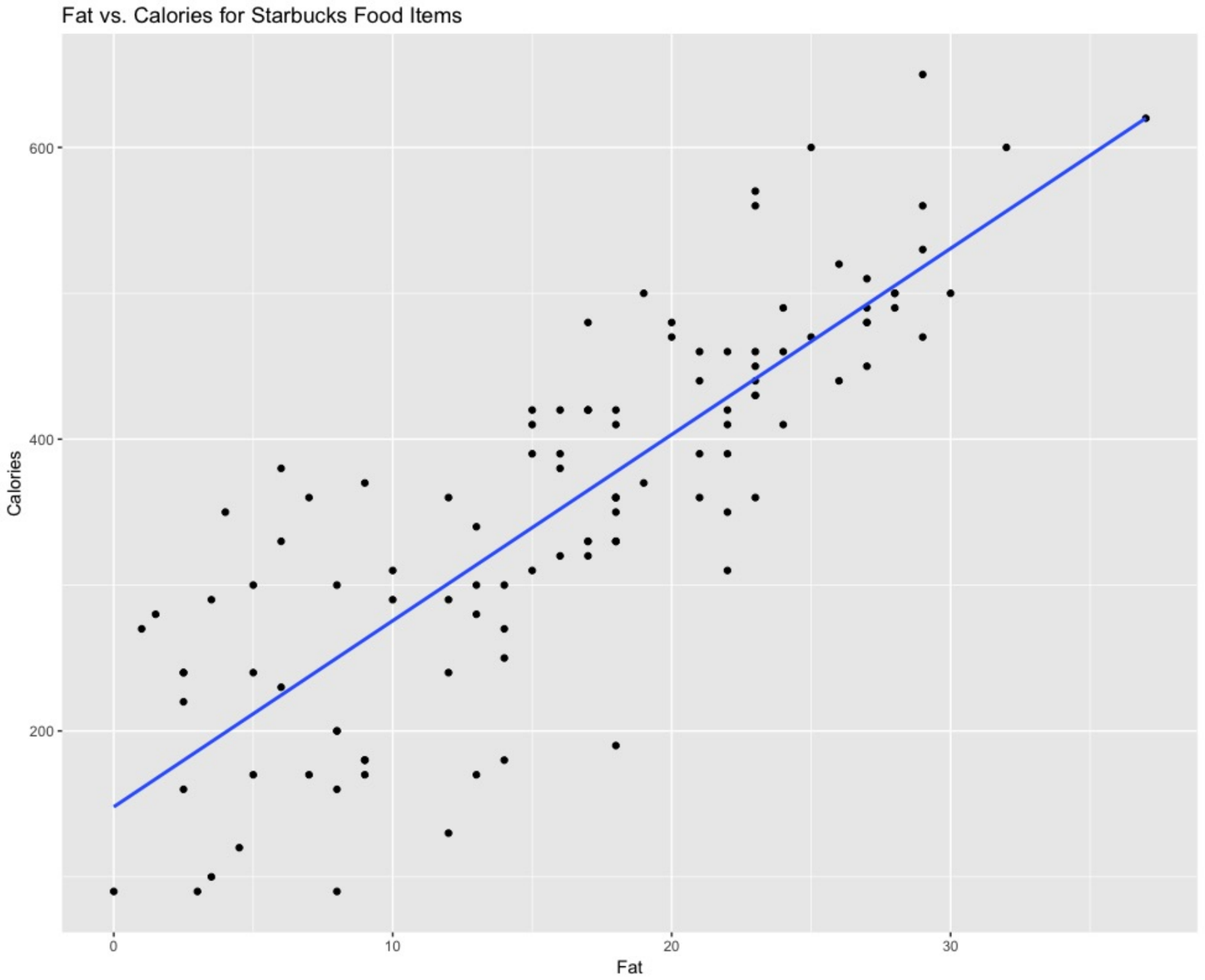
Mathematical approximation

INFERENCE FOR LINEAR REGRESSION IN R

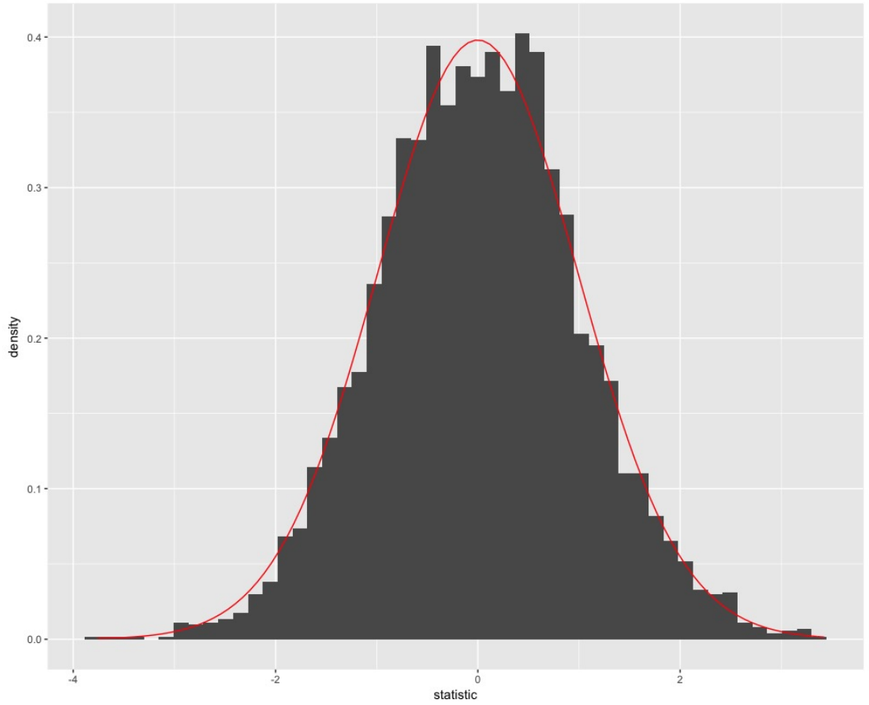
Jo Hardin
Professor, Pomona College

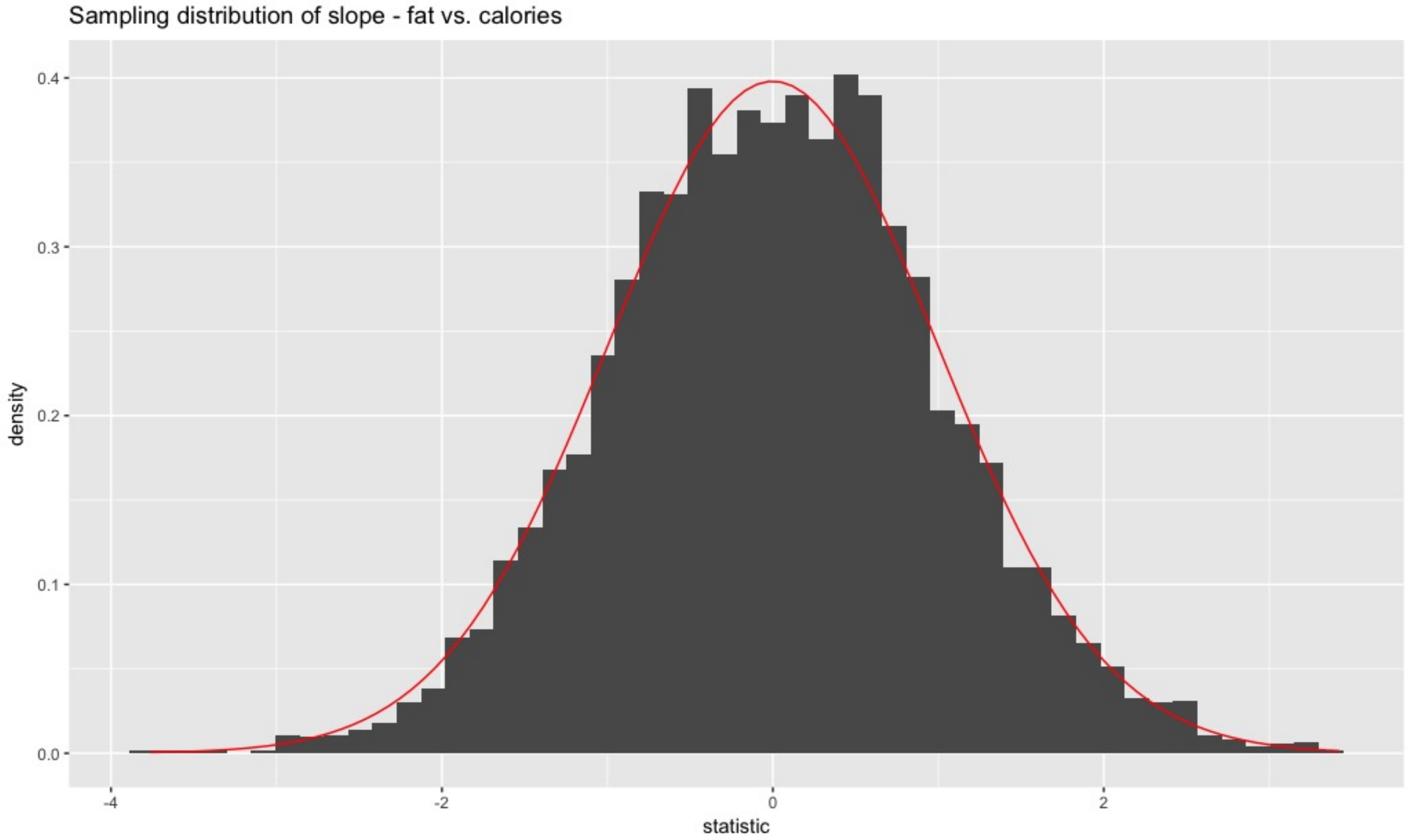
datacamp


Fat vs. Calories for Starbucks Food Items
Calories
600
400
200
0
10
20
30
