Categorical inputs

SUPERVISED LEARNING IN R: REGRESSION

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Example: Effect of Diet on Weight Loss

```
WtLoss24 ~ Diet + Age + BMI
```

Diet	Age	BMI	WtLoss24
Med	59	30.67	-6.7
Low-Carb	48	29.59	8.4
Low-Fat	52	32.9	6.3
Med	53	28.92	8.3
Low-Fat	47	30.20	6.3

model.matrix()

```
model.matrix(WtLoss24 ~ Diet + Age + BMI, data = diet)
```

- All numerical values
- Converts categorical variable with N levels into N - 1 indicator variables

Indicator Variables to Represent Categories

Original Data

Diet	Age	...
Med	59	...
Low-Carb	48	...
Low-Fat	52	...
Med	53	...
Low-Fat	47	...

Model Matrix

(Int)	DietLow-Fat	DietMed	...
1	0	1	...
1	0	0	...
1	1	0	...
1	0	1	...
1	1	0	...

- reference level: "Low-Carb"

Interpreting the Indicator Variables

Linear Model:

$$WtLoss24 = \beta_0 + \beta_{DietLow} x_{DietLow} + \beta_{DietMed} x_{DietMed} + \beta_{Age} x_{Age} + \beta_{BMI} x_{BMI}$$

```
lm(WtLoss24 ~ Diet + Age + BMI, data = diet))
```

```
Coefficients:
      (Intercept)       DietLow-Fat     DietMed
         -1.37149          -2.32130    -0.97883
              Age               BMI
          0.12648           0.01262
```

Issues with one-hot-encoding

- Too many levels can be a problem
  - Example: ZIP code (about 40,000 codes)
- Don't hash with geometric methods!

Let's practice!

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Interactions

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Additive relationships

Example of an additive relationship:

```
plant_height ~ bacteria + sun
```

- Change in height is the sum of the effects of bacteria and sunlight
  - Change in sunlight causes same change in height, independent of bacteria
  - Change in bacteria causes same change in height, independent of sunlight

What is an Interaction?

The simultaneous influence of two variables on the outcome is not additive.

```
plant_height ~ bacteria + sun + bacteria:sun
```

- Change in height is more (or less) than the sum of the effects due to sun/bacteria
- At higher levels of sunlight, 1 unit change in bacteria causes more change in height

What is an Interaction?

The simultaneous influence of two variables on the outcome is not additive.

