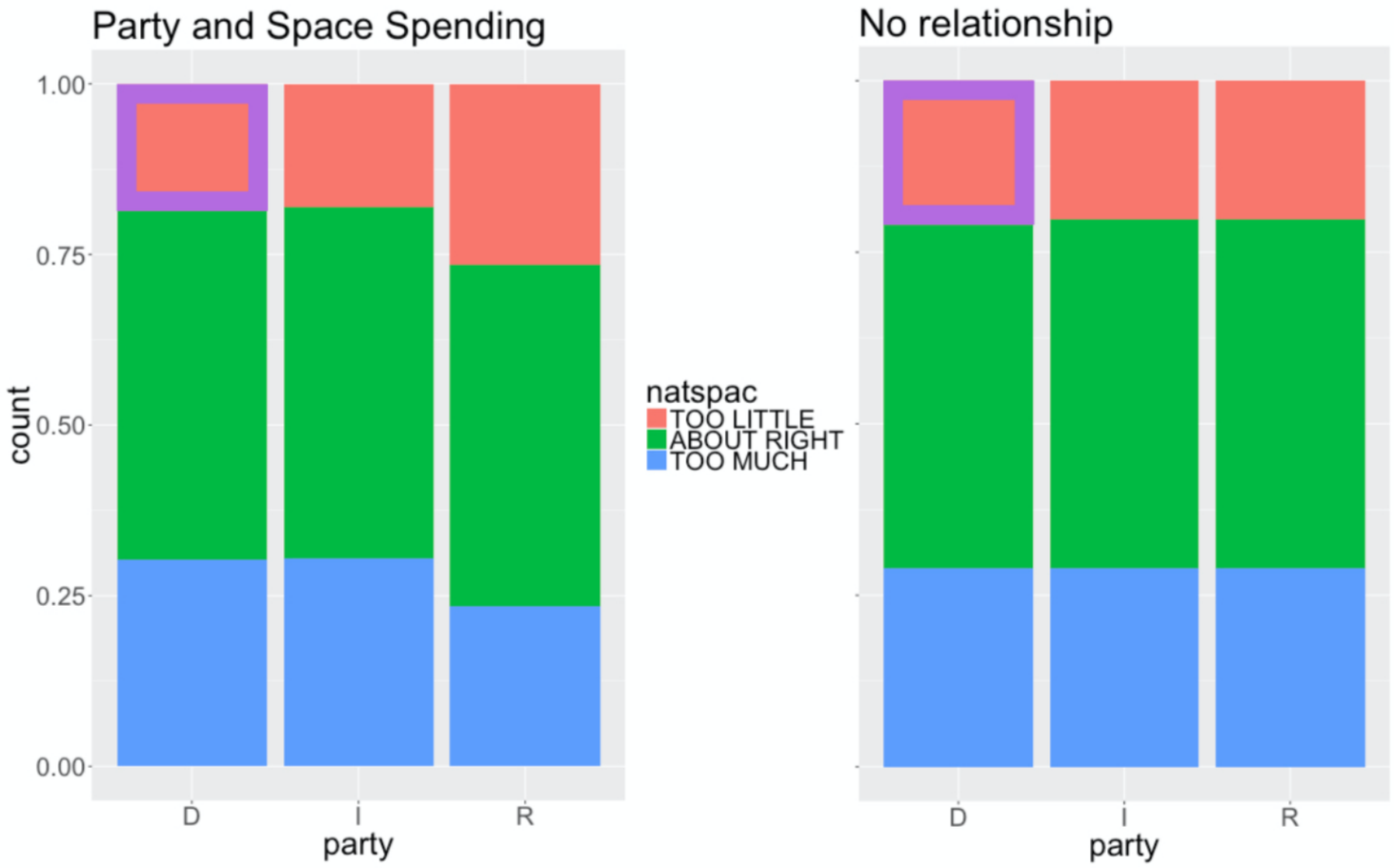
Cell 1: O - E

Chi-squared distance



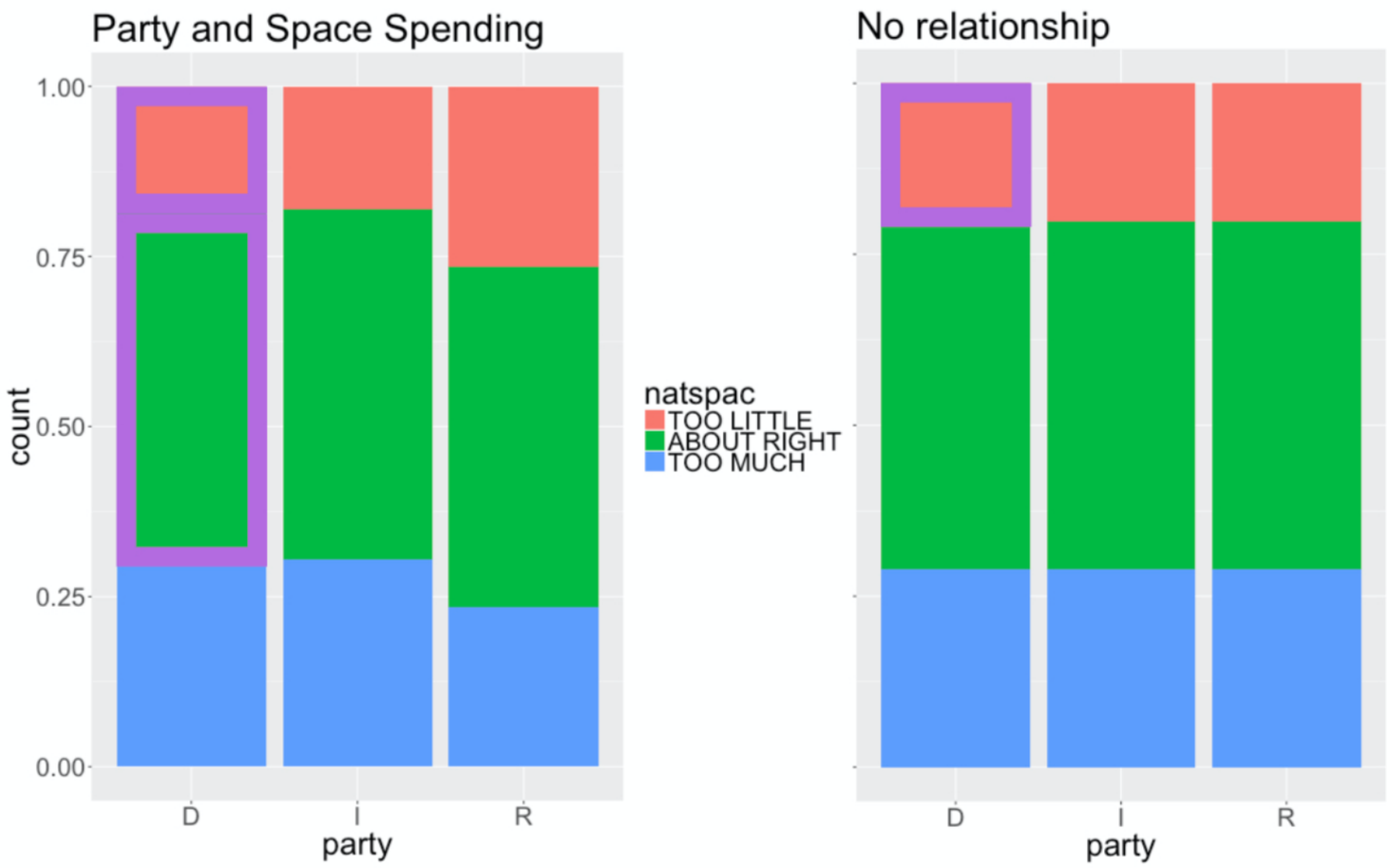
Cell 1: (O - E)^2

Chi-squared distance



Cell 1: $\frac{(O - E)^2}{E}$

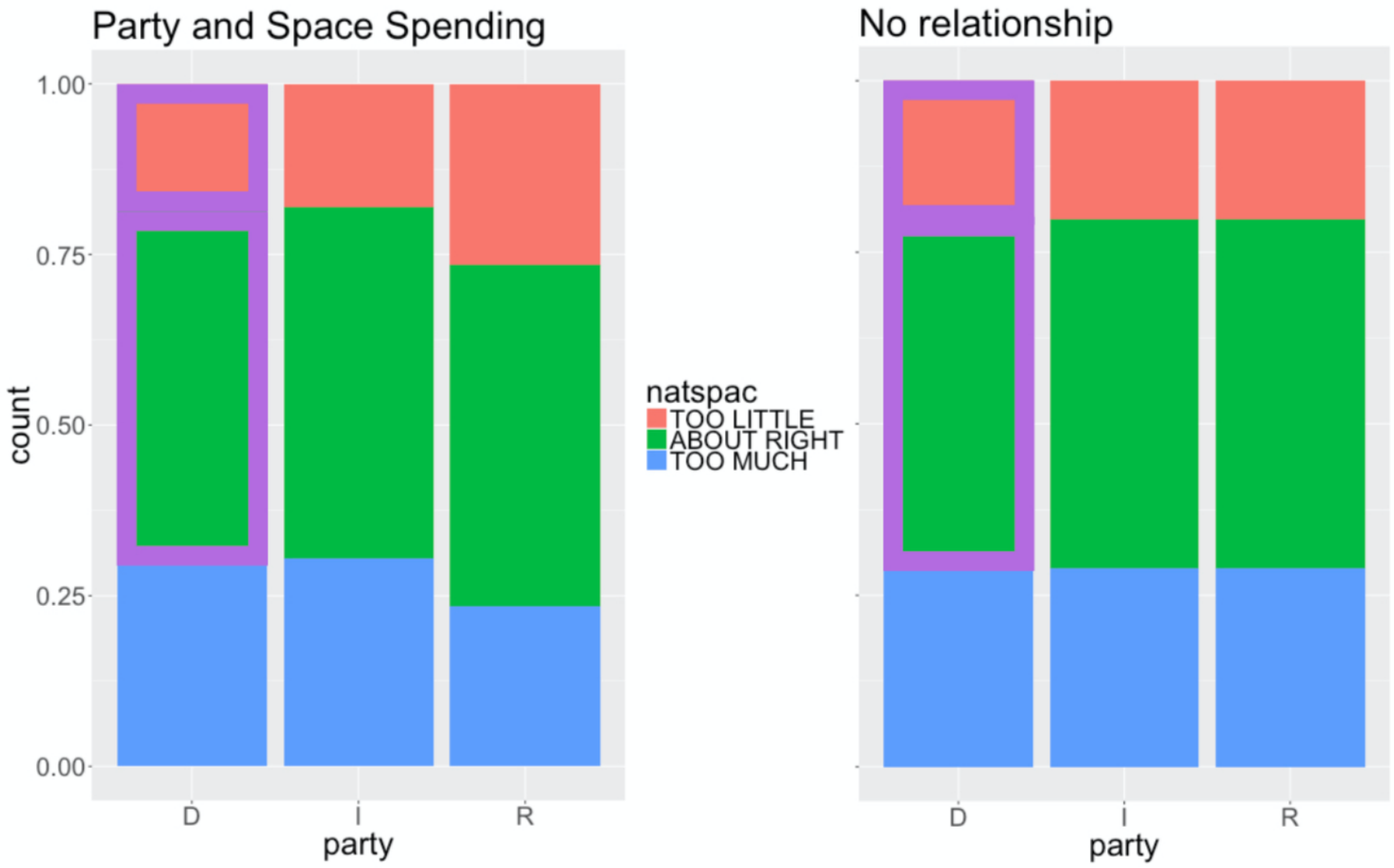
Chi-squared distance



Cell 1: $\frac{(O - E)^2}{E}$

Cell 2: O

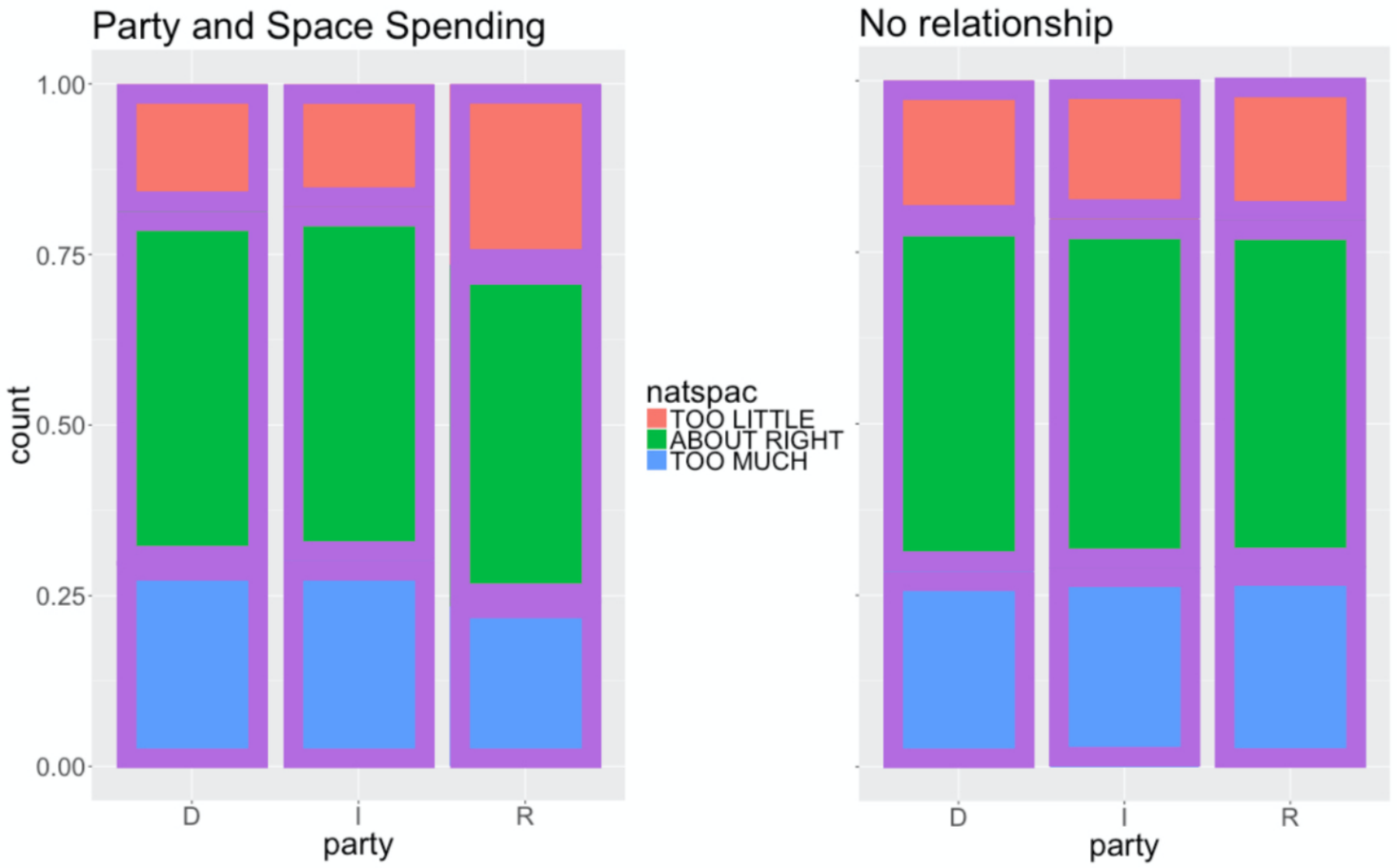
Chi-squared distance



Cell 1: $\frac{(O - E)^2}{E}$

Cell 2: $\frac{(O - E)^2}{E}$

Chi-squared distance