Fat

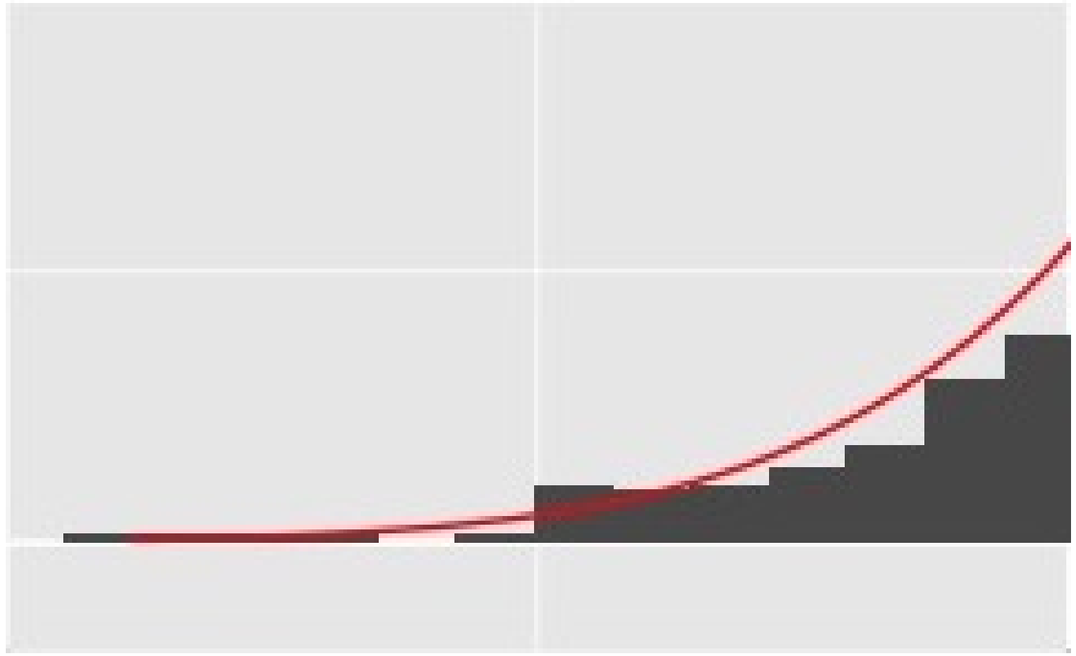
Sampling distribution of slope: good t fit

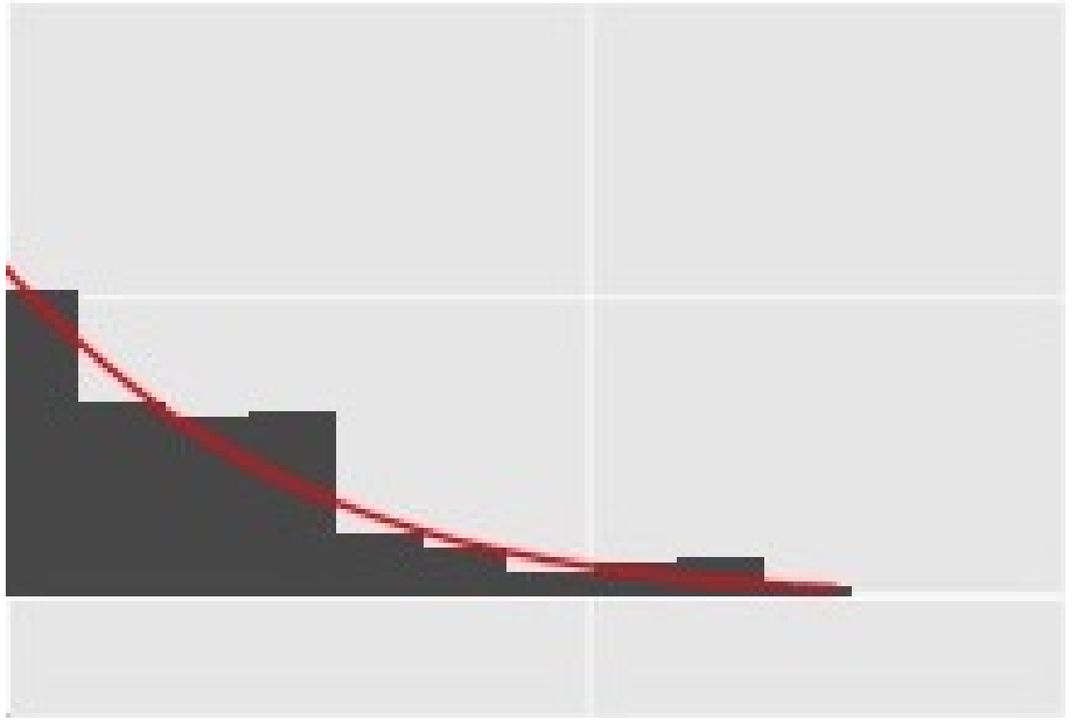
```
ggplot(starFatCal, aes(x = statistic)) +
  geom_histogram(aes(y = ..density..), bins = 50) +
  stat_function(fun = dt, color = "red", args = list(df = nrow(starbucks) - 2))
```