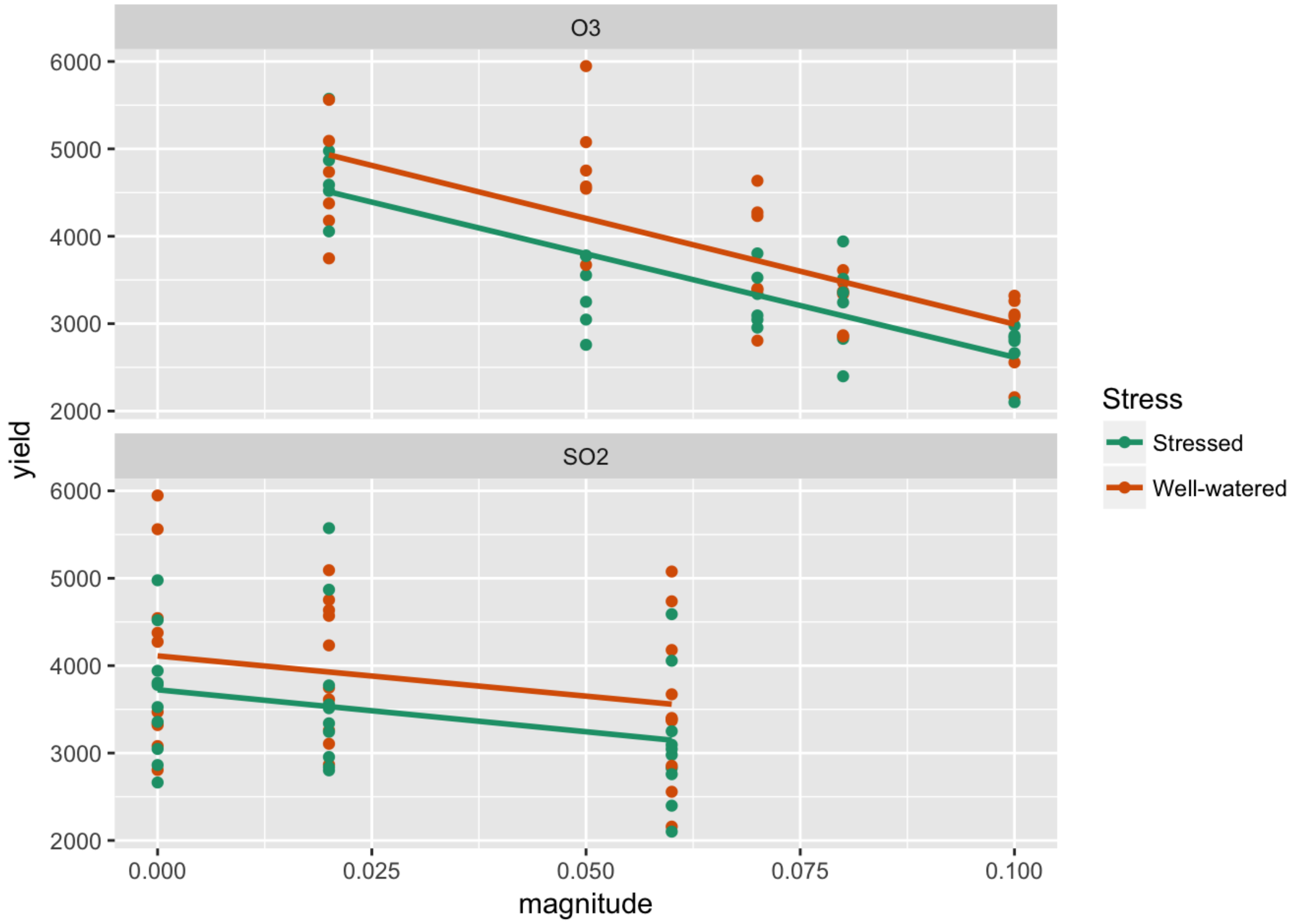
```
plant_height ~ bacteria + sun + bacteria:sun
```

- `sun` : categorical {"sun", "shade"}
- In sun, 1 unit change in bacteria causes *m* units change in height
- In shade, 1 unit change in bacteria causes *n* units change in height

Like two separate models: one for sun, one for shade.

