Trend spotting!

TIME SERIES ANALYSIS IN R

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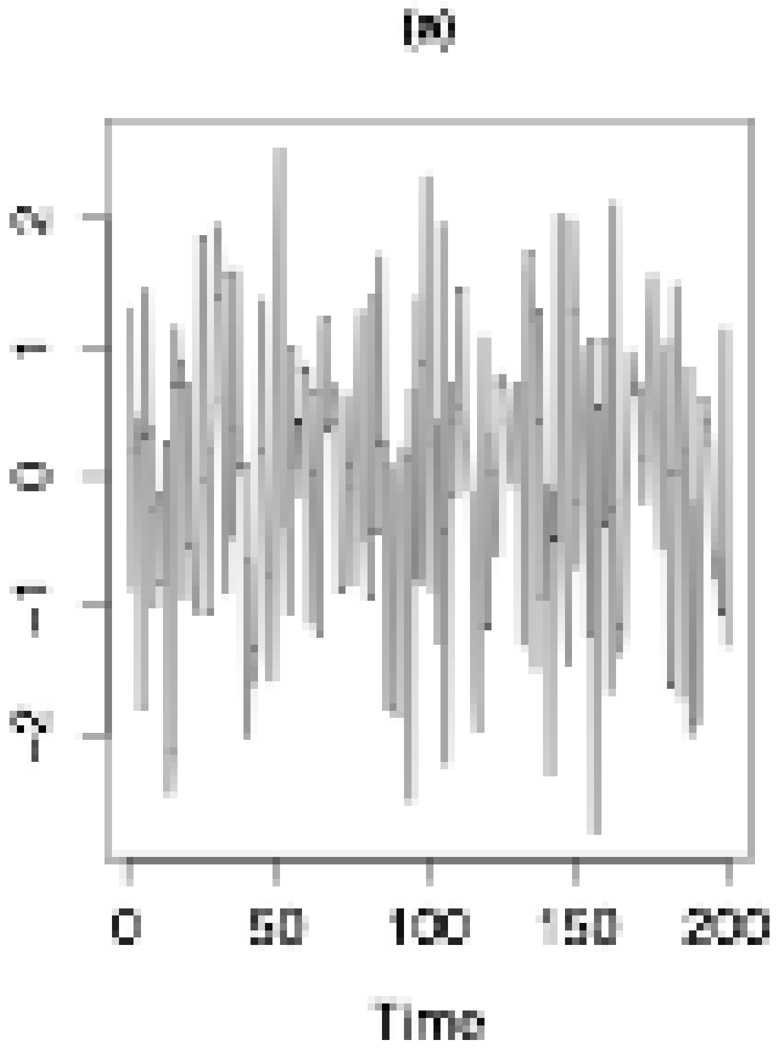
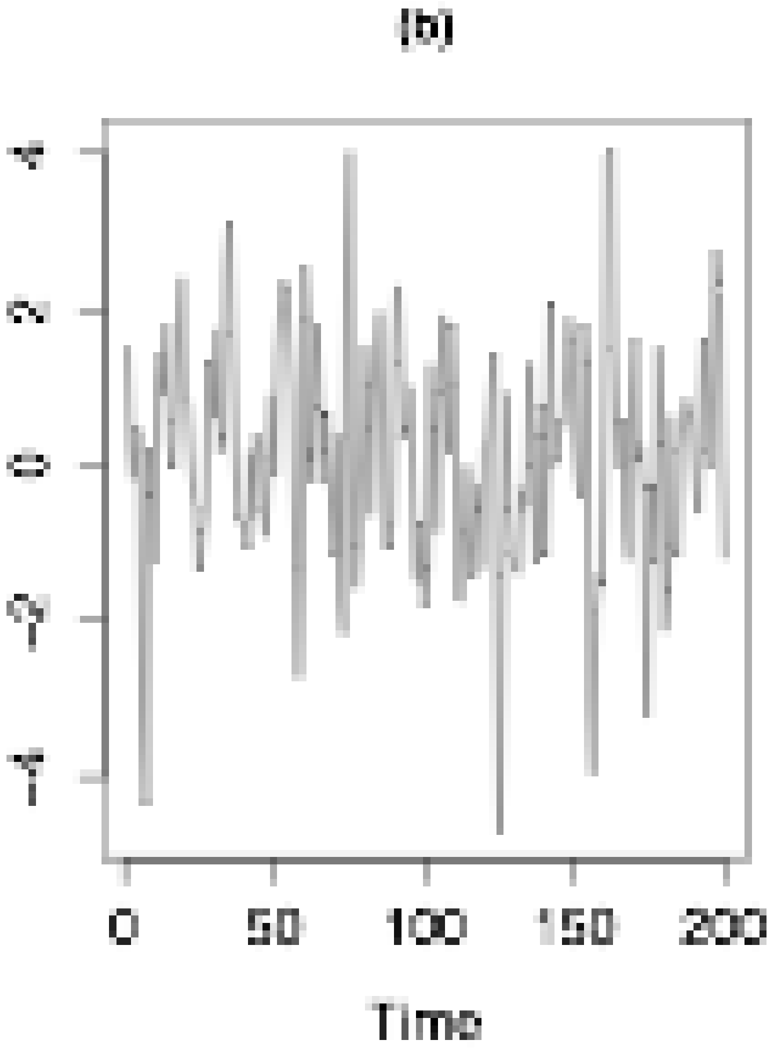
Trends

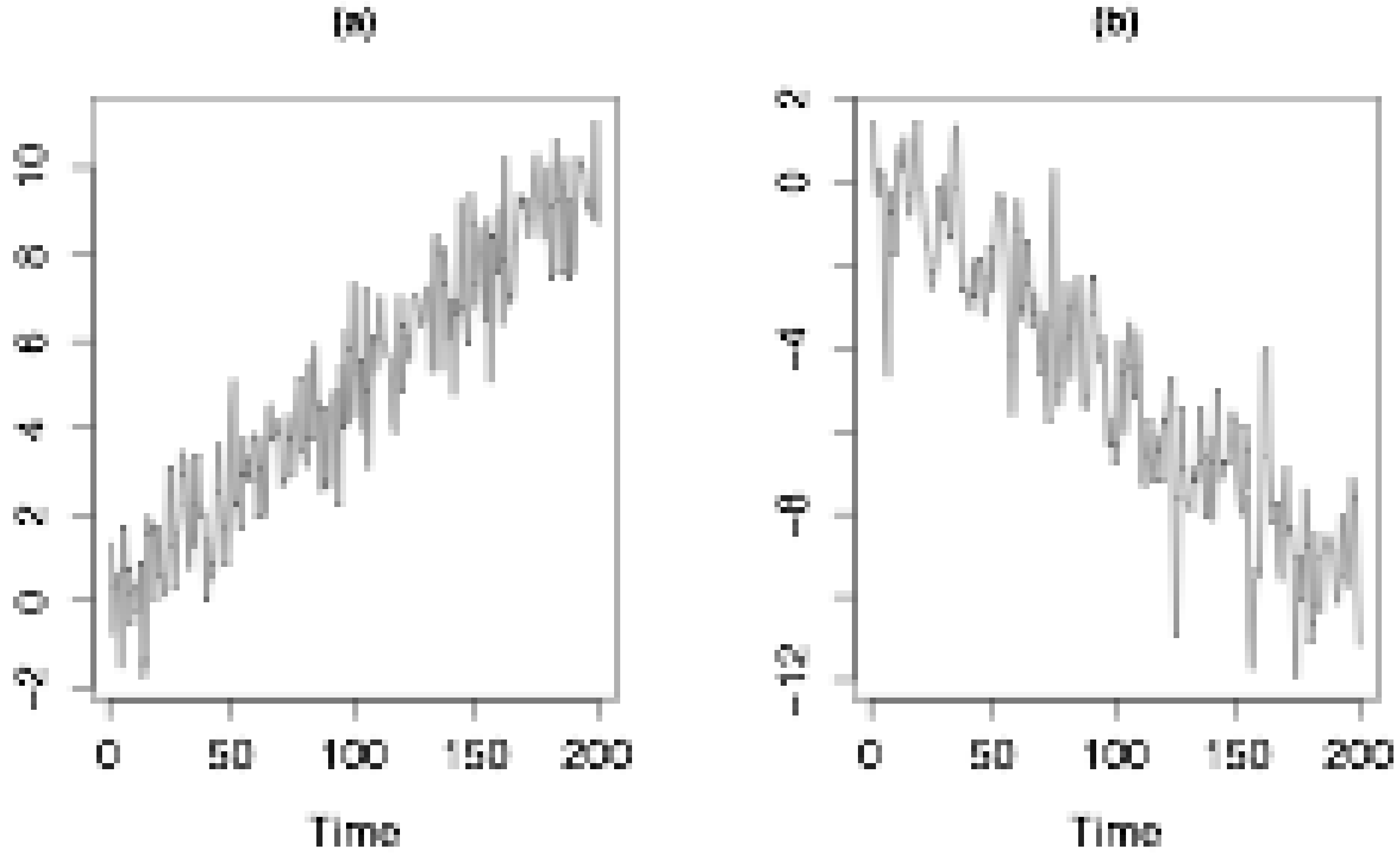
Some time series do not exhibit any clear trends over time:





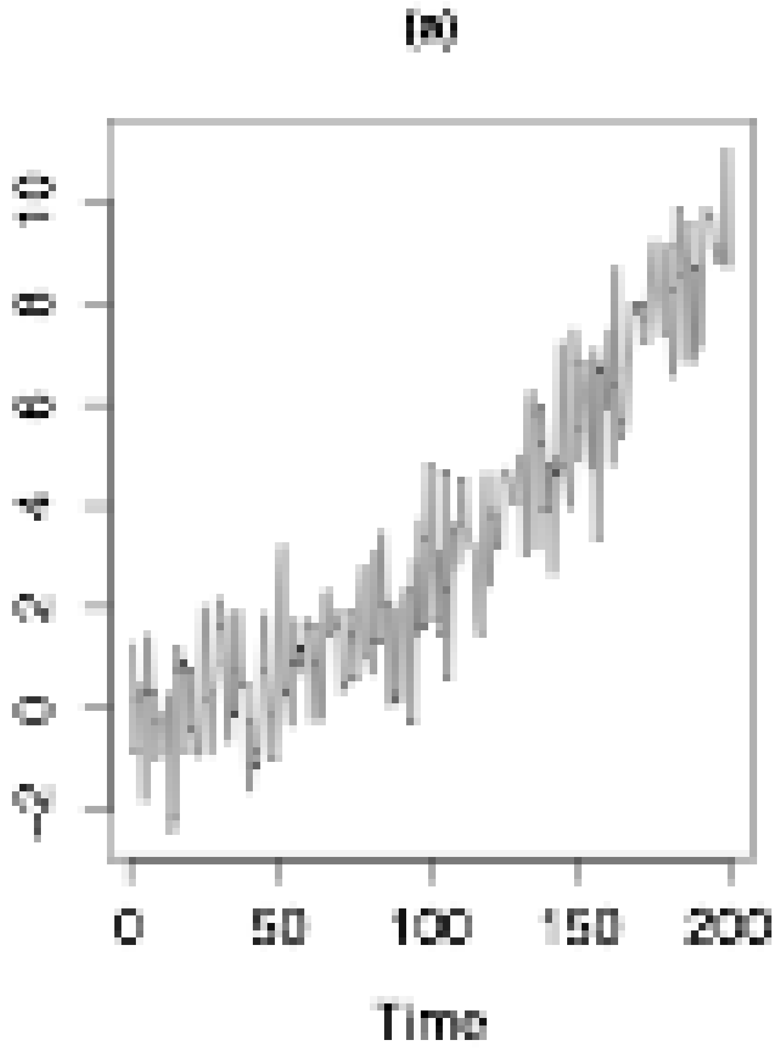
Trends: linear

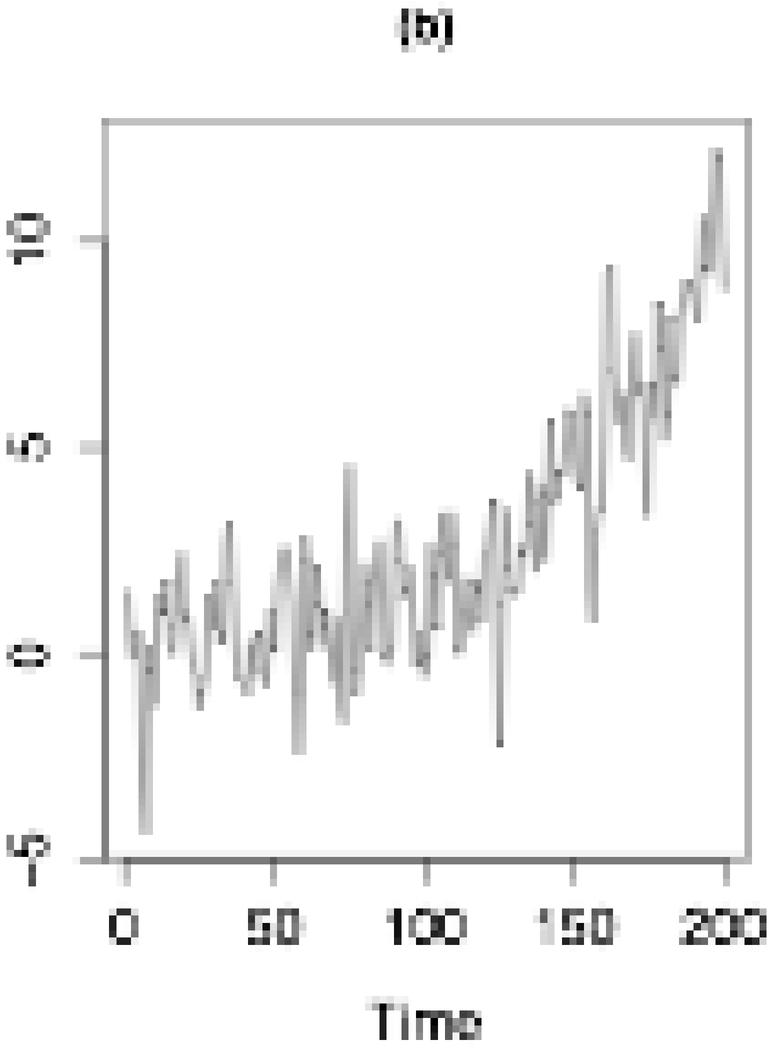
Examples of linear trends over time:



Trends: rapid growth

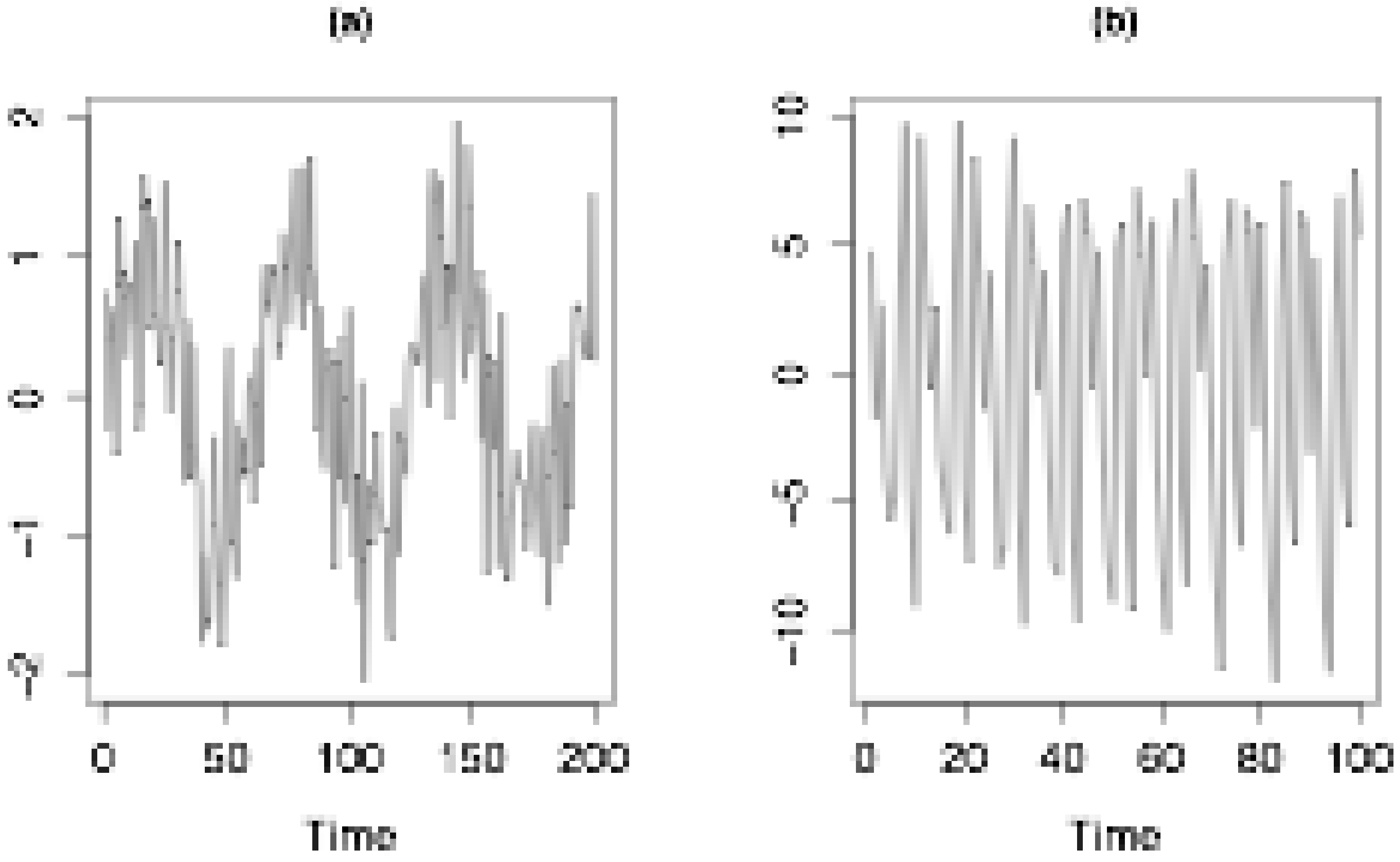
Examples of rapid growth trends over time:





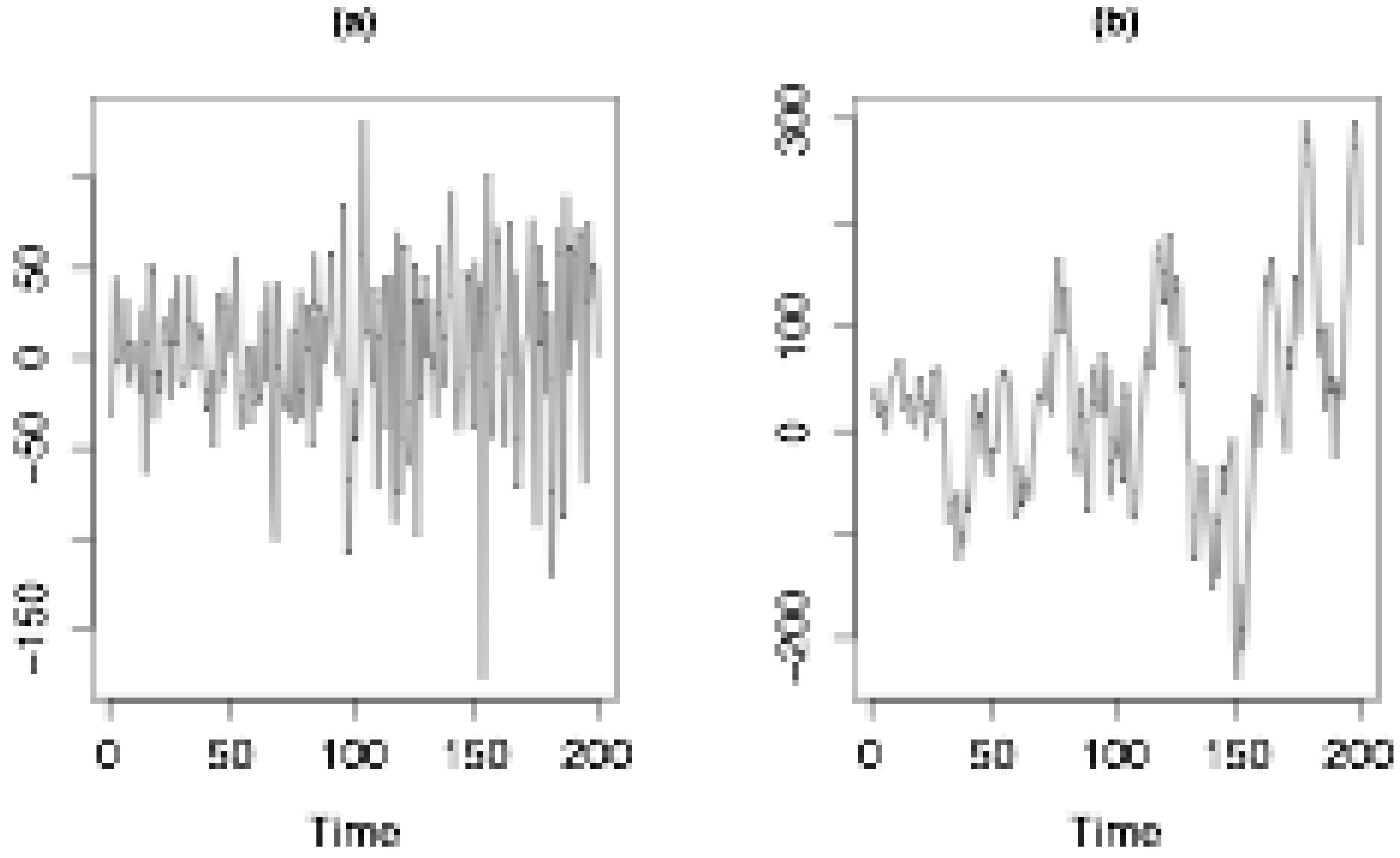
Trends: periodic

Examples of periodic or sinusoidal trends over time:



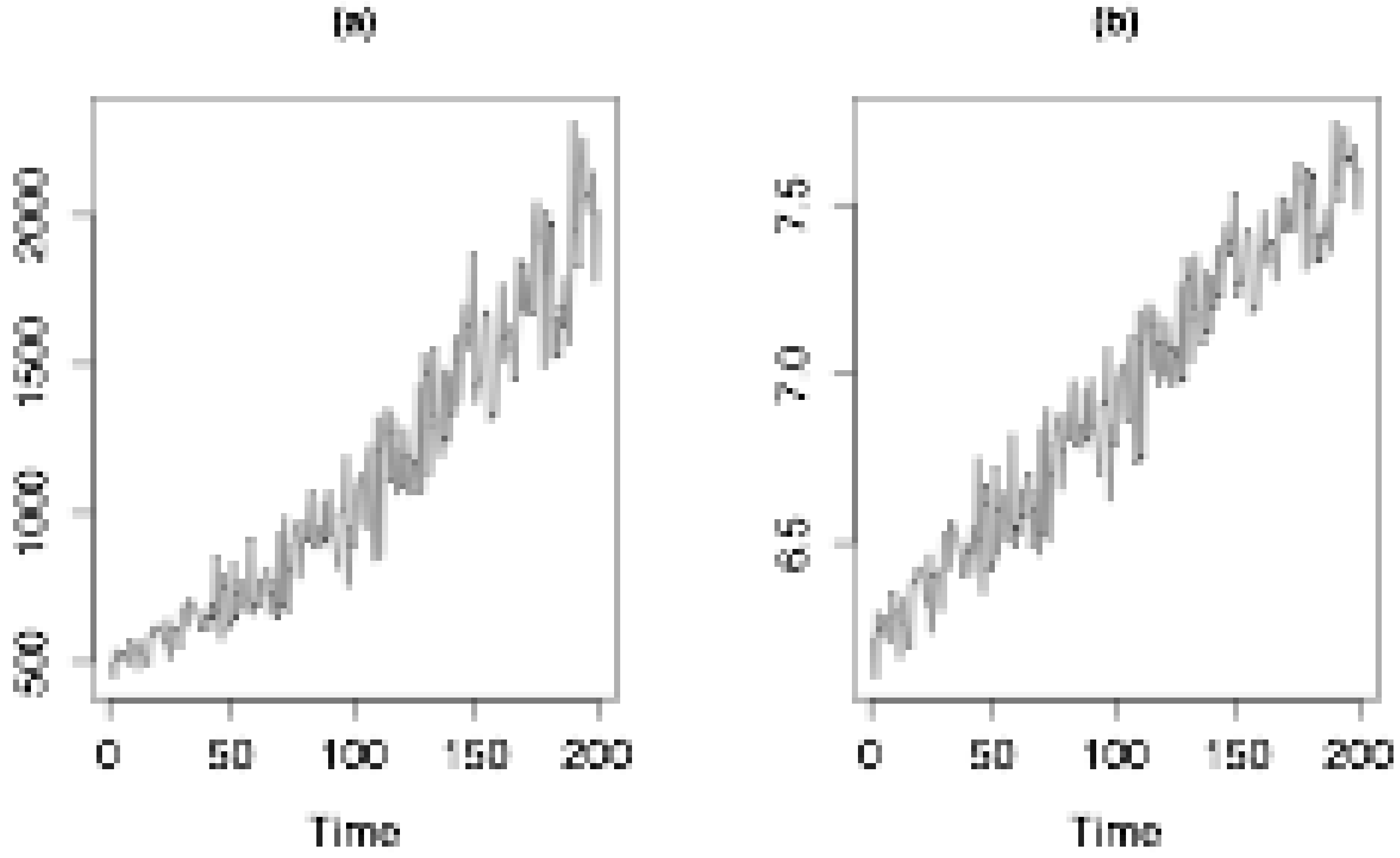
Trends: variance

Examples of increasing variance trends over time:



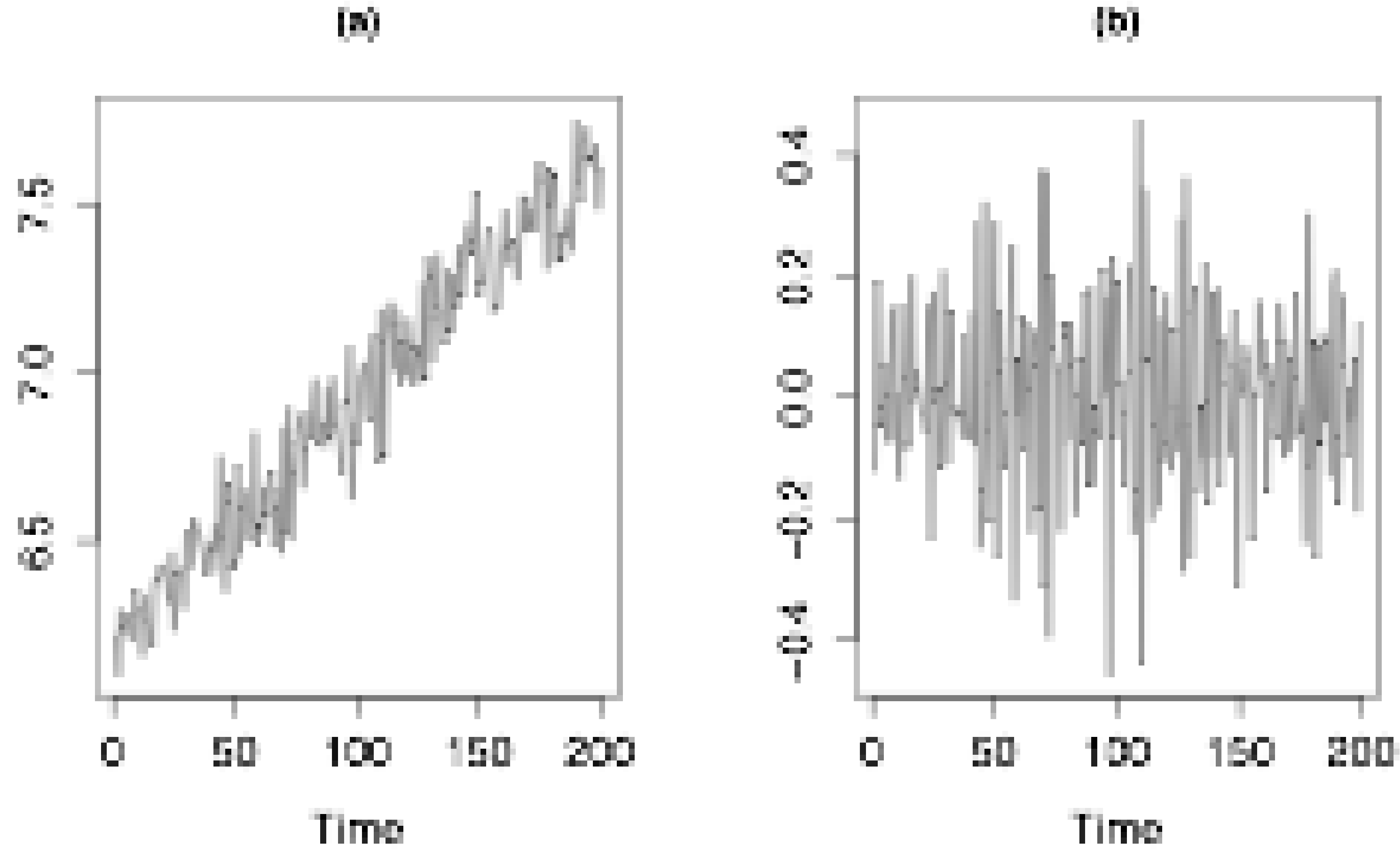
Sample transformations: log()

The `log()` function can linearize a rapid growth trend:



Sample transformations: diff()

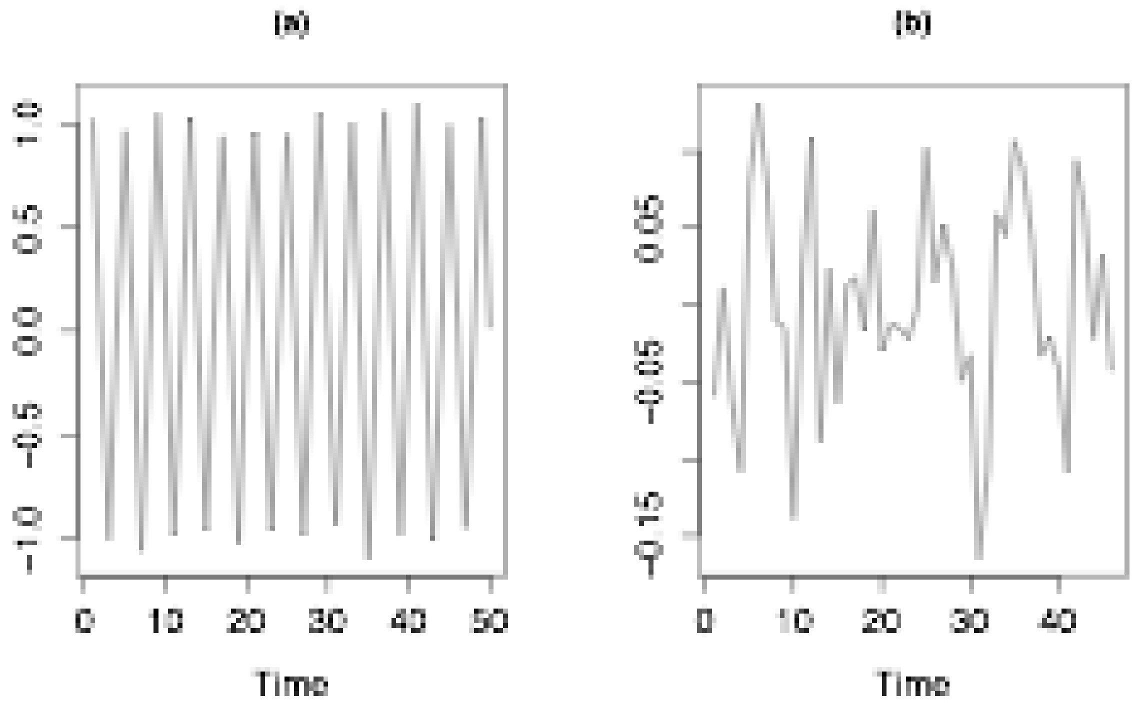
The `diff()` function can remove a linear trend:



Sample transformations: diff(..., s)

The `diff(…, s)` function, or seasonal difference transformation, can remove periodic trends.

```
diff(x, s = 4)
```



Let's practice!