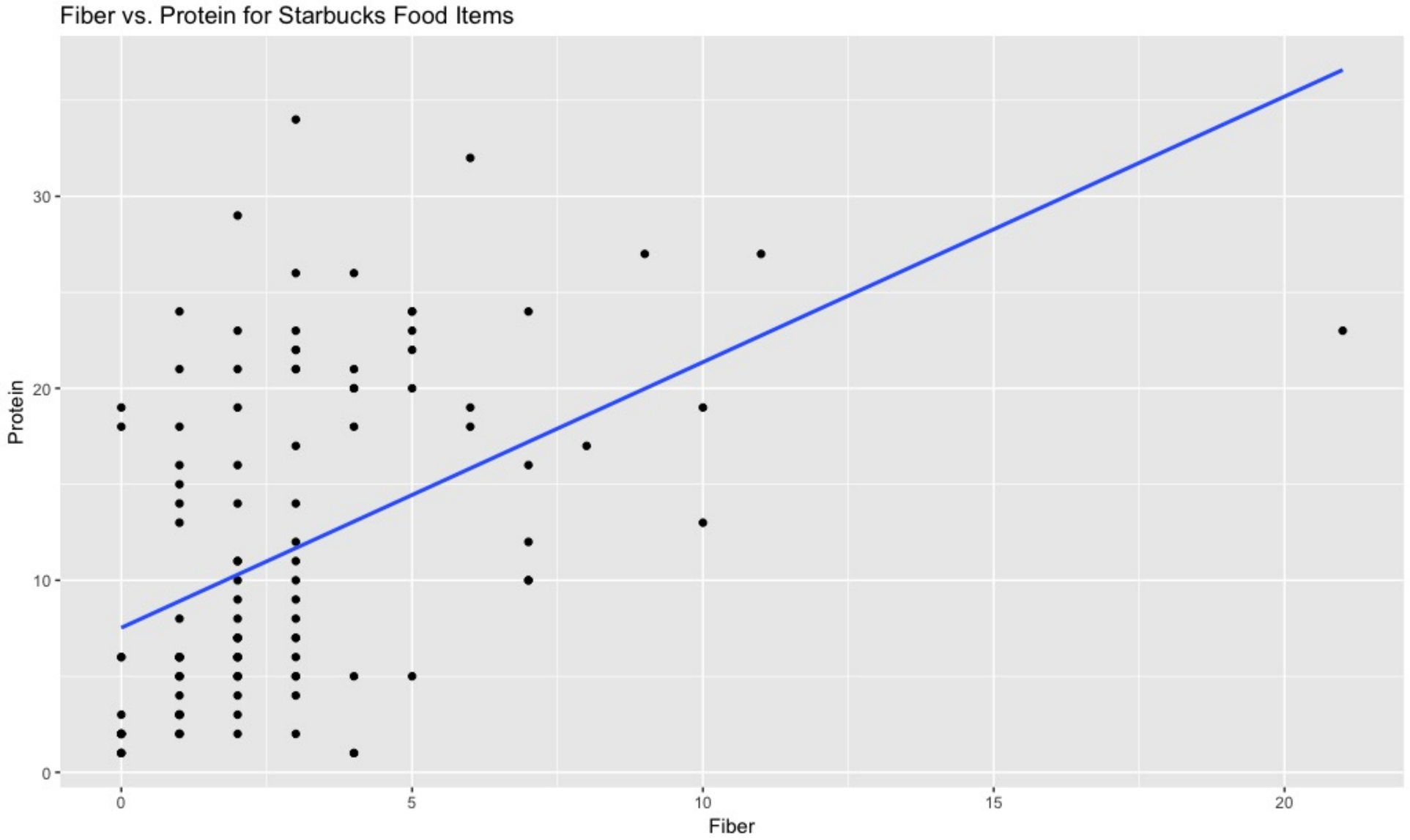



Sampling distribution of slope - fat vs. calories
density
0.4
0.3
0.2
0.1
0.0
-4
-2
0
2
statistic