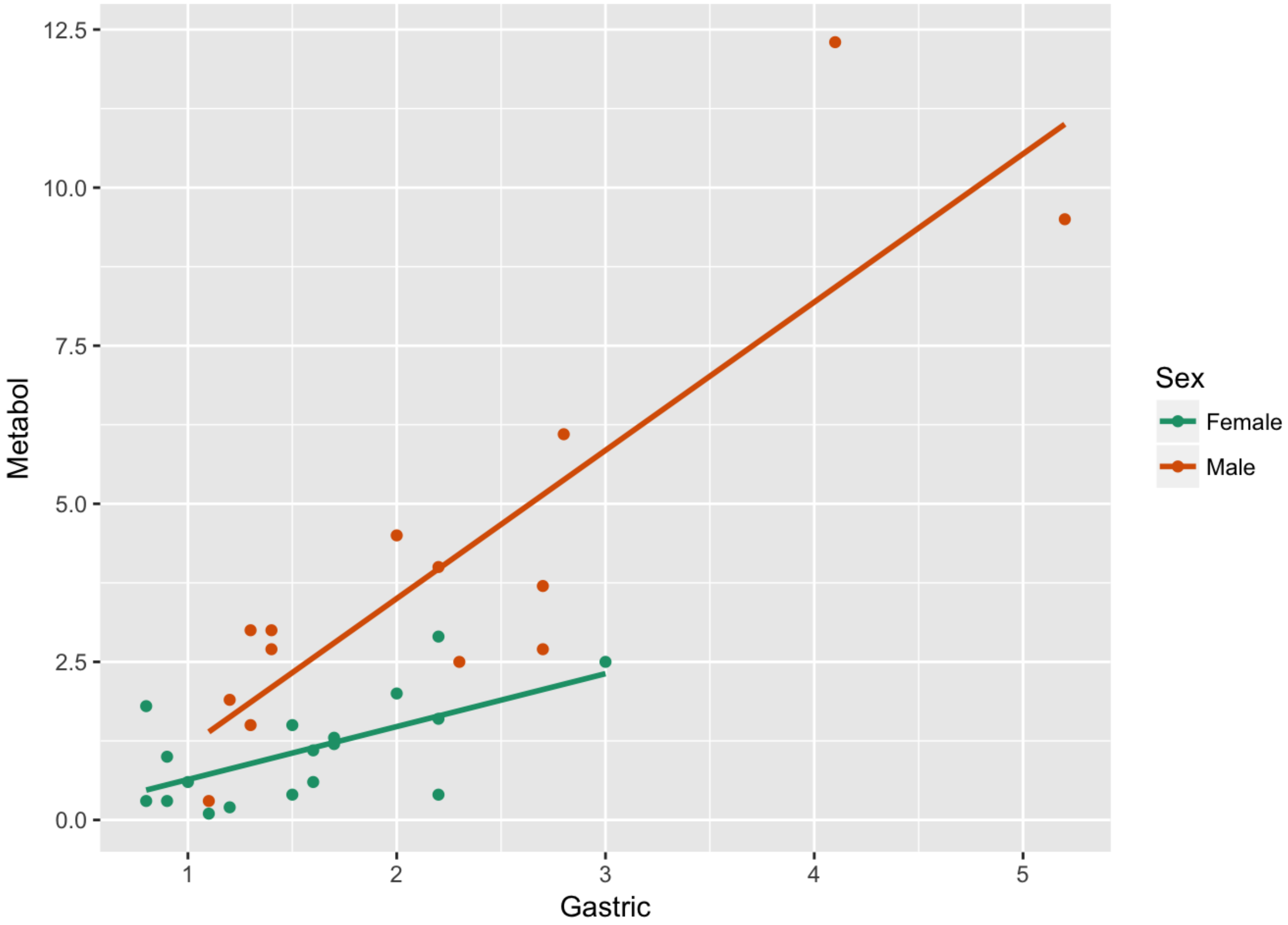
Example of no Interaction: Soybean Yield

```
yield ~ Stress + SO2 + O3
```



Example of an Interaction: Alcohol Metabolism

```
Metabol ~ Gastric + Sex
```



Expressing Interactions in Formulae

- Interaction - Colon (`:`)

```
y ~ a:b
```

- Main effects and interaction - Asterisk (`*`)

```
y ~ a*b
# Both mean the same
y ~ a + b + a:b
```

- Expressing the product of two variables - I

```
y ~ I(a*b)
```

same as $y \propto ab$

Finding the Correct Interaction Pattern

Formula	RMSE (cross validation)
`Metabol ~ Gastric + Sex`	1.46
`Metabol ~ Gastric * Sex`	1.48
`Metabol ~ Gastric + Gastric:Sex`	1.39

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Transforming the response before modeling