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The white noise (WN) model

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White noise

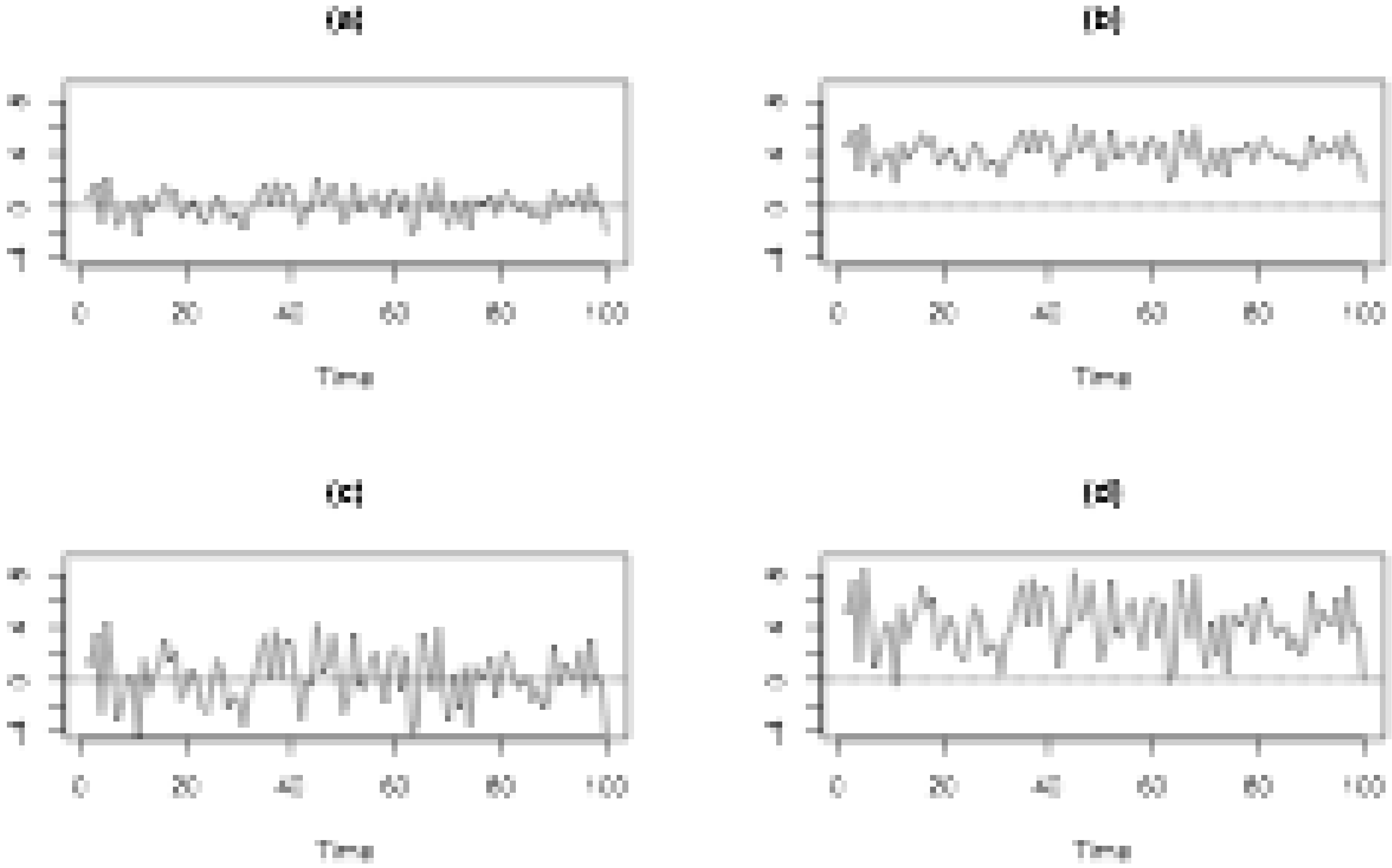
White Noise (WN) is the simplest example of a stationary process.

A *weak white* noise process has:

- A fixed, constant mean.
- A fixed, constant variance.
- No correlation over time.

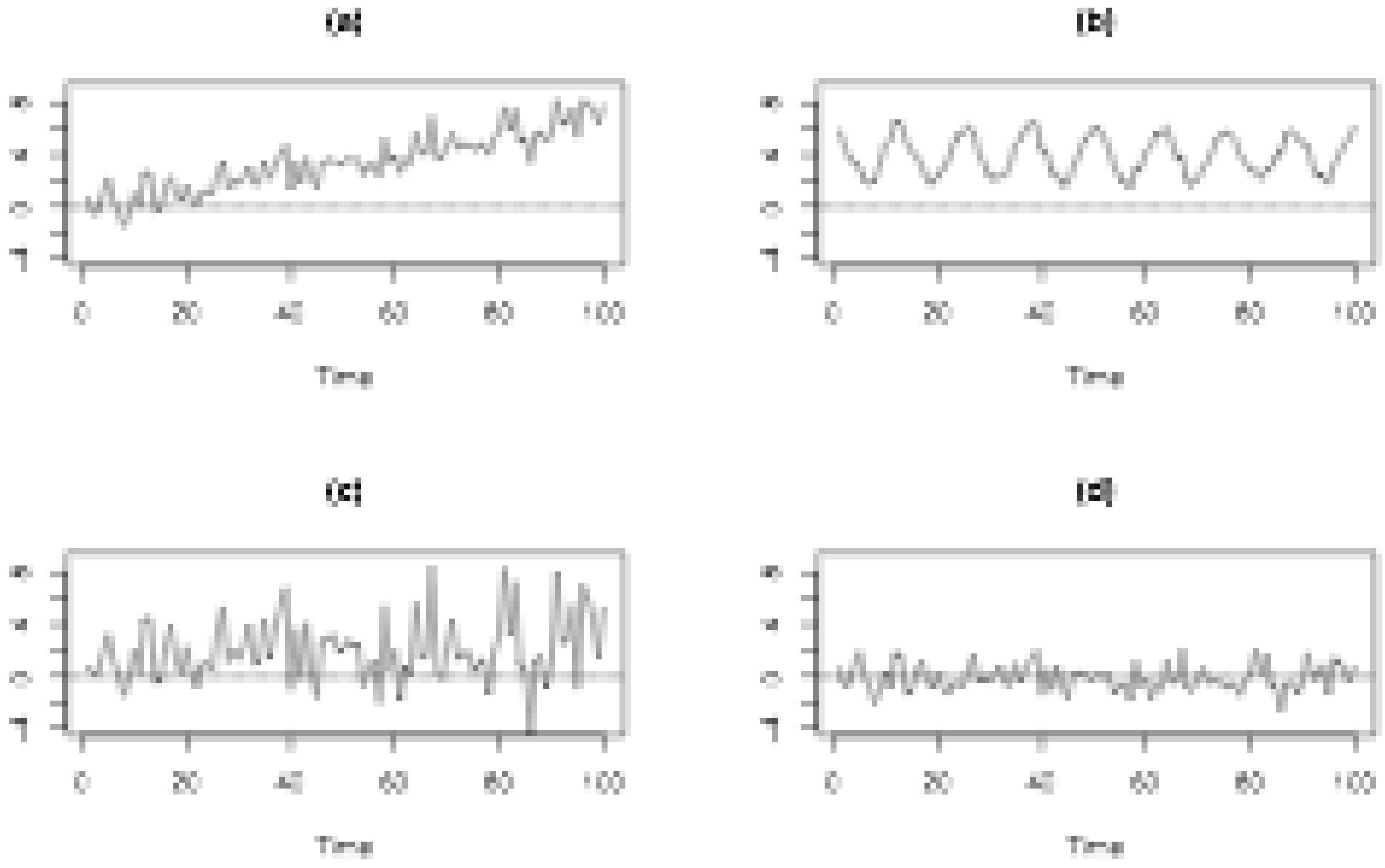
White noise

Time series plots of White Noise:



White noise

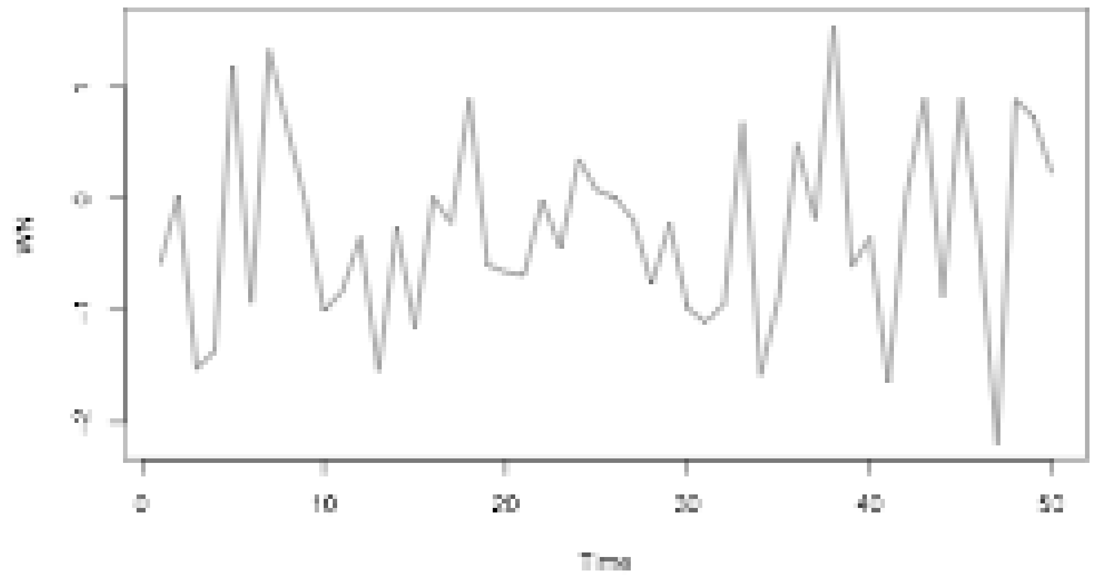
Time series plots of White Noise?



```r
# Simulate n = 50 observations from the WN model
WN_1 <- arima.sim(model = list(order = c(0, 0, 0)), n = 50)
head(WN_1)
```

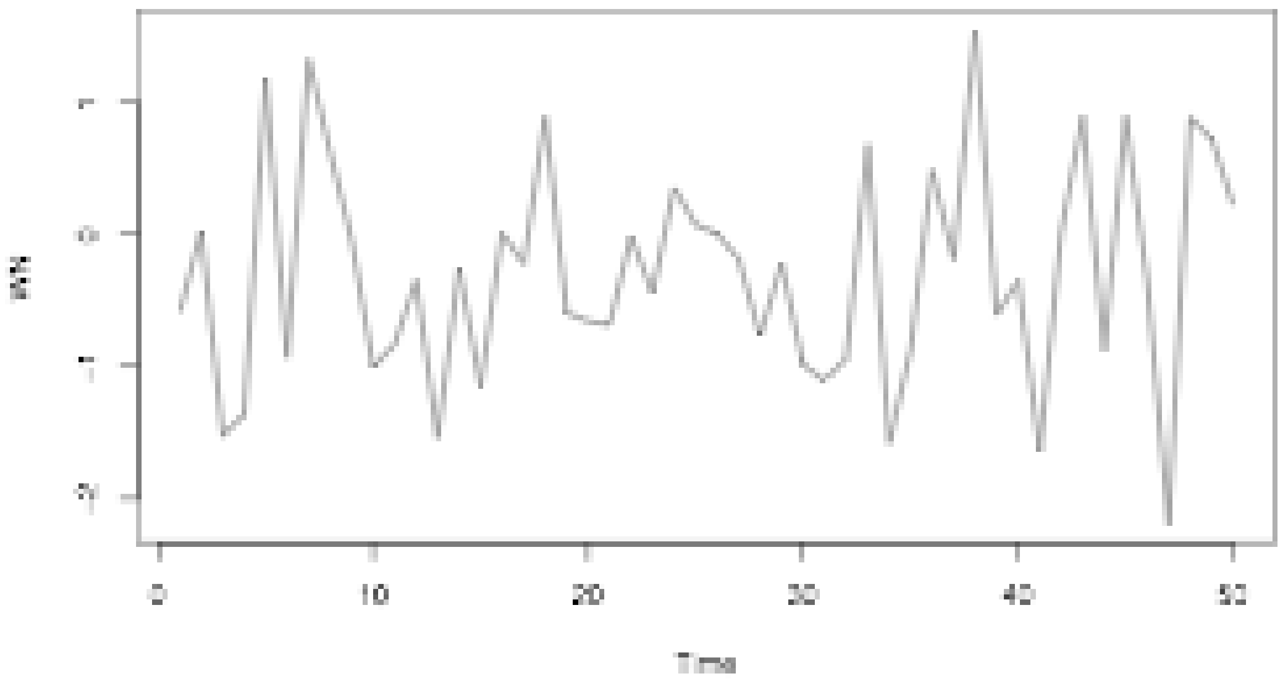
```
-0.005052984  0.042669765  3.261154066
 2.486431235  0.283119322  1.543525773
```

```r
ts.plot(WN_1)
```



```r
# Simulate from the WN model with mean = 4, sd = 2
WN_2 <- arima.sim(model = list(order = c(0, 0, 0)),
                  n = 50, mean = 4, sd = 2)
```

```r
ts.plot(WN_2)
```



Estimating white noise

```r
# Fit the WN model with
# arima()
arima(WN_2,
      order = c(0, 0, 0))
```

```
Coefficients:
      intercept
         4.0739
s.e.     0.2698
sigma^2 estimated as 3.639
```

```r
# Calculate the sample
# mean and sample variance
# of WN
mean(WN_2)
```

```
4.0739
```

```r
var(WN_2)
```

```
3.713
```

Let's practice!