Model fit

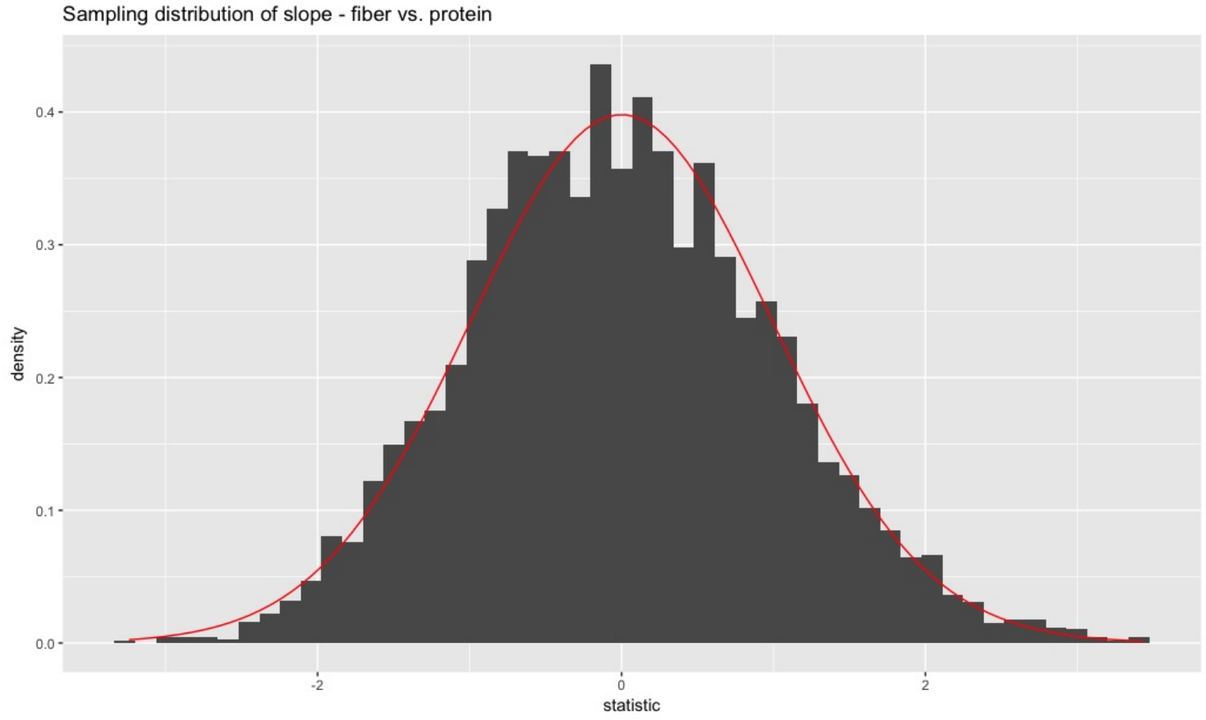




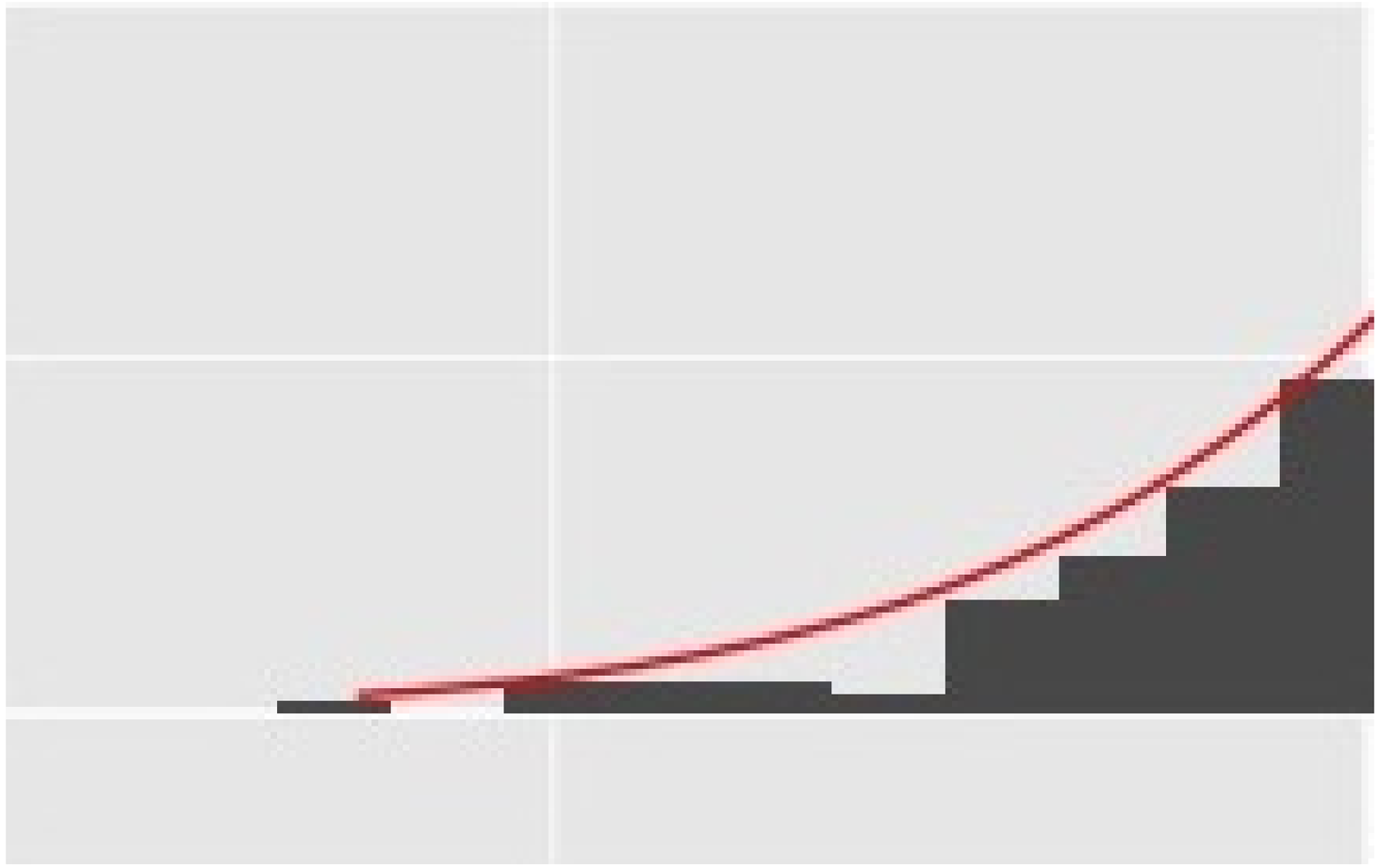

Fiber vs. Protein for Starbucks Food Items
Protein
30
20
10
0
0
5
10
15
20
Fiber

Sampling distribution of slope: poor t fit

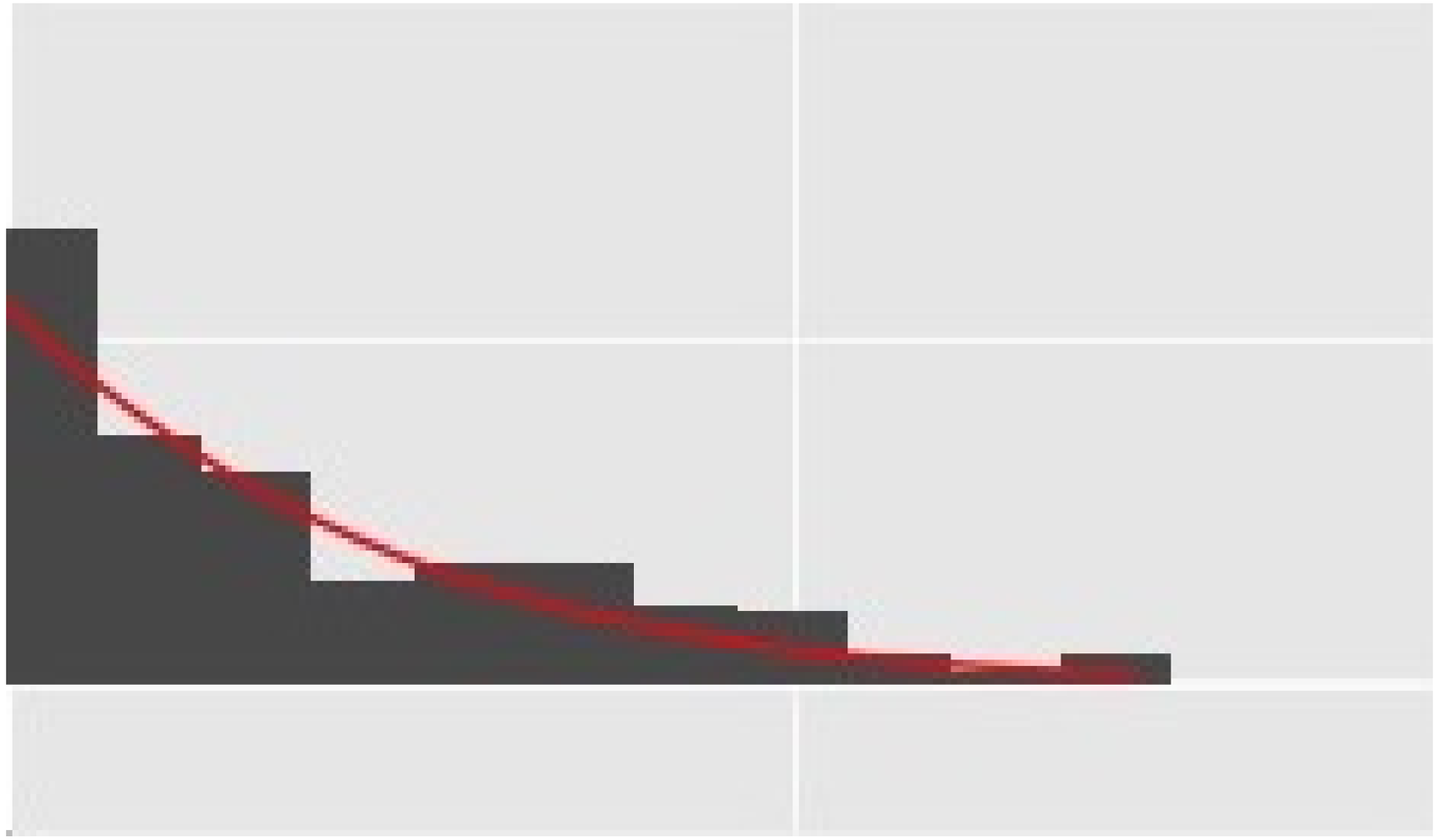
```
ggplot(starProFib, aes(x = statistic)) + geom_histogram(aes(y = ..density..), bins = 50) +