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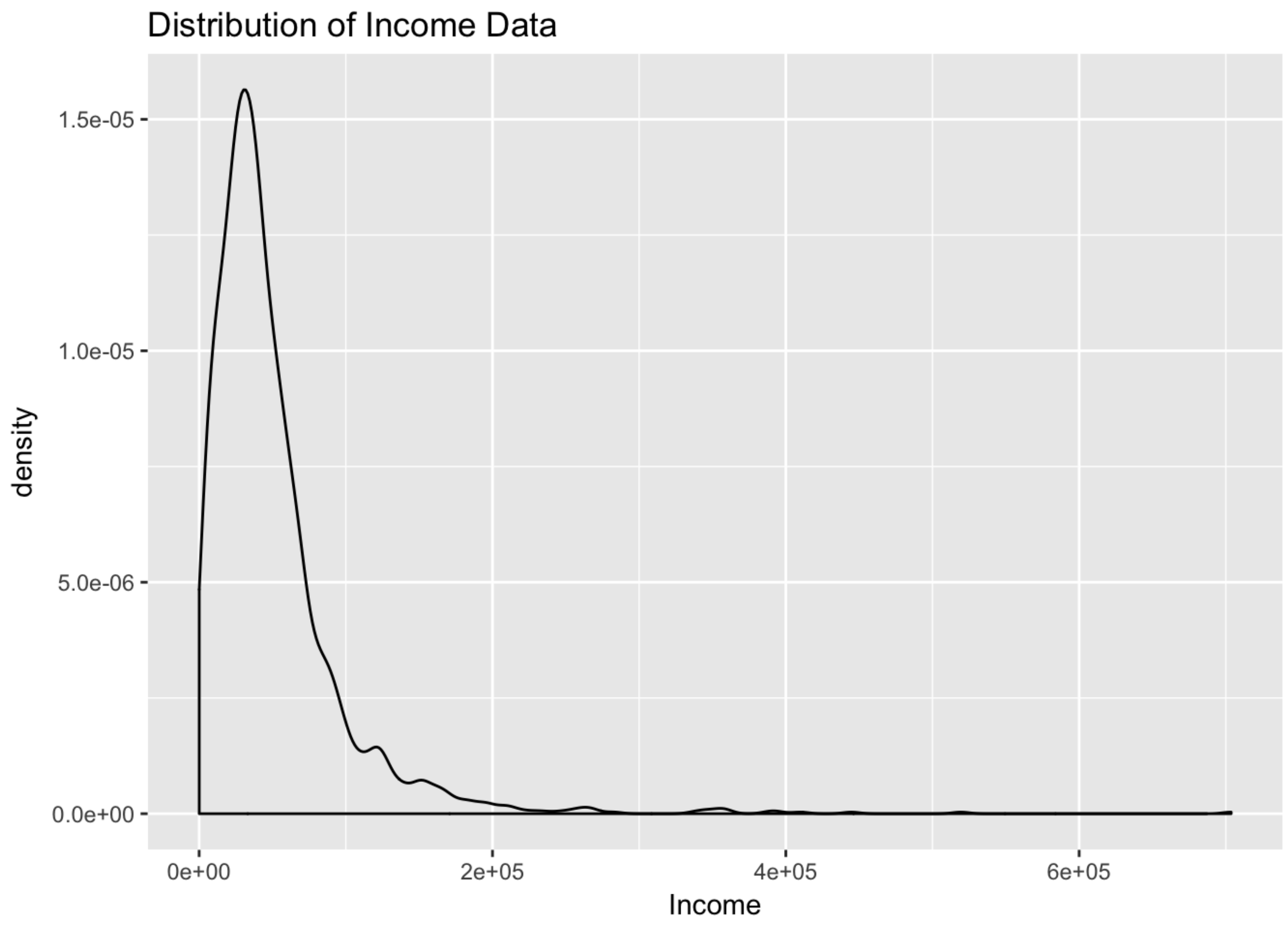
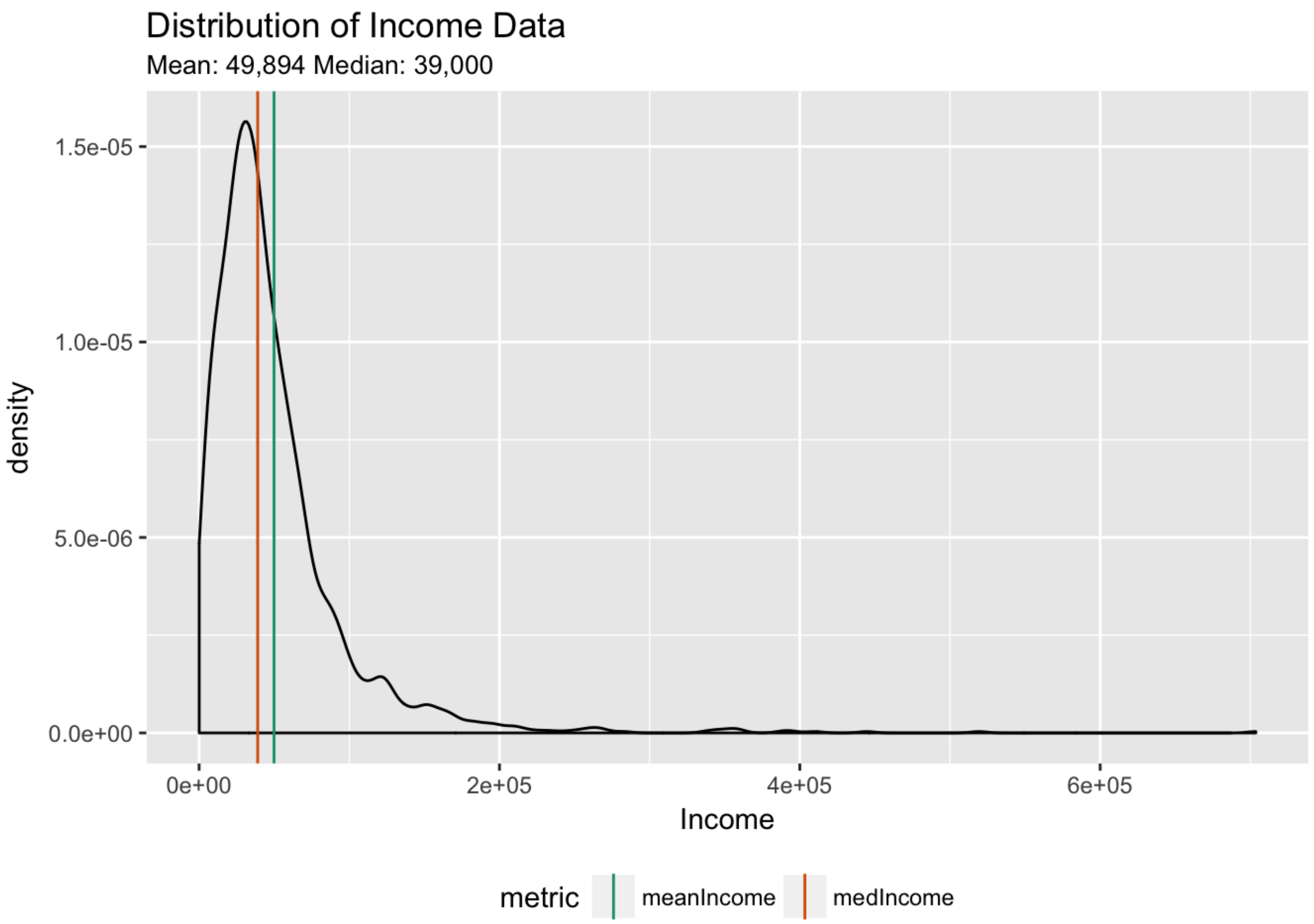
The Log Transform for Monetary Data