Cell 1: $\frac{(O - E)^2}{E}$

Cell 2: $\frac{(O - E)^2}{E}$

↓

Cell 9: $\frac{(O - E)^2}{E}$

$+$

Chi-squared statistic

Chi-squared distance

Cell 1: $\frac{(O - E)^2}{E}$

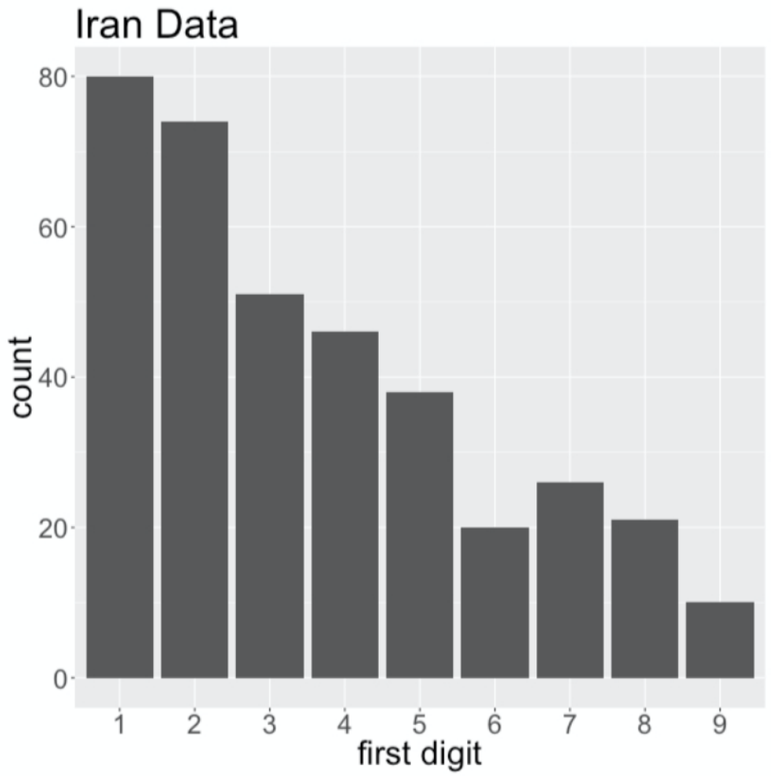
Cell 2: $\frac{(O - E)^2}{E}$

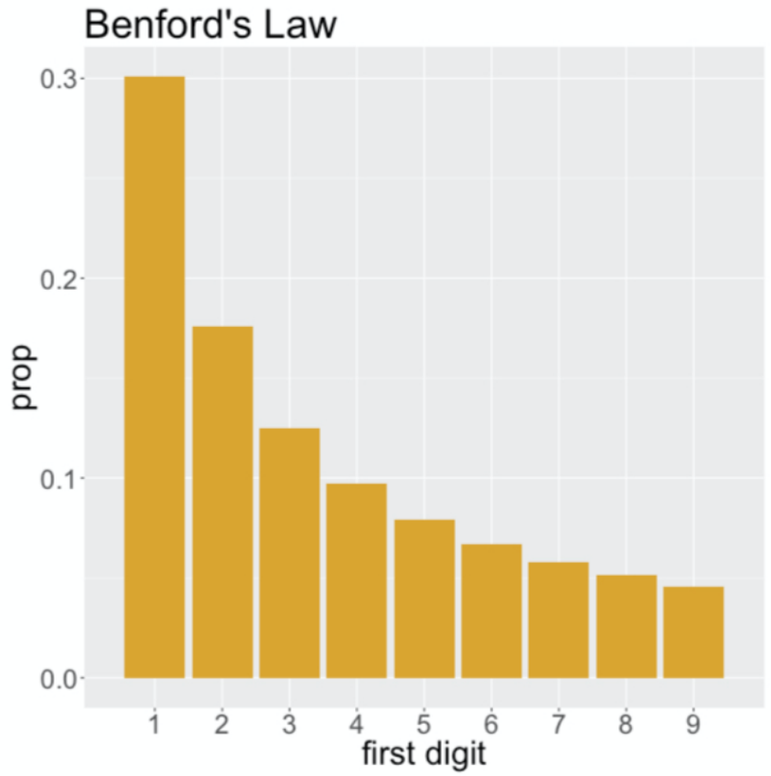
Cell 9: $\frac{(O - E)^2}{E}$

$+$

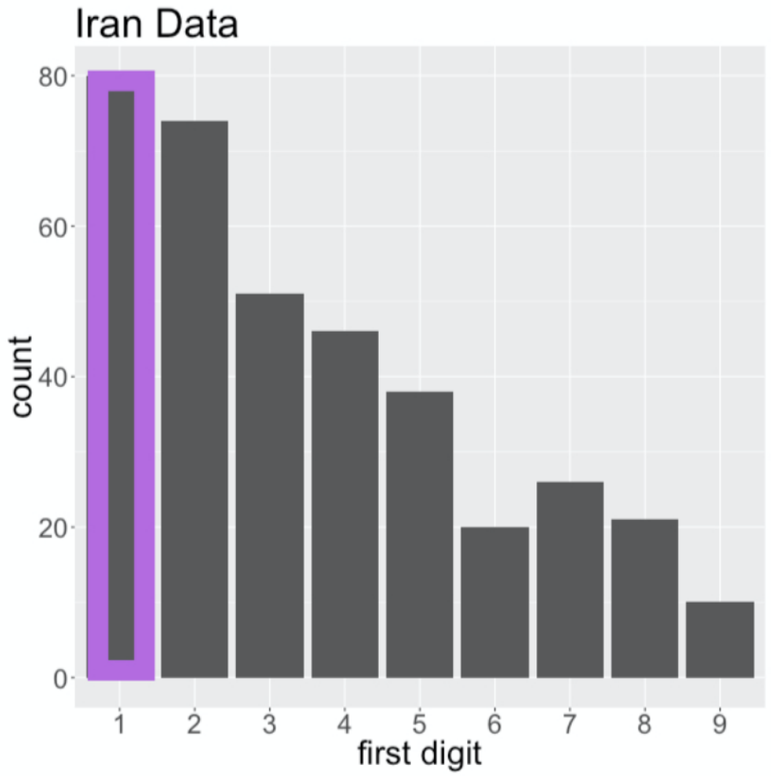
Chi-squared statistic

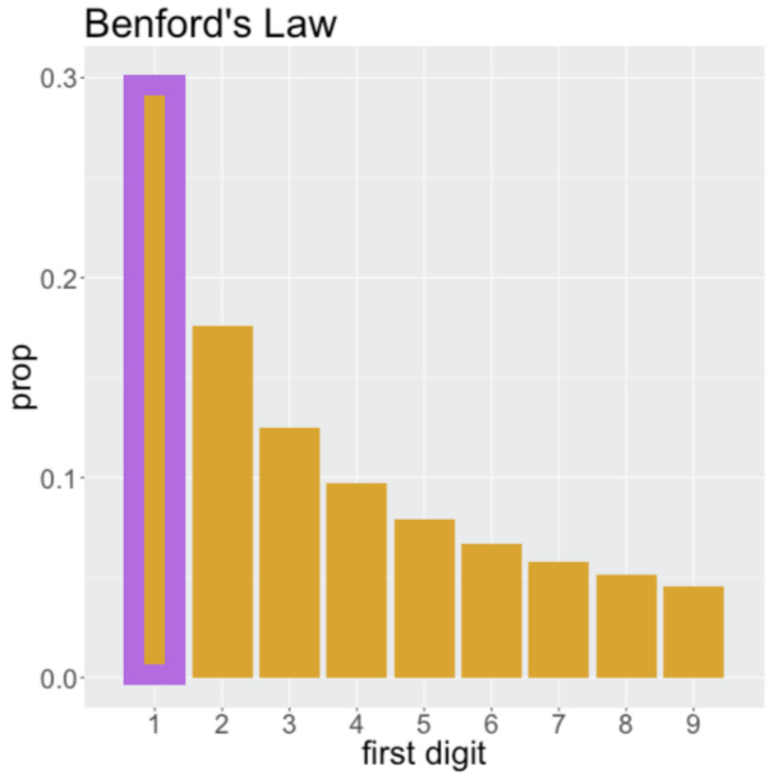
First Digit Distribution