  stat_function(fun = dt, color = "red", args = list(df = nrow(starbucks) - 2))
```



Sampling distribution of slope: poor t fit



Sampling distribution of slope: poor t fit



Let's practice!

INFERENCE FOR LINEAR REGRESSION IN R

Intervals in regression

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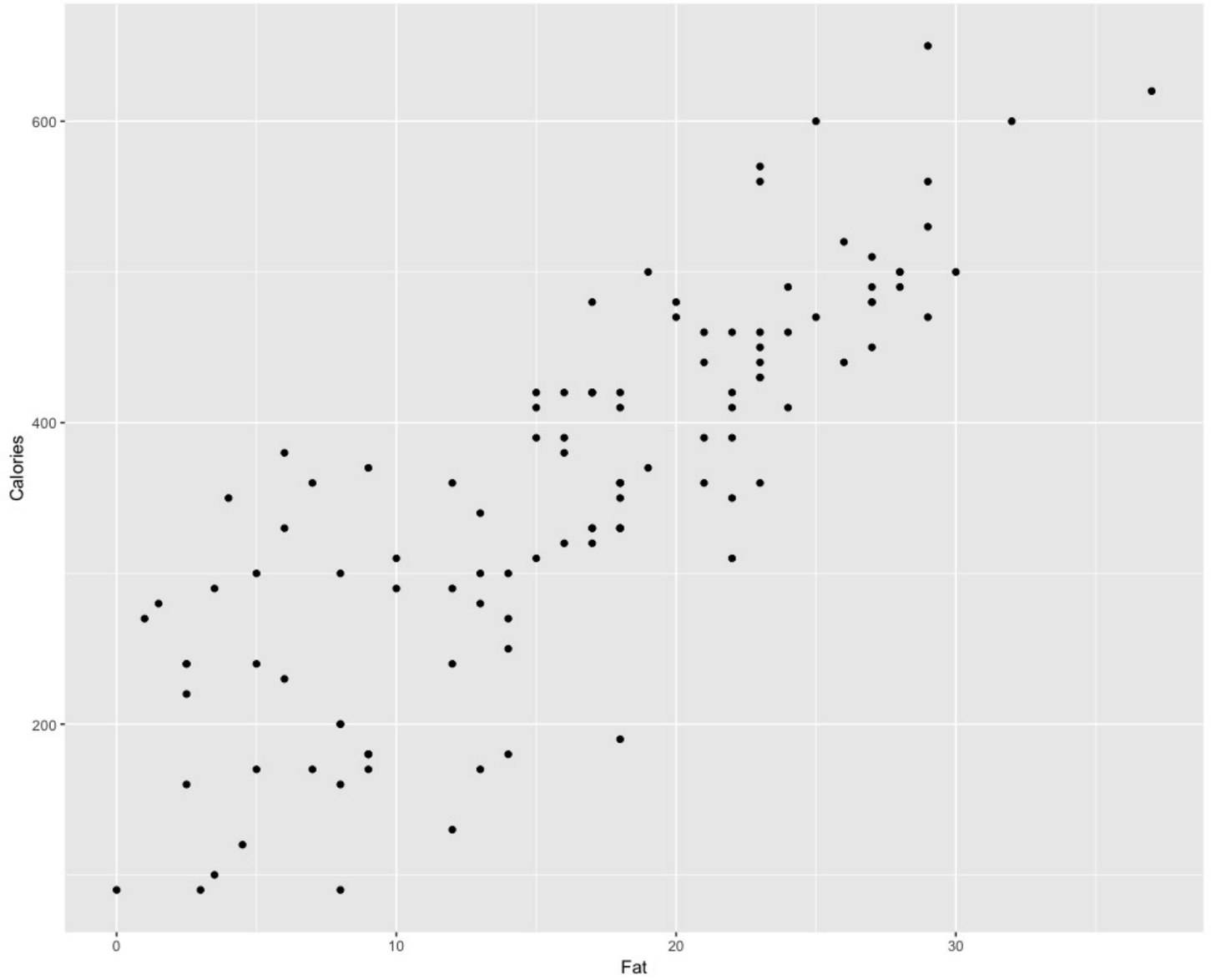
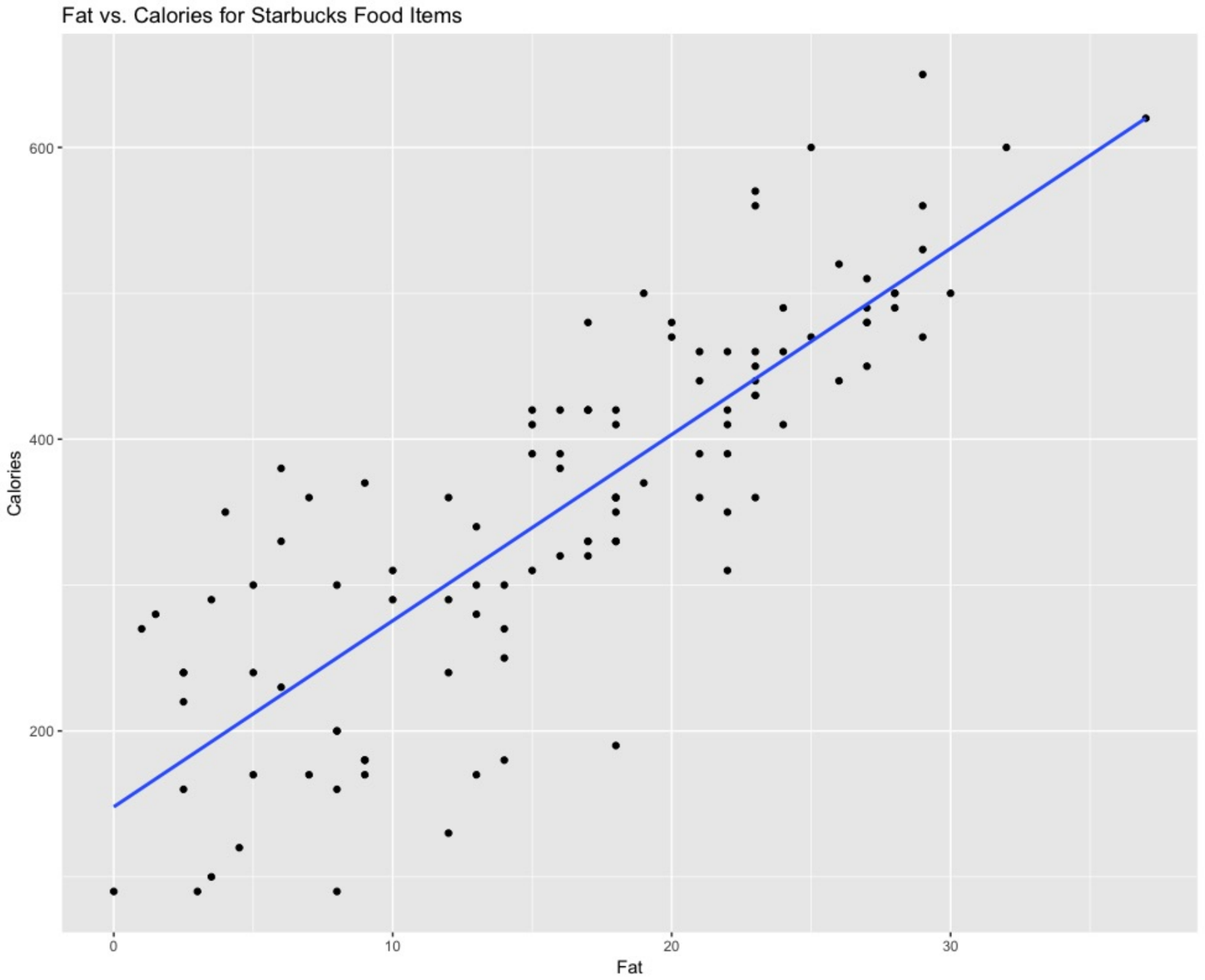