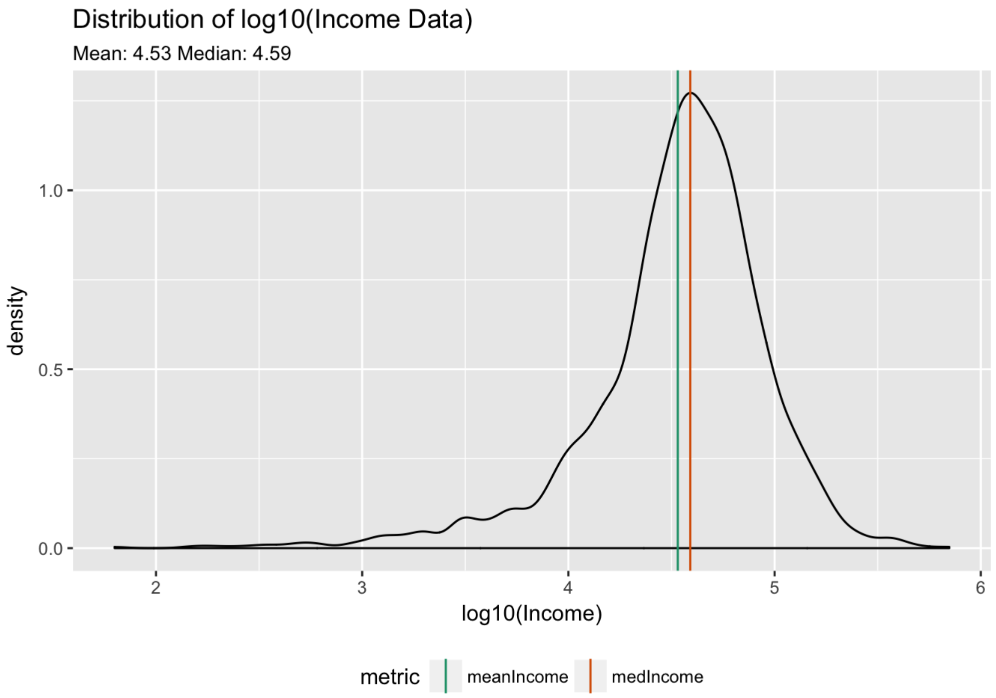
- Monetary values: lognormally distributed
- Long tail, wide dynamic range (60-700K)