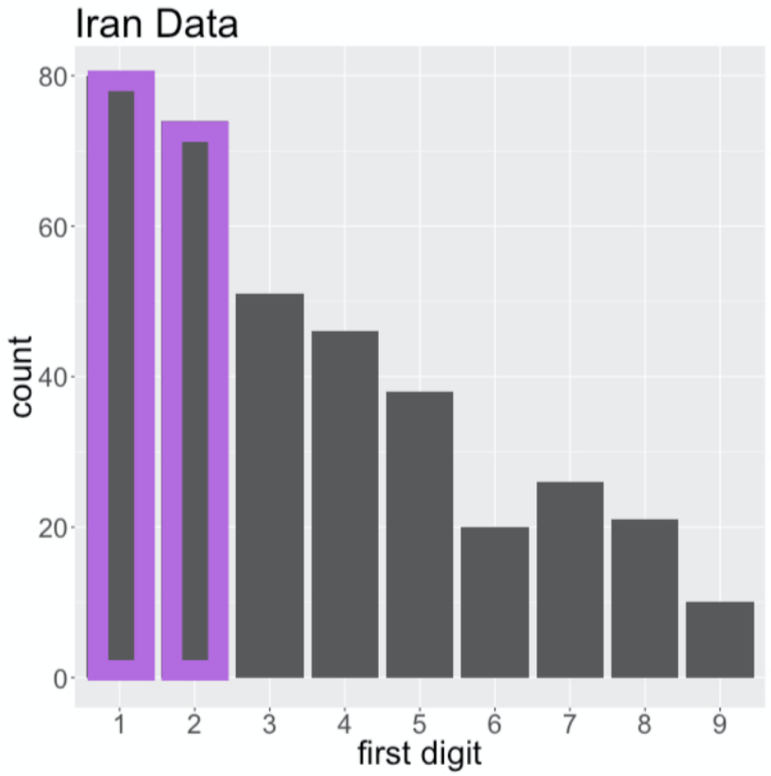
First Digit Distribution

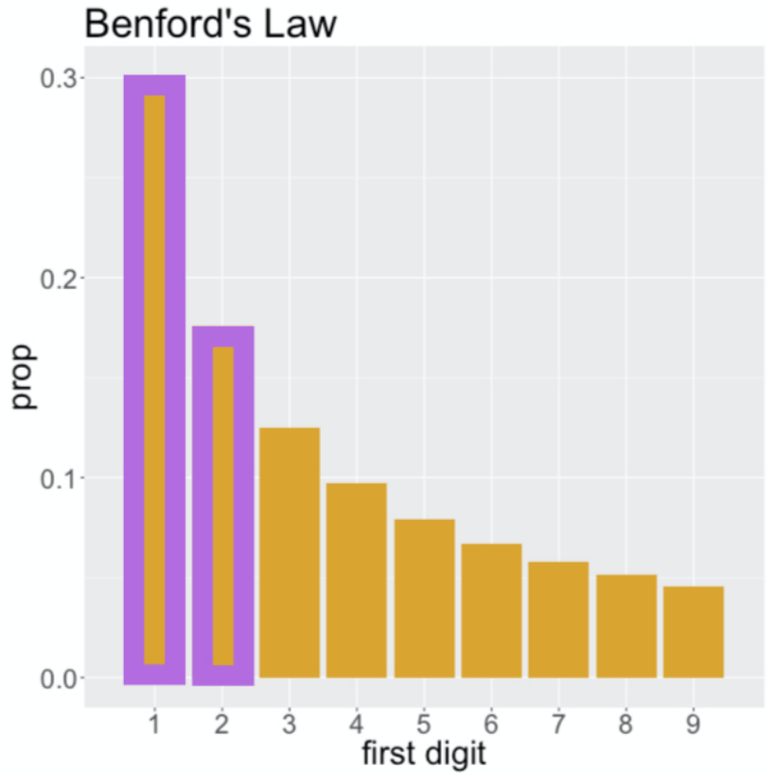




Cell 1: $\frac{(O - E)^2}{E}$

First Digit Distribution

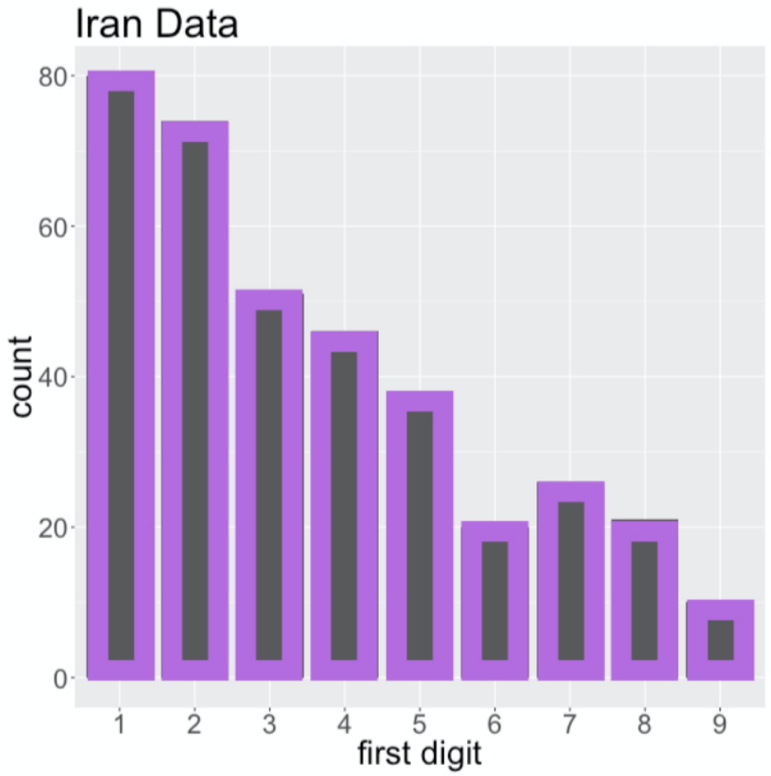


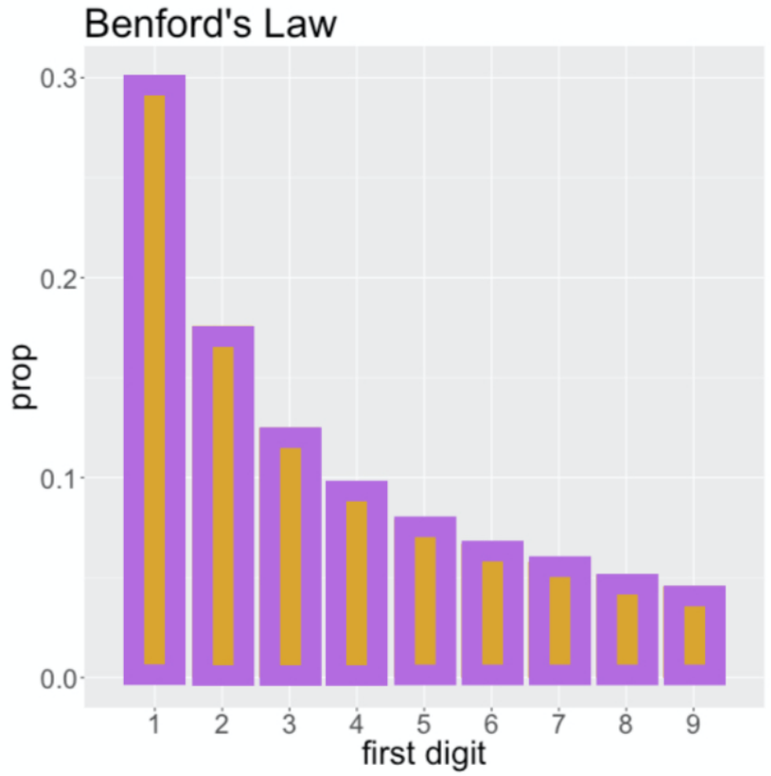


Cell 1: $\frac{(O - E)^2}{E}$

Cell 2: $\frac{(O - E)^2}{E}$

First Digit Distribution





Cell 1: $\frac{(O - E)^2}{E}$

Cell 2: $\frac{(O - E)^2}{E}$

↓

Cell 9: $\frac{(O - E)^2}{E}$