600
400
200
Calories
0
10
20
30
Fat


Fat vs. Calories for Starbucks Food Items
Calories
600
400
200
0
10
20
30
Fat

```
alpha = 0.05
crit_val <- qt((1-alpha/2), df = nrow(starbucks) - 2)
lm(Calories ~ Fat, data=starbucks) %>%
  tidy(conf.int = TRUE, conf.level = 1-alpha)
```

```
        term   estimate  std.error statistic   p.value     conf.low conf.high
1 (Intercept)  147.9833 14.9719851  9.884013 6.630009e-17 118.31530 177.65128
2         Fat   12.7586  0.8171655 15.613236 8.937367e-30 11.13933   14.37787
```

```
lm(Calories ~ Fat, data=starbucks) %>% tidy() %>%
   mutate(lower = estimate - crit_val*std.error,
          upper = estimate + crit_val*std.error)
```

```
        term  estimate  std.error statistic      p.value     lower     upper
1 (Intercept) 147.9833 14.9719851  9.884013 6.630009e-17 118.31530 177.65128
2         Fat  12.7586  0.8171655 15.613236 8.937367e-30 11.13933  14.37787
```

Confidence interval for intercept parameter

```
tidy_mod <- lm(Calories ~ Fat,
               data = starbucks) %>%
  tidy(conf.int = TRUE,
       conf.level = 1-alpha)
tidy_mod
```

```
term estimate  std.error
1 (Intercept) 147.9833 14.9719851
2         Fat  12.7586  0.8171655
statistic      p.value  conf.low
1  9.884013 6.630009e-17 118.31530
2 15.613236 8.937367e-30  11.13933
conf.high
1 177.65128
2  14.37787
```

```
tidy_mod %>%
  filter(term == "(Intercept)") %>%
  select(conf.low, conf.high)
```

```
  conf.low conf.high
1 118.3153  177.6513
```

Confidence interval for slope parameter

```
tidy_mod <- lm(Calories ~ Fat,
               data = starbucks) %>%
  tidy(conf.int = TRUE,
       conf.level = 1-alpha)
tidy_mod
```

```
term estimate  std.error
1 (Intercept) 147.9833 14.9719851
2         Fat  12.7586  0.8171655
statistic      p.value  conf.low
1  9.884013 6.630009e-17 118.31530
2 15.613236 8.937367e-30  11.13933
conf.high
1 177.65128
2  14.37787
```

```
tidy_mod %>%
  filter(term == "Fat") %>%
  select(conf.low, conf.high)
```

```
  conf.low conf.high
1 11.13933  14.37787
```

Bootstrap interval for slope

```
BS_slope <- starbucks %>%
   specify(Calories ~ Fat) %>%
   generate(reps = 1000, type = "bootstrap") %>%
   calculate(stat = "slope")
BS_slope %>%
   summarize(low = quantile(stat, alpha / 2),
             high = quantile(stat, 1 - alpha / 2))
```

```
 A tibble: 1 x 2
       low     high
     <dbl>    <dbl>
1 11.16712 14.34817
```

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Different types of intervals