Lognormal Distributions



- mean > median (~ 50K vs 39K)
- Predicting the mean will overpredict typical values

Back to the Normal Distribution



For a Normal Distribution:

- mean = median (here: 4.53 vs 4.59)
- more reasonable dynamic range (1.8 - 5.8)

The Procedure

1. Log the outcome and fit a model

```
model <- lm(log(y) ~ x, data = train)
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model <- lm(log(y) ~ x, data = train)
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```
logpred <- predict(model, data = test)
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The Procedure

1. Log the outcome and fit a model

```
model <- lm(log(y) ~ x, data = train)
```

2. Make the predictions in log space

```
logpred <- predict(model, data = test)
```

3. Transform the predictions to outcome space

```
pred <- exp(logpred)
```

Predicting Log-transformed Outcomes: Multiplicative Error

$$log(a) + log(b) = log(ab)$$

$$log(a) - log(b) = log(a/b)$$

- Multiplicative error: $pred/y$
- Relative error: $(pred - y)/y = \frac{pred}{y} - 1$

Reducing multiplicative error reduces relative error.

Root Mean Squared Relative Error

RMS-relative error = $\sqrt{\overline{(\frac{pred-y}{y})^2}}$

- Predicting log-outcome reduces RMS-relative error
- But the model will often have larger RMSE

Example: Model Income Directly

```
modIncome <- lm(Income ~ AFQT + Educ, data = train)
```

- `AFQT` : Score on proficiency test 25 years before survey
- `Educ` : Years of education to time of survey
- `Income` : Income at time of survey

Model Performance

```
test %>%
+     mutate(pred = predict(modIncome, newdata = test),
+            err = pred - Income) %>%
+     summarize(rmse = sqrt(mean(err^2)),
+               rms.relerr = sqrt(mean((err/Income)^2)))
```

RMSE	RMS-relative error
36,819.39	3.295189

Model log(Income)

```
modLogIncome <- lm(log(Income) ~ AFQT + Educ, data = train)
```

Model Performance