\+

Chi-squared statistic

Example: uniformity of party

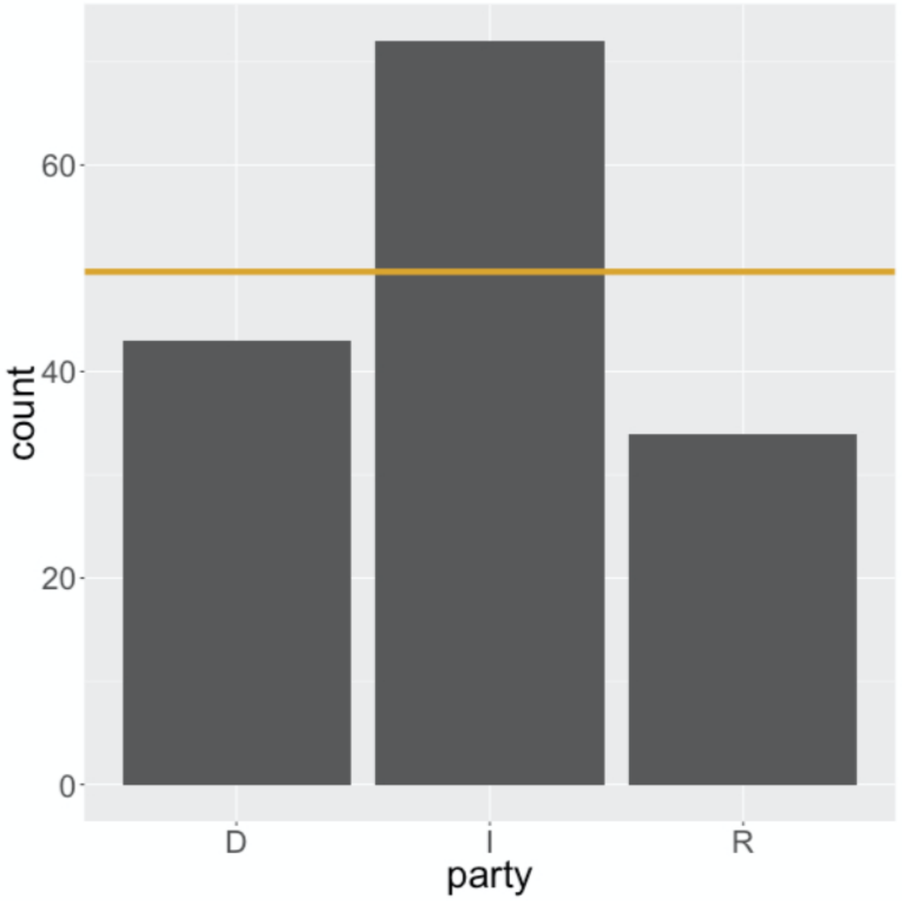
```
ggplot(gss2016, aes(x = party)) +
  geom_bar() +
  geom_hline(yintercept = 149/3, color = "goldenrod", size = 2)
```

```
tab <- gss2016 %>%
     select(party) %>%
     table()
tab
```

```
Dem Ind Rep
 43  72  34
```

```
p_uniform <- c(Dem = 1/3, Ind = 1/3, Rep = 1/3)
chisq.test(tab, p = p_uniform)$stat
```

```
X-squared
 15.87919
```