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The random walk (RW) model

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Random walk

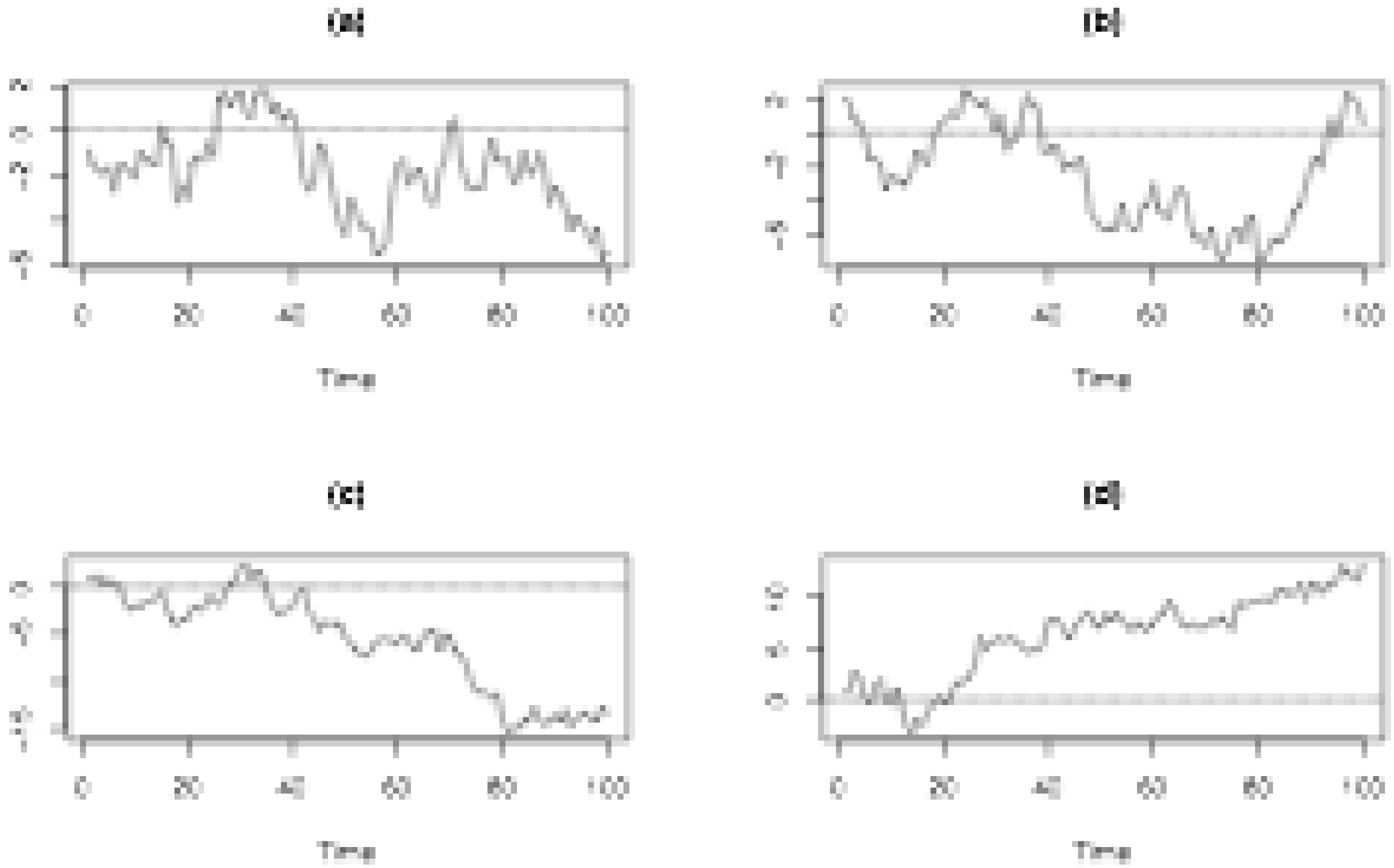
Random Walk (RW) is a simple example of a non-stationary process.

A random walk has:

- No specified mean or variance.
- Strong dependence over time.
- Its changes or increments are white noise (WN).

Random walk

Time series plots of Random Walk:



Random walk

The random walk recursion:

$$Today = Yesterday + Noise$$

More formally:

$$Y_t = Y_{t-1} + \epsilon_t$$

where ϵ_t is mean zero white noise (WN).

- Simulation requires an initial point Y_0.
- Only one parameter, the WN variance σ_{ϵ}^2.

Random walk - I

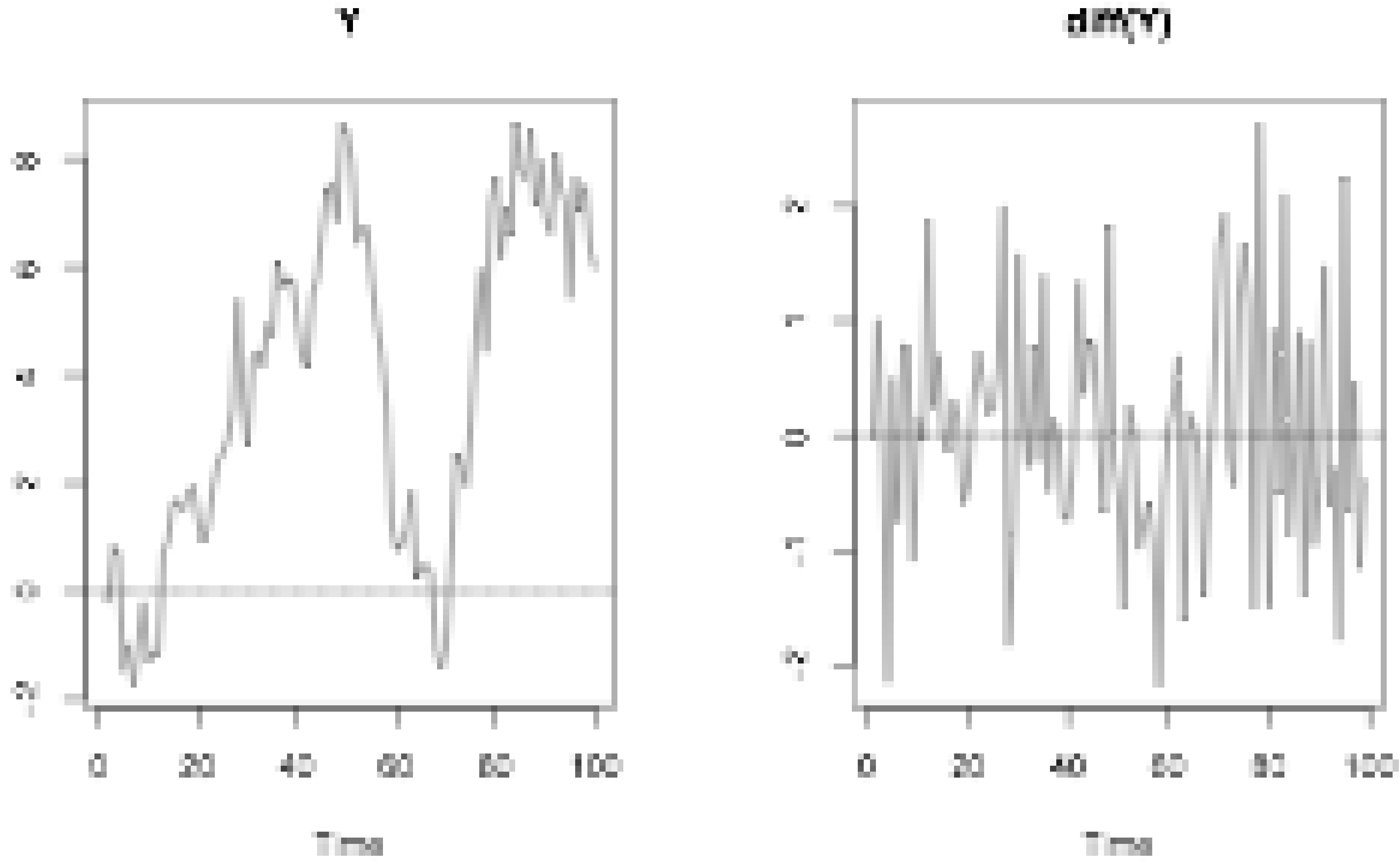
The random walk process:

$$Y_t = Y_{t-1} + \epsilon_t$$

where ϵ_t is mean zero WN

As $Y_t - Y_{t-1} = \epsilon_t \rightarrow$ `diff(Y)` is WN

Random walk - II



Random walk with drift - I

The random walk with a drift:

$$Today = Constant + Yesterday + Noise$$

More formally:

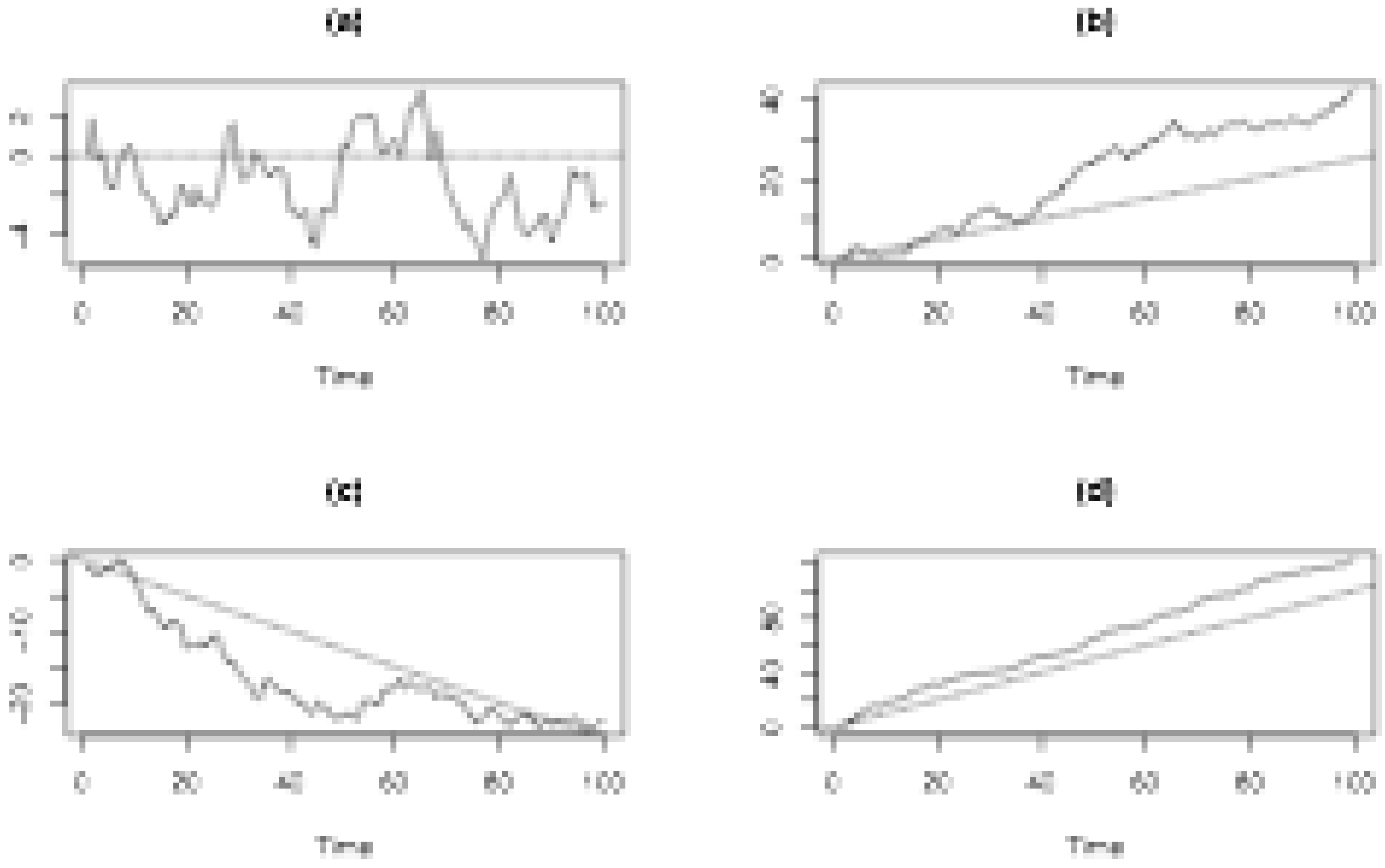
$$Y_t = c + Y_{t-1} + \epsilon_t$$

where ϵ_t is mean zero white noise (WN).

- Two parameters, the constant c , and the WN variance σ^2_ϵ.
- $Y_t - Y_{t-1} =$? $\rightarrow$ WN with mean c!

Random walk with drift - II

Time series plots of Random Walk with drift:



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Stationary processes

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Stationarity

- Stationary models are parsimonious.
- Stationary processes have distributional stability over time.

Observed time series:

- Fluctuate randomly.
- But behave similarly from one time period to the next.

Weak stationarity - I

Weak stationary: mean, variance, covariance constant over time.

Y_1, Y_2, ...is a *weakly stationary* process if:

- Mean μ of Y_t is same (constant) for all t.
- Variance σ^2 of Y_t is same (constant) for all t.
- And....

Weak stationarity - II

Covariance of Y_t and Y_s is same (constant) for all $|t - s| = h$, for all$h$.

$$Cov(Y_2, Y_5) = Cov(Y_7, Y_{10})$$

since each pair is separated by three units of time.

Stationarity: why?

A stationary process can be modeled with **fewer parameters**.

For example, we do not need a different expectation for each Y_t; rather they all have a common expectation, μ.

- Estimate μ accurately by $\bar{y}$.

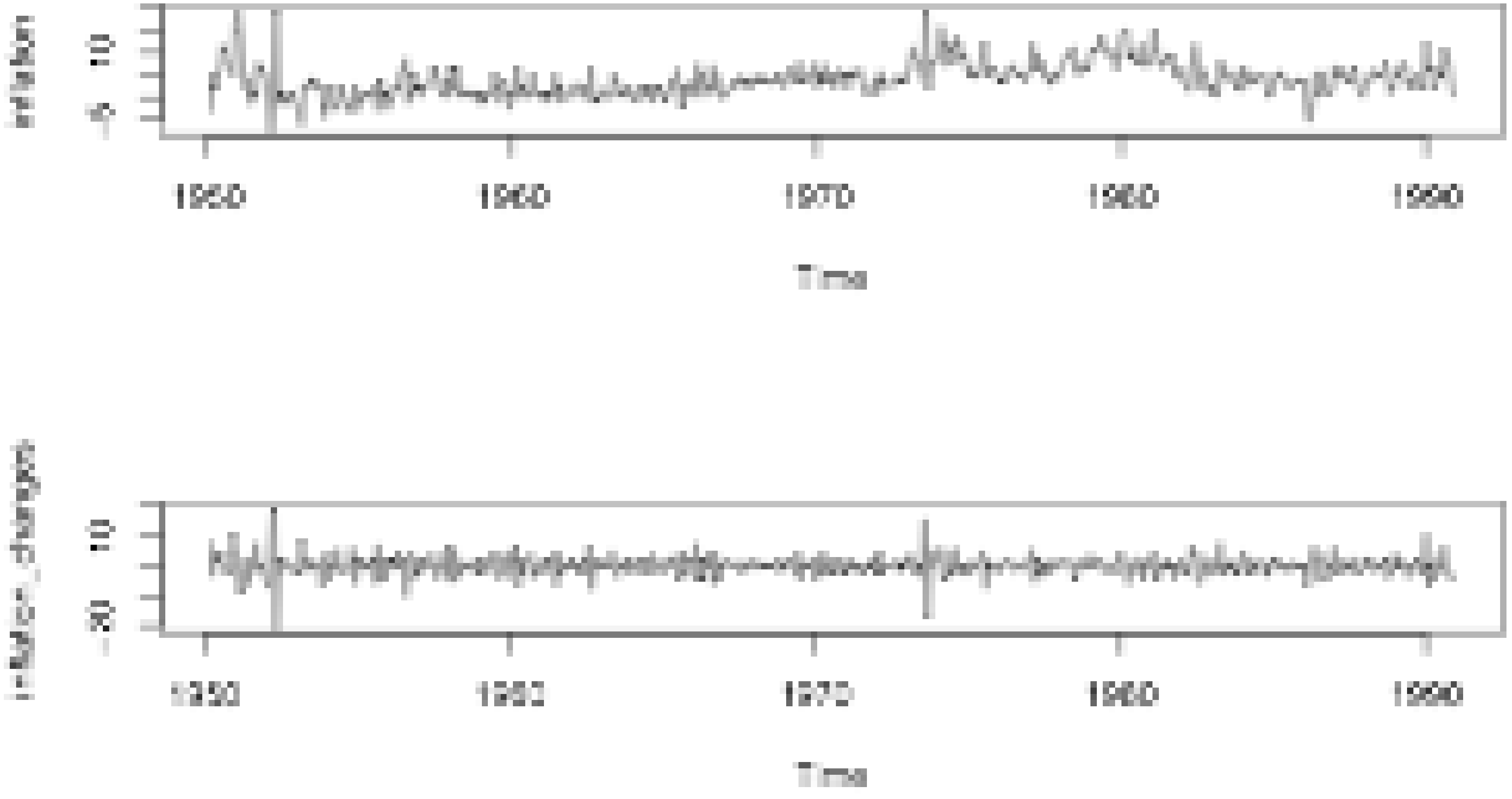
Stationarity: when?

Many financial time series do not exhibit stationarity, however:

- The **changes** in the series are often approximately stationary.
- A stationary series should show random oscillation around some fixed level; a phenomenon called **mean-reversion**.

Stationarity example

Inflation rates and *changes* in inflation rates:



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