```
test %>%
+     mutate(predlog = predict(modLogIncome, newdata = test),
+              pred = exp(predlog),
+              err = pred - Income) %>%
+     summarize(rmse = sqrt(mean(err^2)),
+                rms.relerr = sqrt(mean((err/Income)^2)))
```

RMSE	RMS-relative error
38,906.61	2.276865

Compare Errors

`log(Income)` model: smaller RMS-relative error, larger RMSE

Model	RMSE	RMS-relative error
On `Income`	36,819.39	3.295189
On `log(Income)`	38,906.61	2.276865

Let's practice!

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Why To Transform Input Variables

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  - $Intelligence \sim \frac{mass.brain}{mass.body^{2/3}}$

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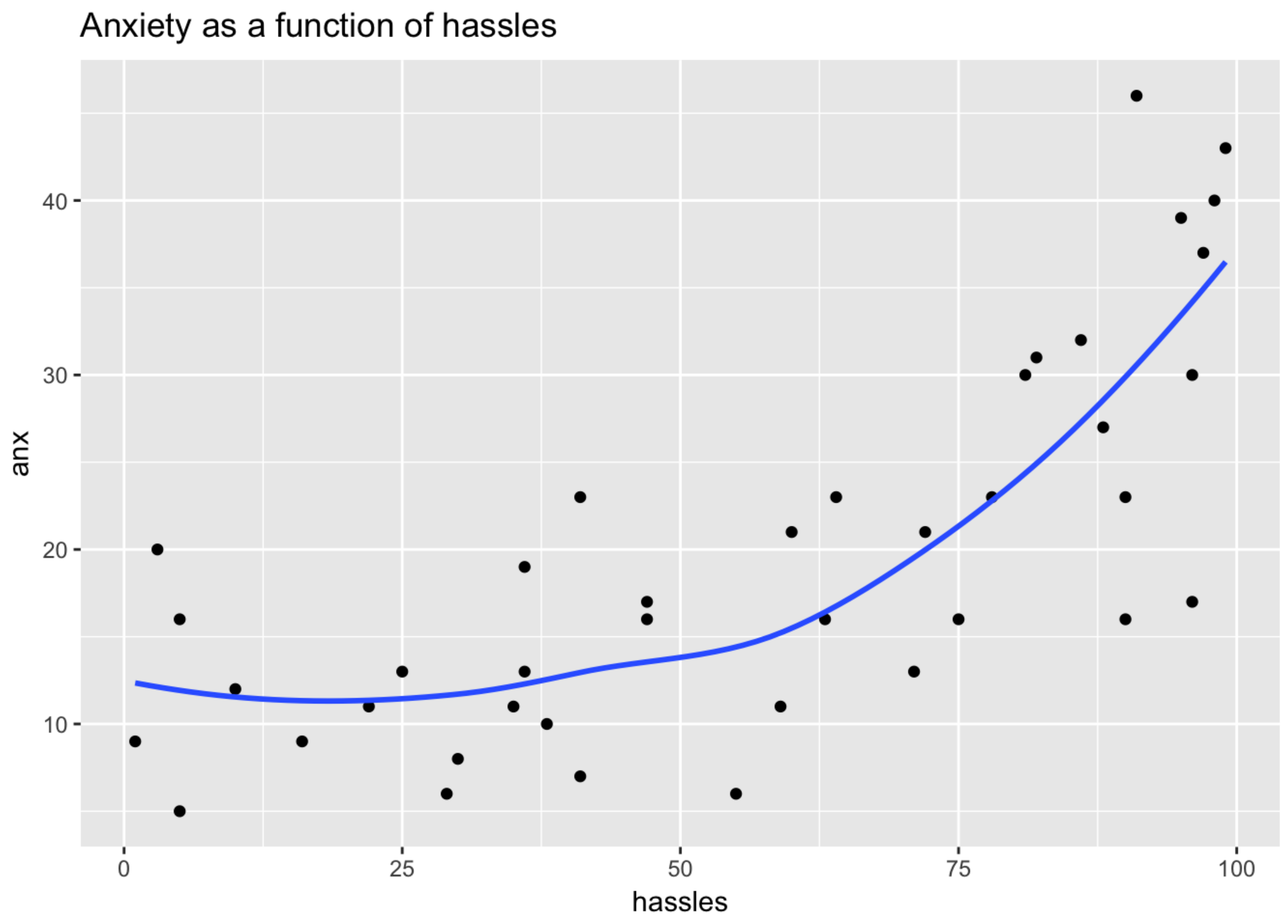
  - $Intelligence \sim \frac{mass.brain}{mass.body^{2/3}}$
- Pragmatic reasons
  - Log transform to reduce dynamic range
  - Log transform because meaningful changes in variable are multiplicative

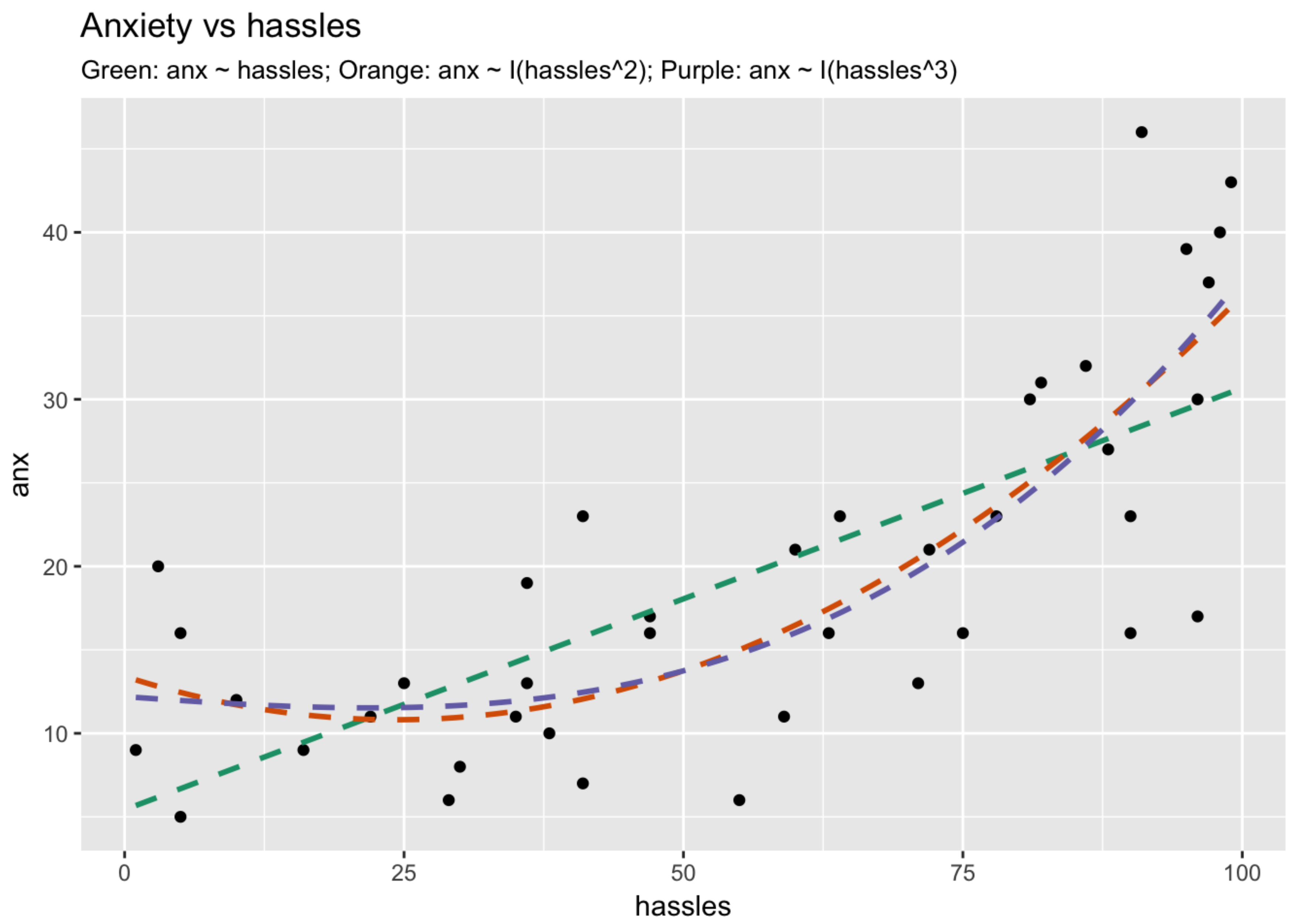
Why To Transform Input Variables

- Domain knowledge/synthetic variables