Simulating the null

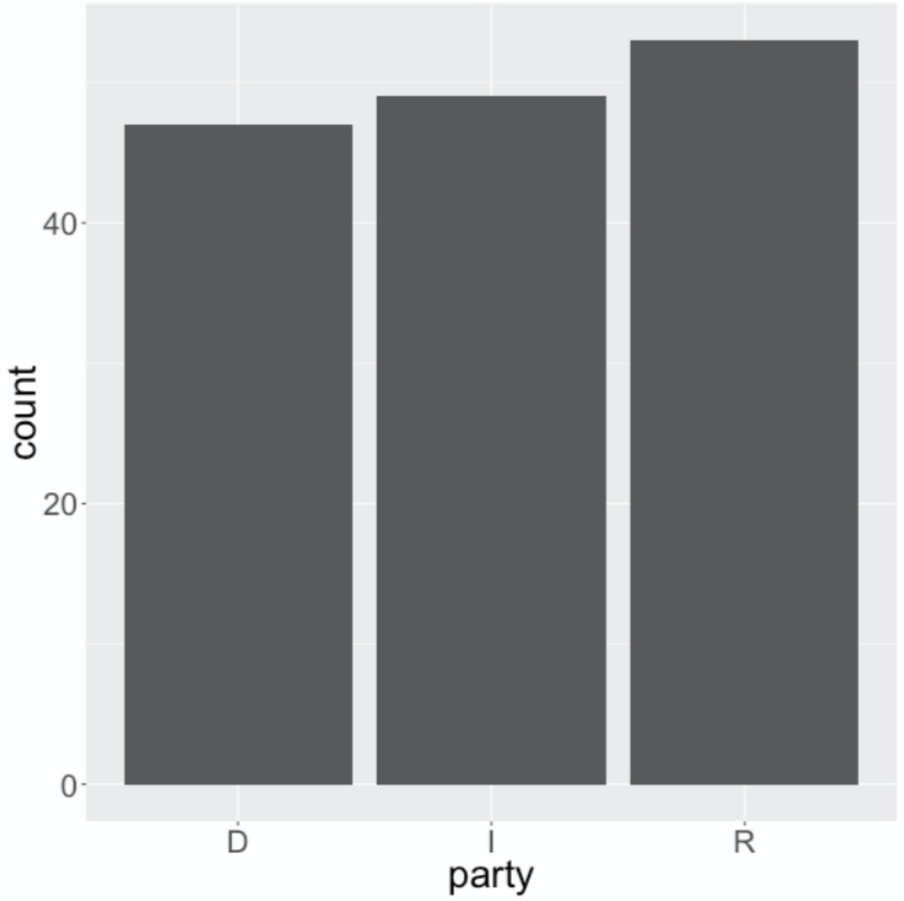
```
gss2016 %>%
  specify(response = party) %>%
  hypothesize(null = "point", p = p_uniform) %>%
  generate(reps = 1, type = "simulate")
```

```
# A tibble: 149 x 2
# Groups:   replicate [1]
   party replicate
   <fct> <fct>
 1 I         1
 2 D         1
 3 I         1
 4 I         1
 5 D         1
 6 R         1
 7 I         1
 8 R         1
 9 D         1
10 I         1
# ... with 139 more rows
```

Simulating the null

```
sim_1 <- gss2016 %>%
   specify(response = party) %>%
   hypothesize(null = “point”, p = p_uniform) %>%
   generate(reps = 1, type = "simulate")
```

```
ggplot(sim_1, aes(x = party)) +
   geom_bar()
```



Let's practice!

INFERENCE FOR CATEGORICAL DATA IN R

And now to US

INFERENCE FOR CATEGORICAL DATA IN R

Andrew Bray

Assistant Professor of Statistics at Reed College

Iran election fraud

H_0: the election was fair (Benford's Law holds)

Iran election fraud

H_0: the election was fair (Benford's Law holds)

H_A: the election was fraudulent (Benford's Law does *not* hold)

Iran election fraud

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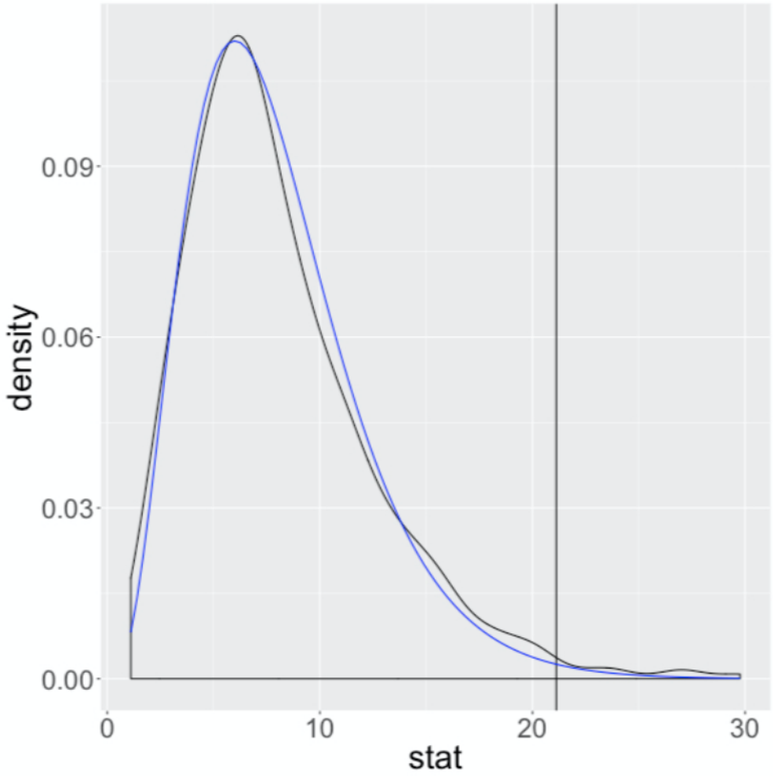

Iran election fraud

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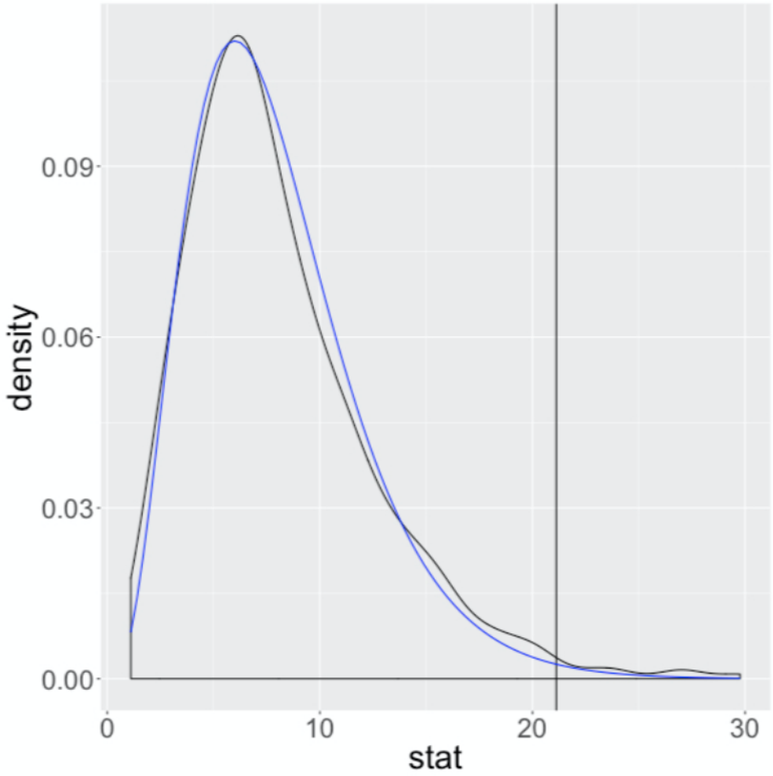

U.S.A. 2016 election

H_0 : the election was fair (Benford's Law holds)

H_A : the election was fraudulent (Benford's Law does not hold)

Iowa vote totals

[1] By TUBS [CC BY SA 3.0], from Wikimedia Commons

Iowa vote totals

```
iowa
```

```
# A tibble: 1,386 x 5
   office              candidate                          party              county votes
   <chr>               <chr>                              <chr>              <chr>  <dbl>
 1 President/Vice Pre… Evan McMullin / Nathan Johnson     Nominated by Peti… Adair     10
 2 President/Vice Pre… Under Votes                        NA                 Adair     32
 3 President/Vice Pre… Gary Johnson / Bill Weld           Libertarian        Adair    127
 4 President/Vice Pre… Over Votes                         NA                 Adair      5
 5 President/Vice Pre… Gloria La Riva / Dennis J. Banks   Socialism and Lib… Adair      0
 6 President/Vice Pre… Darrell L. Castle / Scott N. Bra… Constitution       Adair     10
 7 President/Vice Pre… Hillary Clinton / Tim Kaine        Democratic         Adair   1133
 8 President/Vice Pre… Jill Stein / Ajamu Baraka          Green              Adair     14
 9 President/Vice Pre… Rocky Roque De La Fuente / Micha… Nominated by Peti… Adair      3
10 President/Vice Pre… Donald Trump / Mike Pence          Republican         Adair   2461
# … with 1,376 more rows
```

Let's practice!