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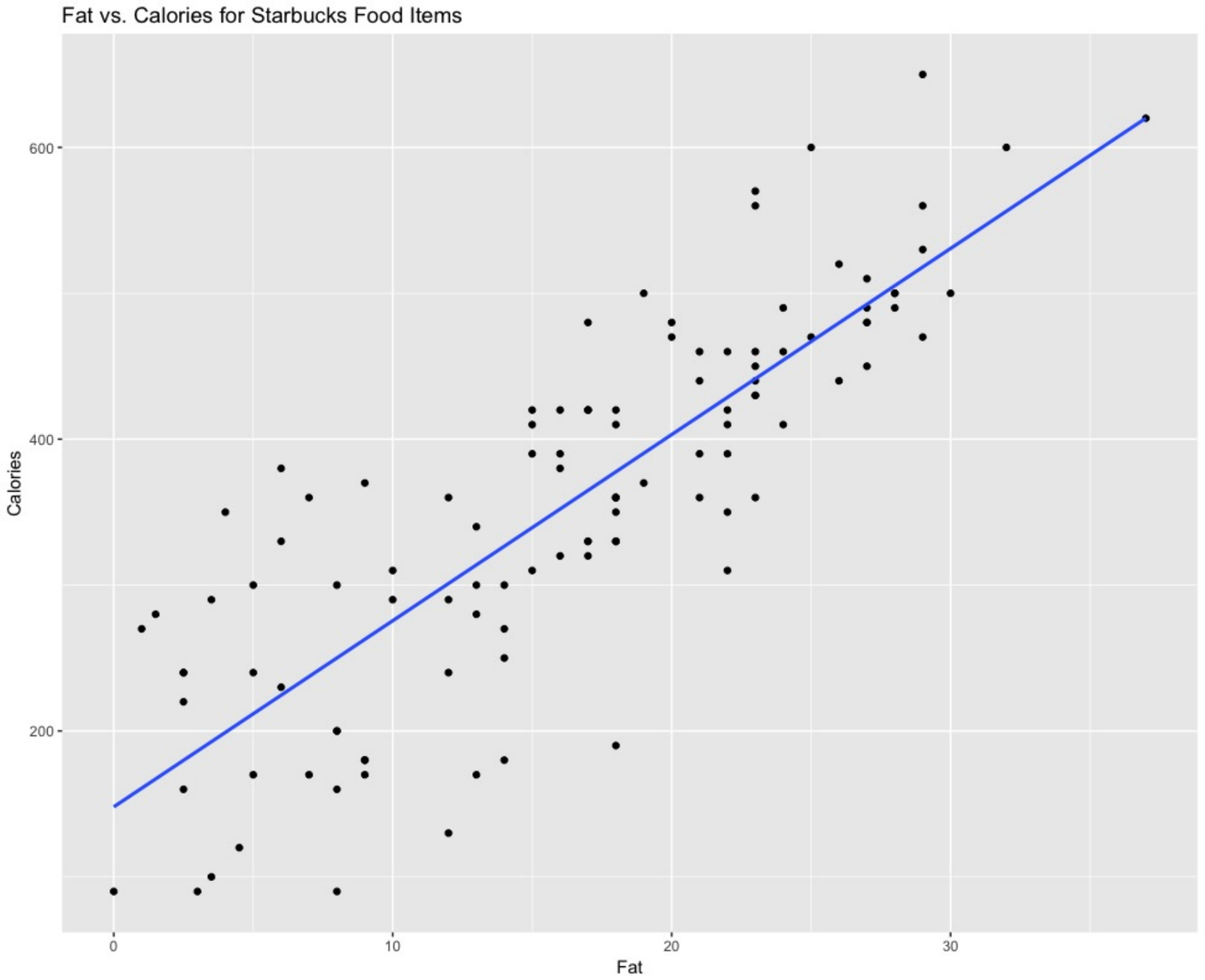
Fat vs. Calories for Starbucks Food Items
Calories
600
400
200
0
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20
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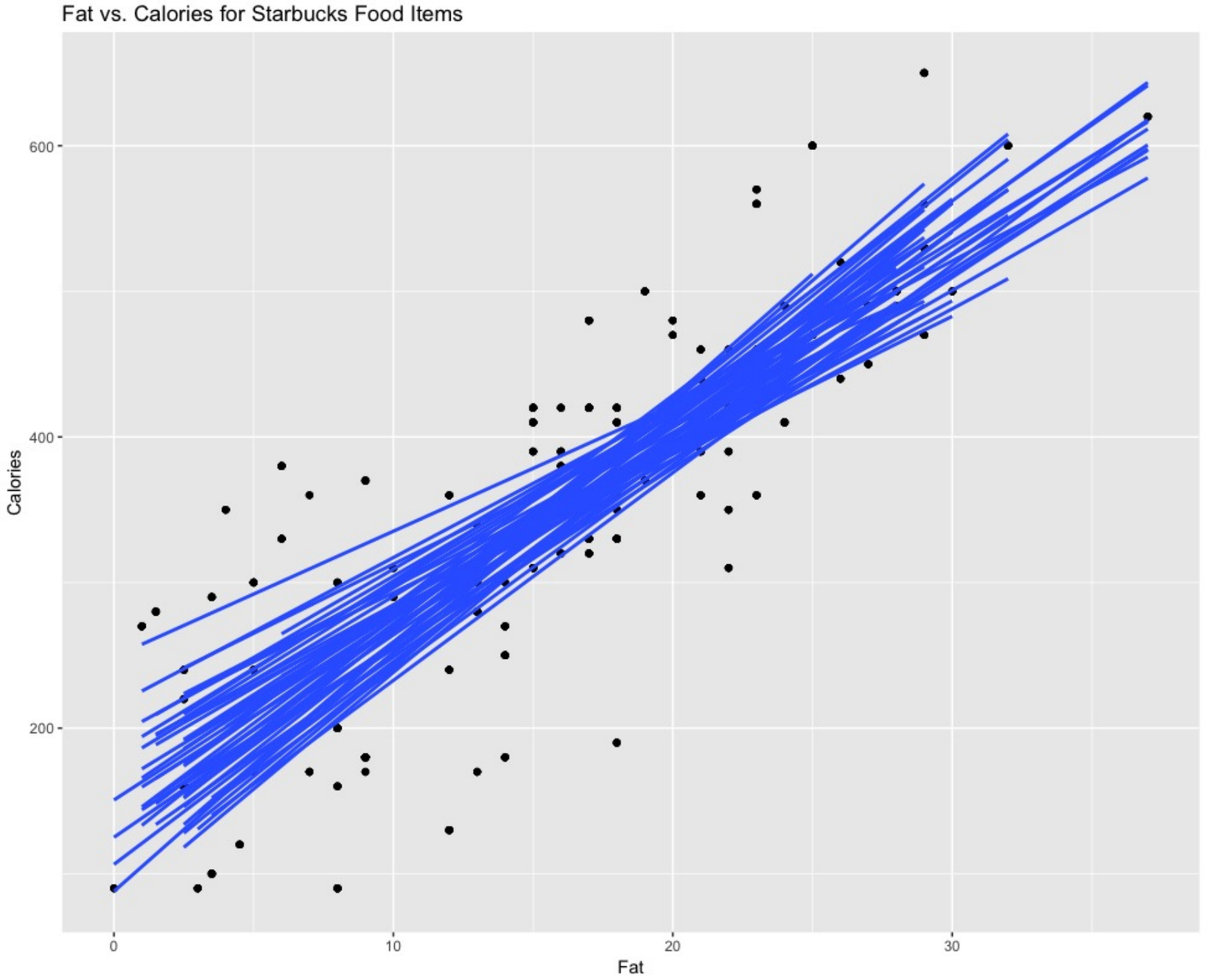
Fat