  - $Intelligence \sim \frac{mass.brain}{mass.body^{2/3}}$
- Pragmatic reasons
  - Log transform to reduce dynamic range
  - Log transform because meaningful changes in variable are multiplicative
  - y approximately linear in $f(x)$ rather than in x

Example: Predicting Anxiety



Transforming the hassles variable



Different possible fits

Which is best?

- `anx ~ I(hassles^2)`
- `anx ~ I(hassles^3)`
- `anx ~ I(hassles^2) + I(hassles^3)`
- `anx ~ exp(hassles)`
- ...

`I()` : treat an expression literally (not as an interaction)

Compare different models

Linear, Quadratic, and Cubic models

```
mod_lin <- lm(anx ~ hassles, hassleframe)
summary(mod_lin)$r.squared
```

```
0.5334847
```

```
mod_quad <- lm(anx ~ I(hassles^2), hassleframe)
summary(mod_quad)$r.squared
```

```
0.6241029
```

```
mod_tritic <- lm(anx ~ I(hassles^3), hassleframe)
summary(mod_tritic)$r.squared
```

```
0.6474421
```

Compare different models

Use cross-validation to evaluate the models

Model	RMSE
Linear ($hassles$)	7.69
Quadratic ($hassles^2$)	6.89
Cubic ($hassles^3$)	6.70

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