INFERENCE FOR CATEGORICAL DATA IN R

Election fraud in Iran and Iowa: debrief

INFERENCE FOR CATEGORICAL DATA IN R

Andrew Bray

Assistant Professor of Statistics at Reed College

datacamp

Iowa election fraud

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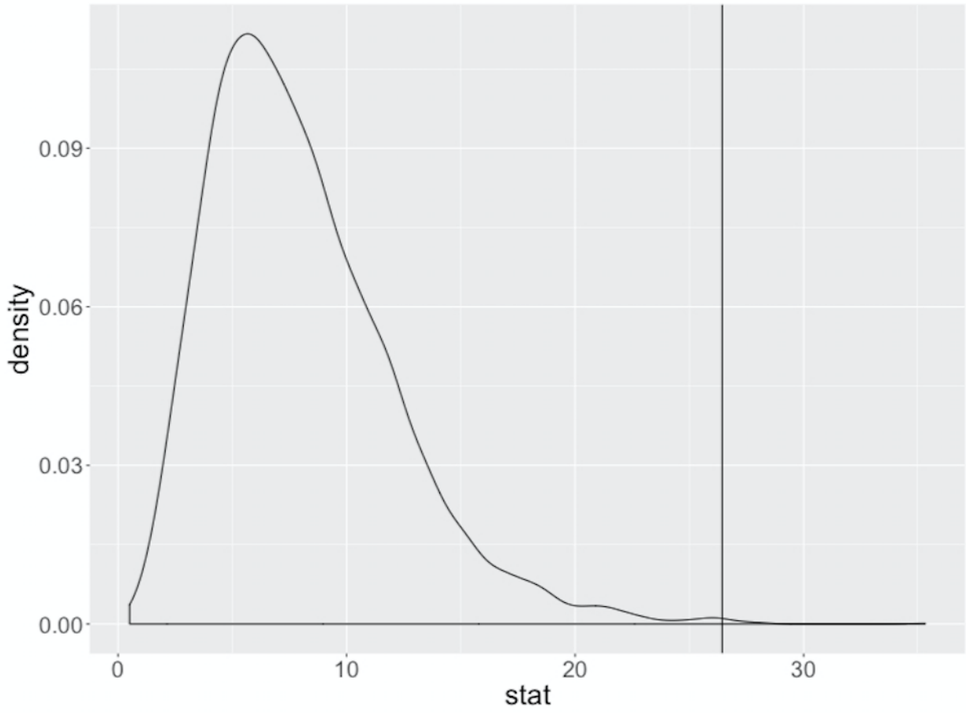
H_A: the election was fraudulent (Benford's Law does not hold)

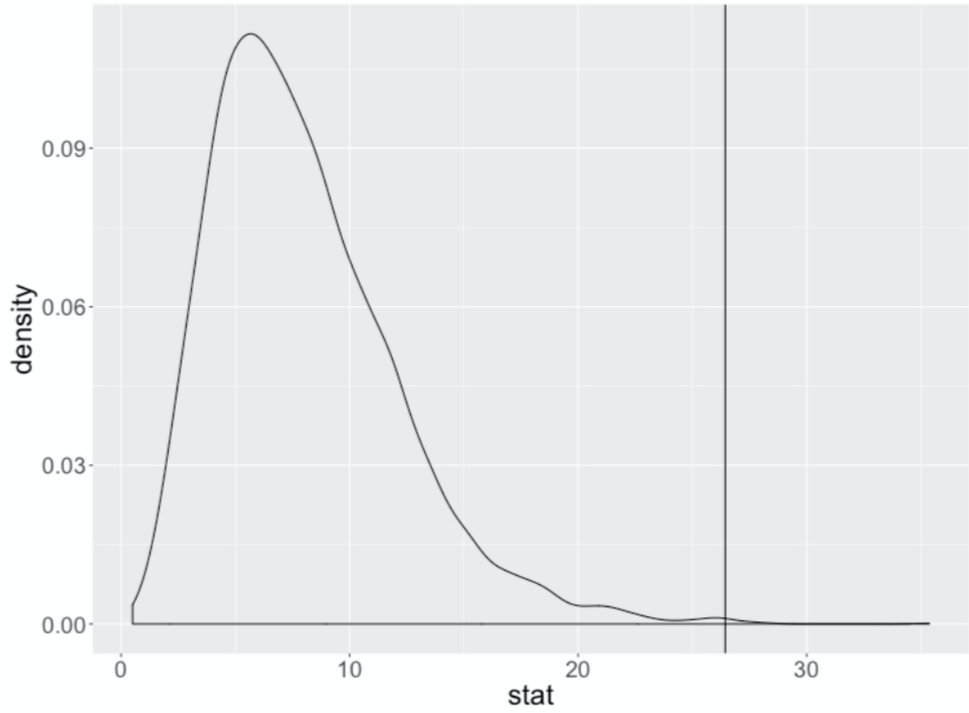


Iowa election fraud

Rejected

H_0: the election was fair (Benford's Law holds)

H_A: the election was fraudulent (Benford's Law does not hold)



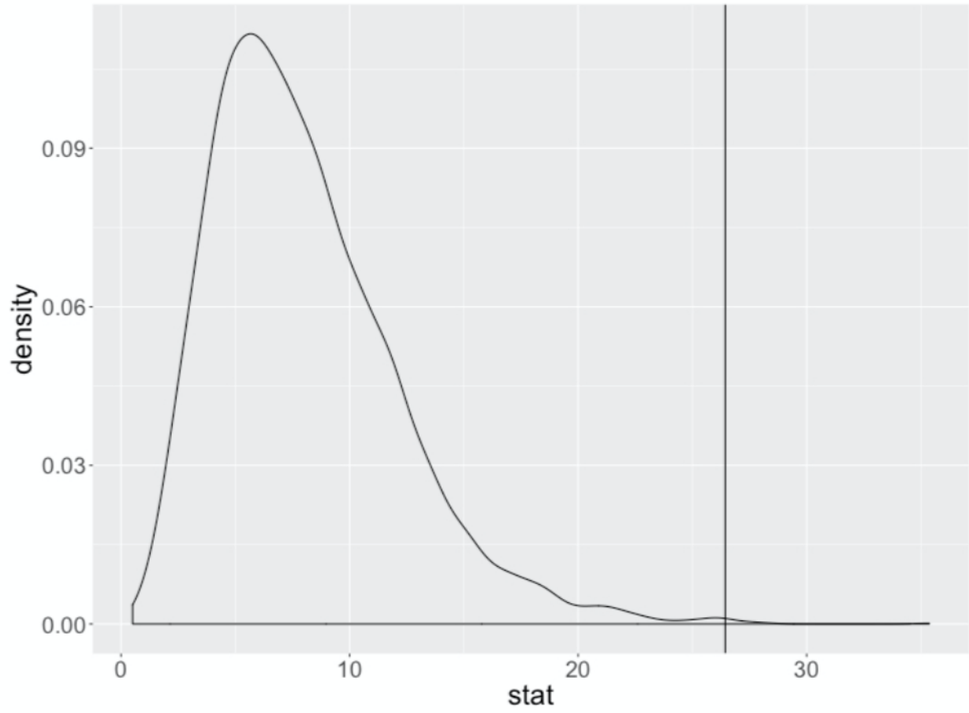
Iowa election fraud

Rejected

H_0: the election was fair (Benford's Law holds)

H_A: the election was fraudulent (Benford's Law does not hold)

What went wrong?



What went wrong?

Possibility 1: type I error

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- Rejecting the null hypothesis when it is true, just due to chance

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Possibility 2: Bedford's Law doesn't apply to vote totals

What went wrong?