Fat vs. Calories for Starbucks Food Items
Calories
600
400
200
0
10
20
30
Fat

Predicting average calories at specific fat

```
library(broom)
alpha <- .05
crit_val <- qt((1-alpha/2), df = nrow(starbucks) - 2)
newfood <- data.frame(Fat = c(0,10,20,30))
augment(lm(Calories ~ Fat, data=starbucks), newdata = newfood) %>%
   mutate(lowMean = .fitted - crit_val*.se.fit,
          upMean = .fitted + crit_val*.se.fit)
```

```
  Fat   .fitted    .se.fit  lowMean   upMean
1   0 147.9833 14.971985 118.3153 177.6513
2  10 275.5693  8.516206 258.6938 292.4447
3  20 403.1552  7.378555 388.5341 417.7763
4  30 530.7412 13.035040 504.9114 556.5710
```

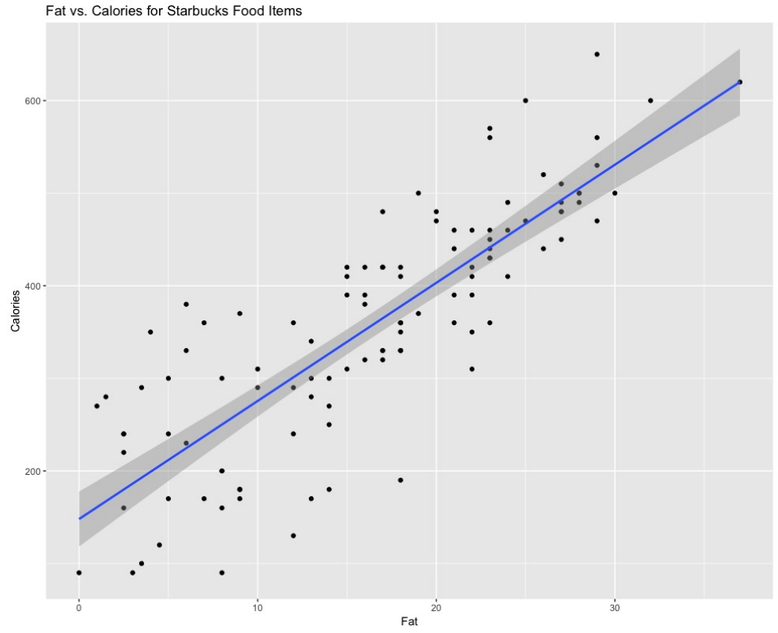
Creating CI for average response

```
predMeans <- augment(lm(Calories ~ Fat, data = starbucks)) %>%
   select(Calories, Fat, .fitted, .se.fit) %>%
   mutate(lowMean = .fitted - crit_val*.se.fit,
          upMean =  .fitted + crit_val*.se.fit)
head(predMeans)
```

```
  Calories Fat  .fitted   .se.fit  lowMean   upMean
1      300   5 211.7763 11.473843 189.0401 234.5125
2      380   6 224.5349 10.823741 203.0869 245.9828
3      410  22 428.6724  8.176354 412.4704 444.8744
4      460  23 441.4310  8.663769 424.2632 458.5989
5      420  22 428.6724  8.176354 412.4704 444.8744
6      380  16 352.1209  6.756473 338.7324 365.5093
```

Plotting CI for average response

```
ggplot(predMeans, aes(x = Fat, y = Calories)) +
   geom_point() +
   stat_smooth(method = "lm", se = FALSE) +
   geom_ribbon(aes(ymin = lowMean, ymax = upMean), alpha=.2)
```

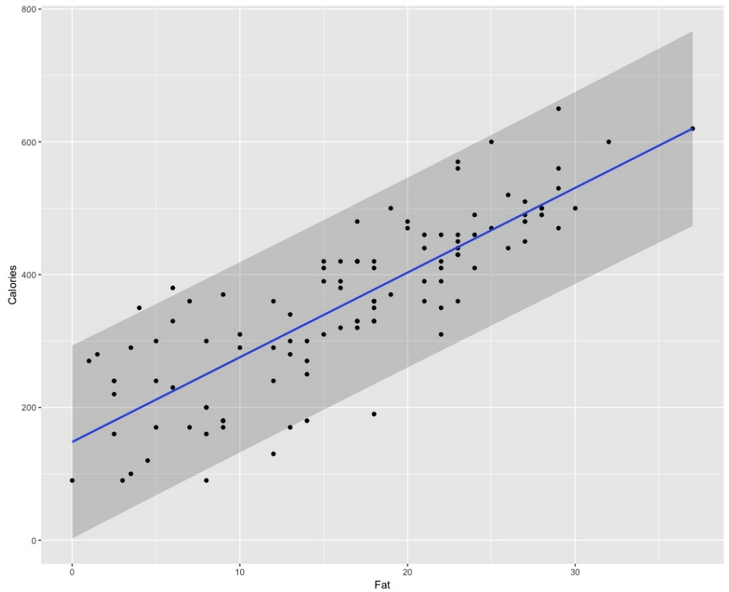


```
alpha <- .05
crit_val <- qt((1-alpha/2), df = nrow(twins) - 2)
FatCal_lm <- lm(Calories ~ Fat, data = starbucks)
FatCal_gl <- glance(FatCal_lm)
FatCal_sig <- pull(FatCal_gl, sigma)
FatCal_pred <- augment(FatCal_lm) %>%
    mutate(.se.pred = sqrt(FatCal_sig^2 + .se.fit^2))
predResp <- FatCal_pred %>%
    mutate(lowResp = .fitted - crit_val*.se.pred, upResp = .fitted + crit_val*.se.pred)
predResp
```

```
A tibble: 113 x 12
   Calories   Fat  .fitted   .se.fit     .resid        .hat   .sigma
      <int> <dbl>    <dbl>     <dbl>      <dbl>       <dbl>    <dbl>
 1      300     5 211.7763 11.473843  88.223722 0.025567957 71.57640
 2      380     6 224.5349 10.823741 155.465125 0.022752704 70.50502
 3      410    22 428.6724  8.176354 -18.672436 0.012983674 72.05959
... with 103 more rows, and 5 more variables: .cooksd <dbl>, .std.resid <dbl>, .se.pred <dbl>,
    lowResp <dbl>, upResp bl>
```

Plotting prediction intervals

```
ggplot(predResp, aes(x = Fat, y = Calories)) +
  geom_point() +
  stat_smooth(method = "lm", se = FALSE) +
  geom_ribbon(aes(ymin = lowResp, ymax = upResp), alpha = .2)
```



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