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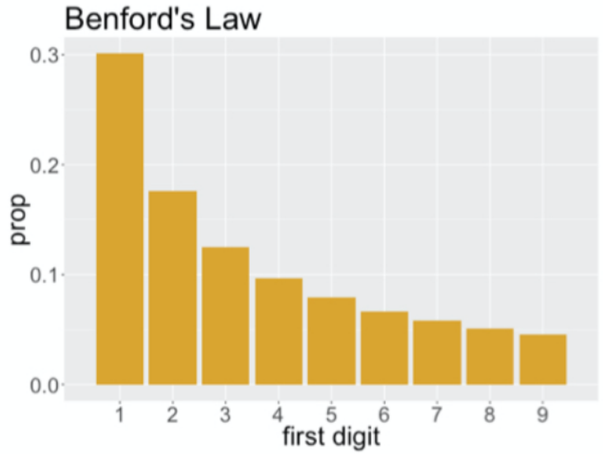
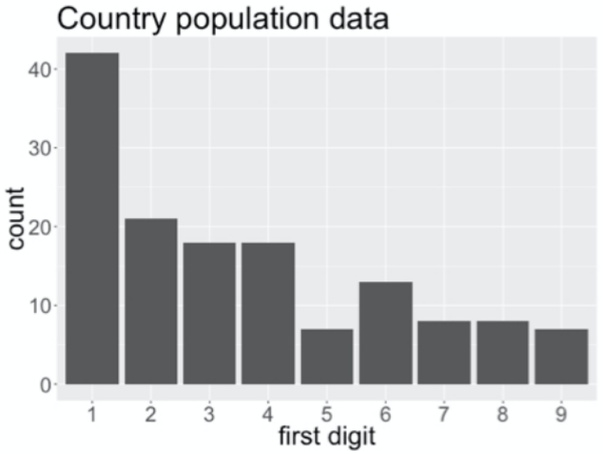
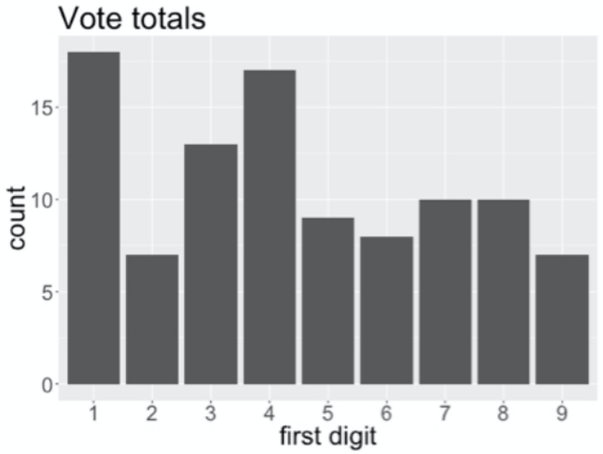
- Rejecting the null hypothesis when it is true, just due to chance



Possibility 2: Bedford's Law doesn't apply to vote totals





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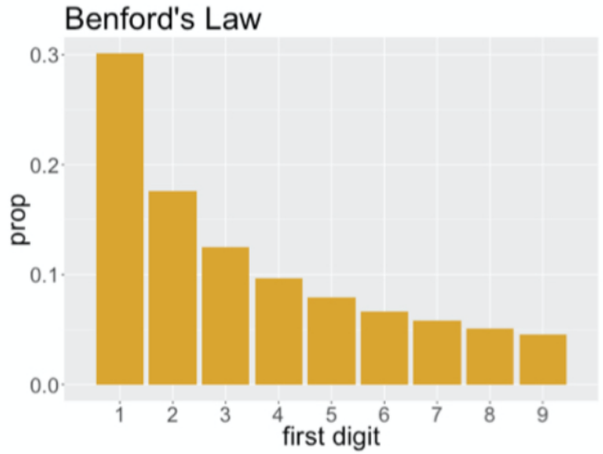
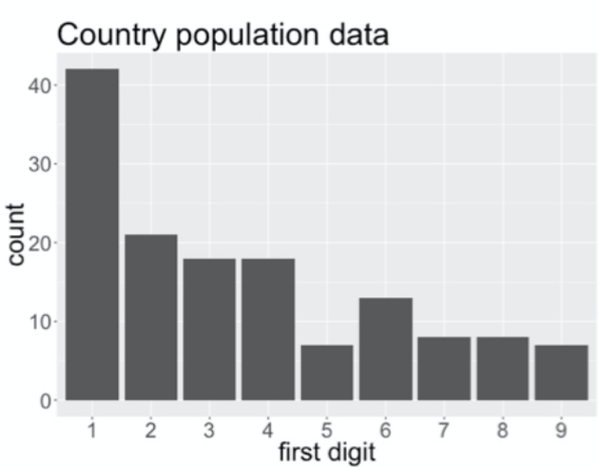
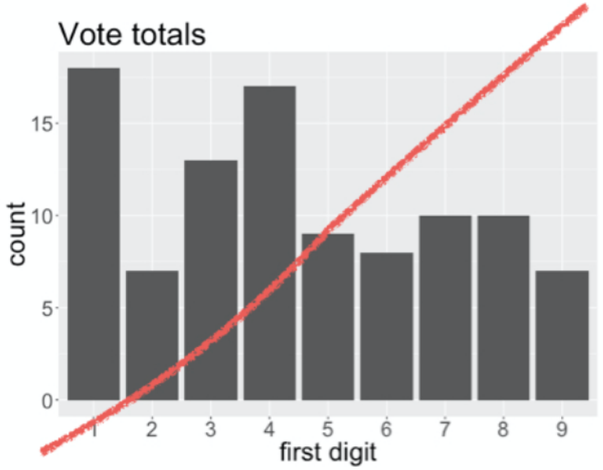


Possibility 2: Bedford's Law doesn't apply to vote totals





Take-home lesson

The statistical tool must be appropriate for the task.

[1] By TUBS [CC BY-SA 3.0], from Wikimedia Commons [2] By P30Carl [GFDL] or [CC BY-SA 3.0], from Wikimedia Commons

Methods for categorical data

Confidence Intervals

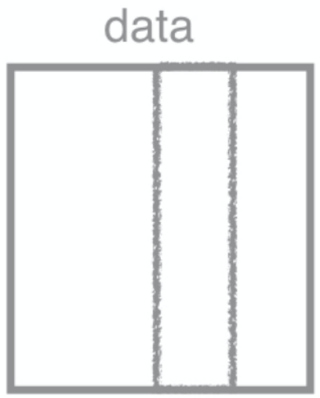
- One proportion
- Difference in proportions

Hypothesis tests

- One proportion
- Difference in proportions
- Test of independence
- Goodness of fit

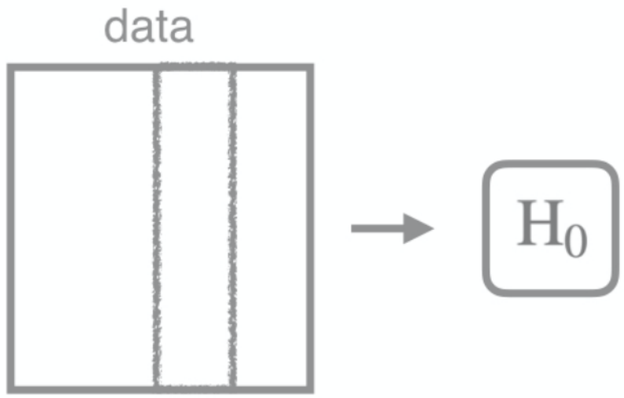
Hypothesis Test

Hypothesis Test



specify()

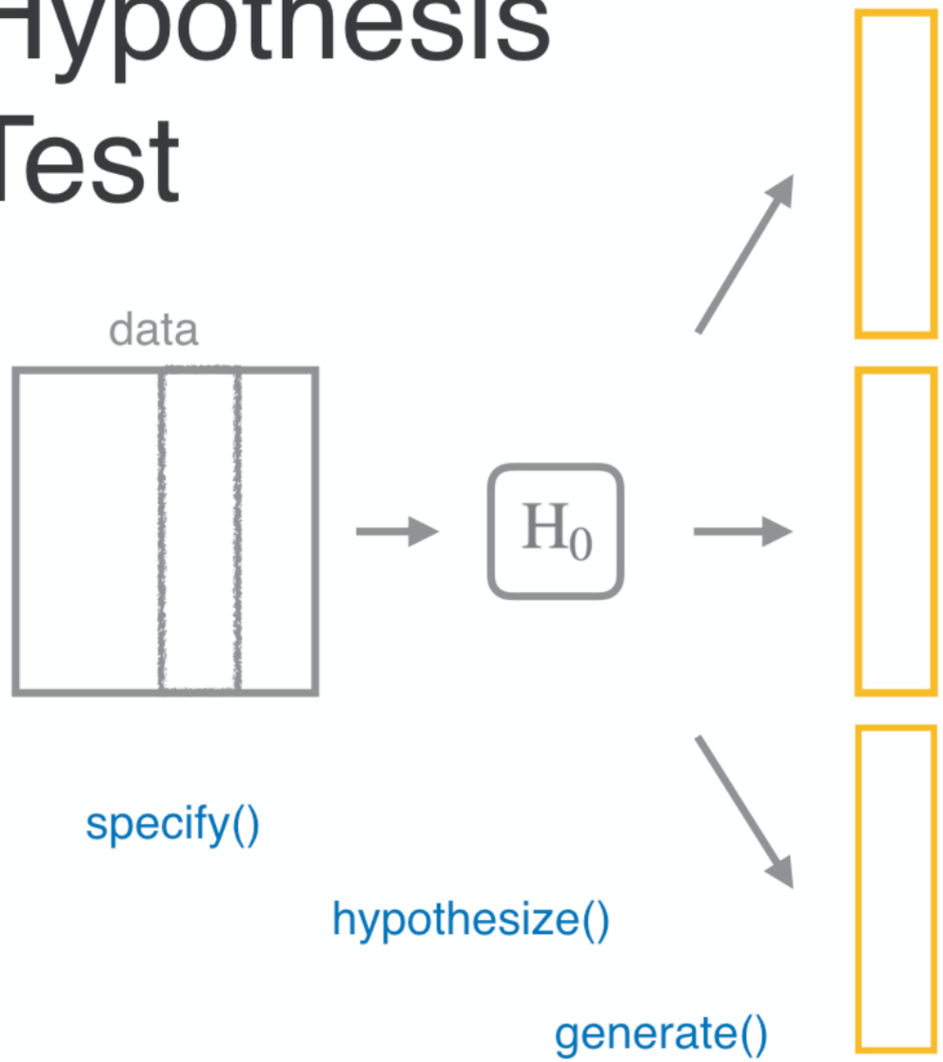
Hypothesis Test



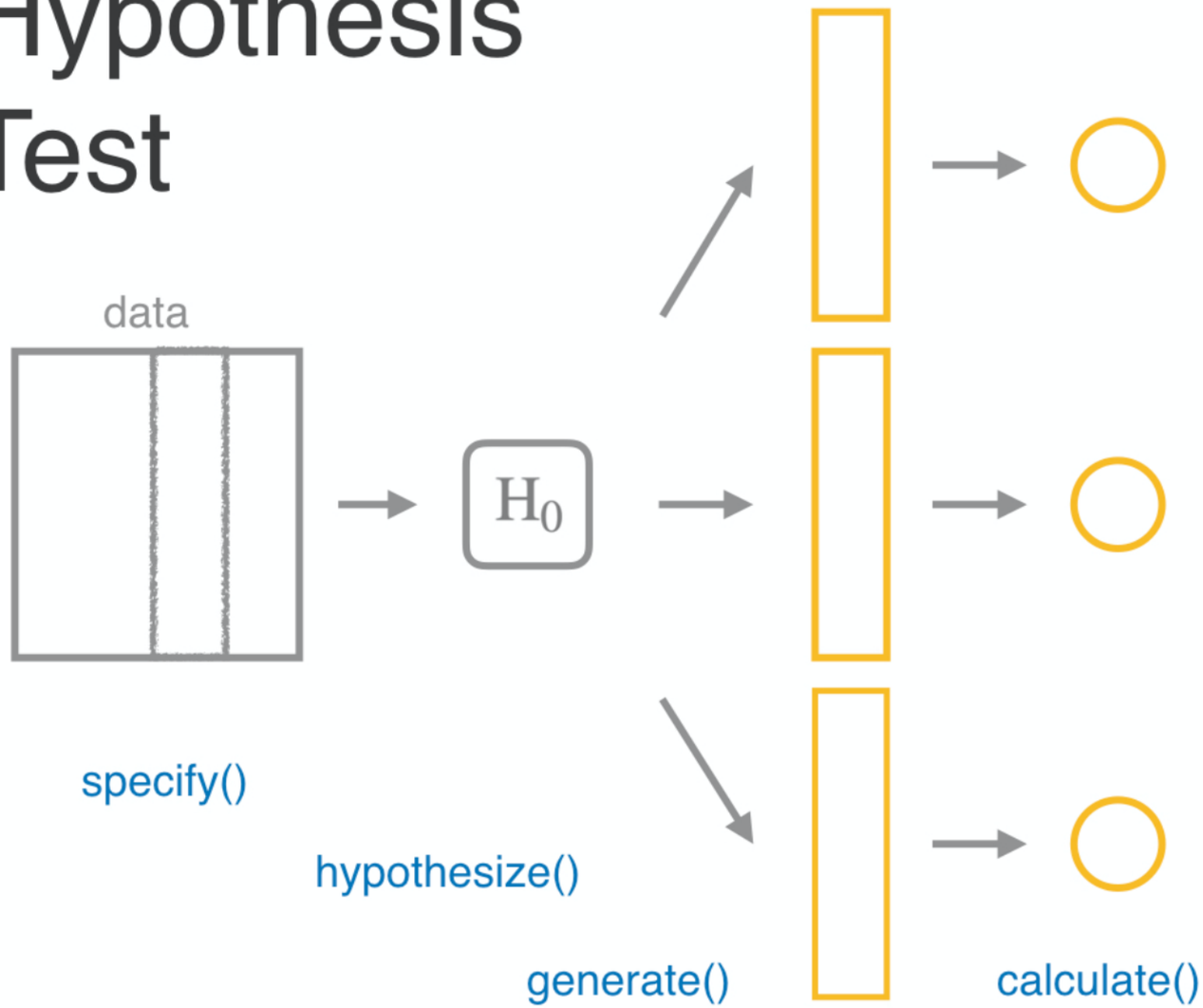
specify()

hypothesize()

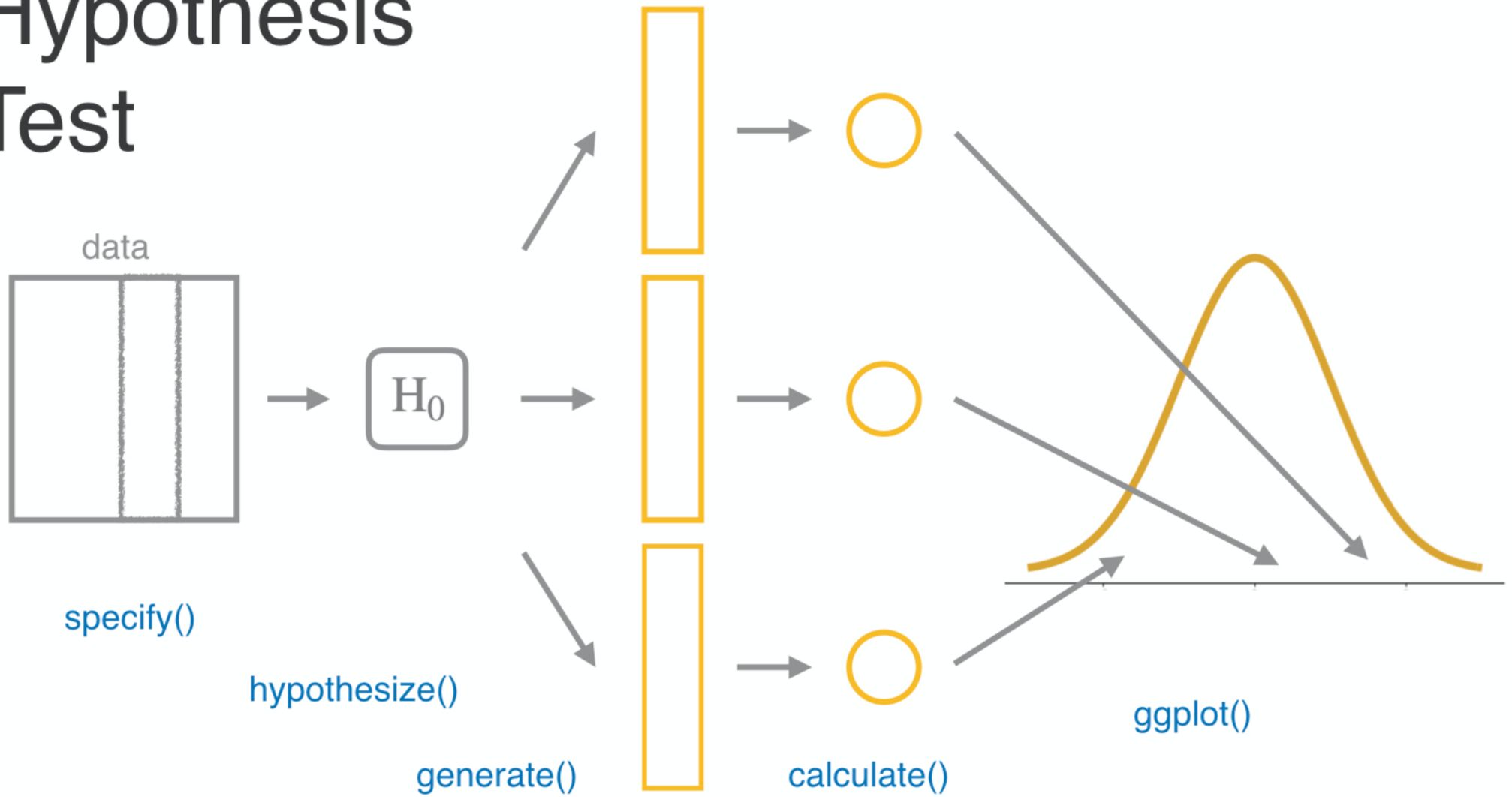
Hypothesis Test



Hypothesis Test



Hypothesis Test



Let's practice!

INFERENCE FOR CATEGORICAL DATA IN R