The autoregressive model

TIME SERIES ANALYSIS IN R

David S. Matteson
Associate Professor at Cornell University

The autoregressive model - I

The Autoregressive (AR) recursion:

$$Today = Constant + Slope * Yesterday + Noise$$

Mean centered version:

$$(Today - Mean) =$$

$$Slope * (Yesterday - Mean) + Noise$$

The autoregressive model - II

$$(Today - Mean) =$$

$$Slope * (Yesterday - Mean) + Noise$$

More formally:

$$Y_t - \mu = \phi(Y_{t-1} - \mu) + \epsilon_t$$

where ϵ_t is mean zero white noise (WN).

- The mean μ
- The slope ϕ
- The WN variance σ^2

AR processes - I

$$Y_t - \mu = \phi(Y_{t-1} - \mu) + \epsilon_t$$

- If slope $\phi = 0$ then: $Y_t = \mu + \epsilon_t$ and

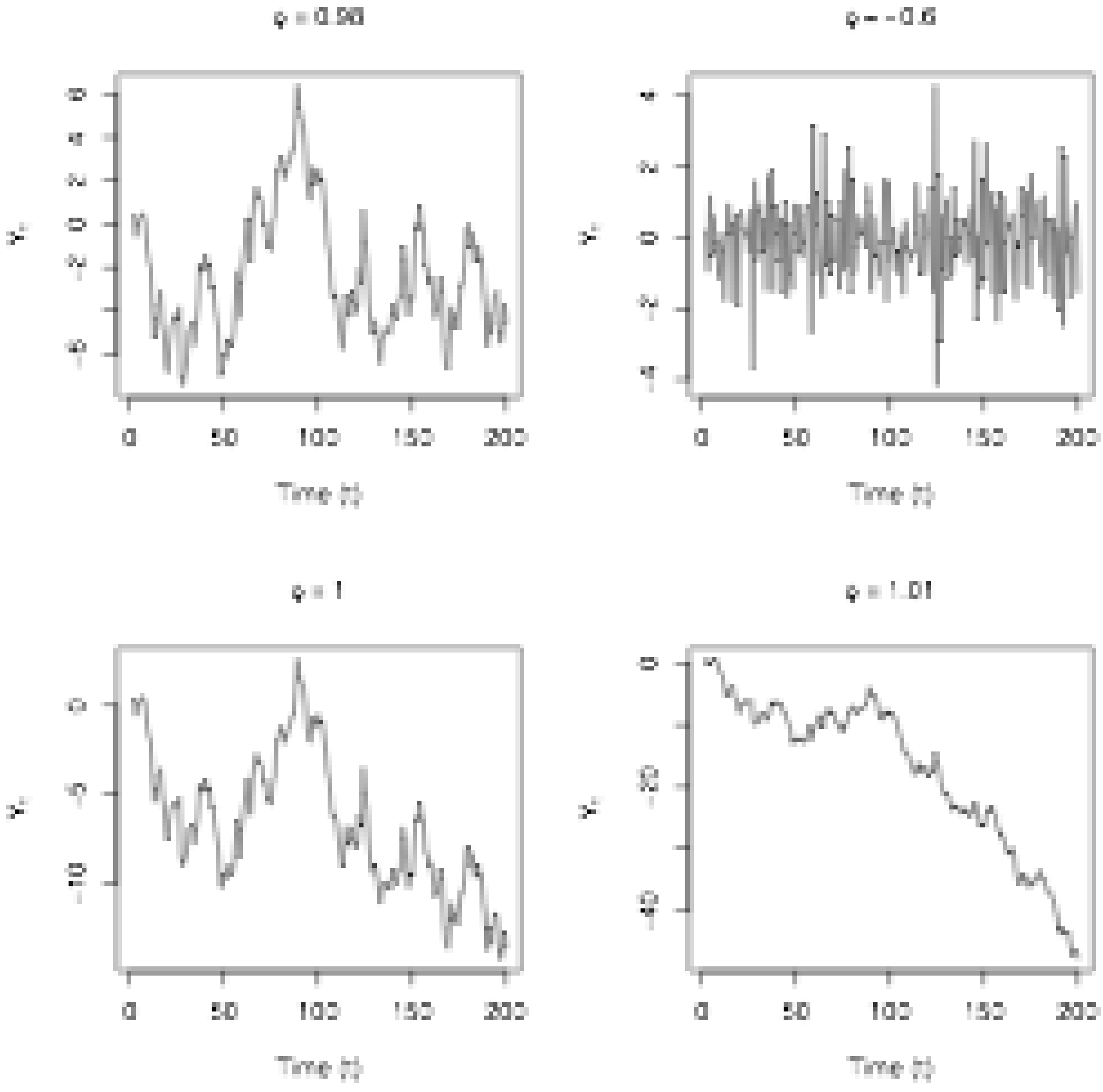
And Y_t is white noise: (μ, σ_ϵ^2)

- If slope $\phi \neq 0$ then: Y_t depends on both ϵ_t and Y_{t-1}

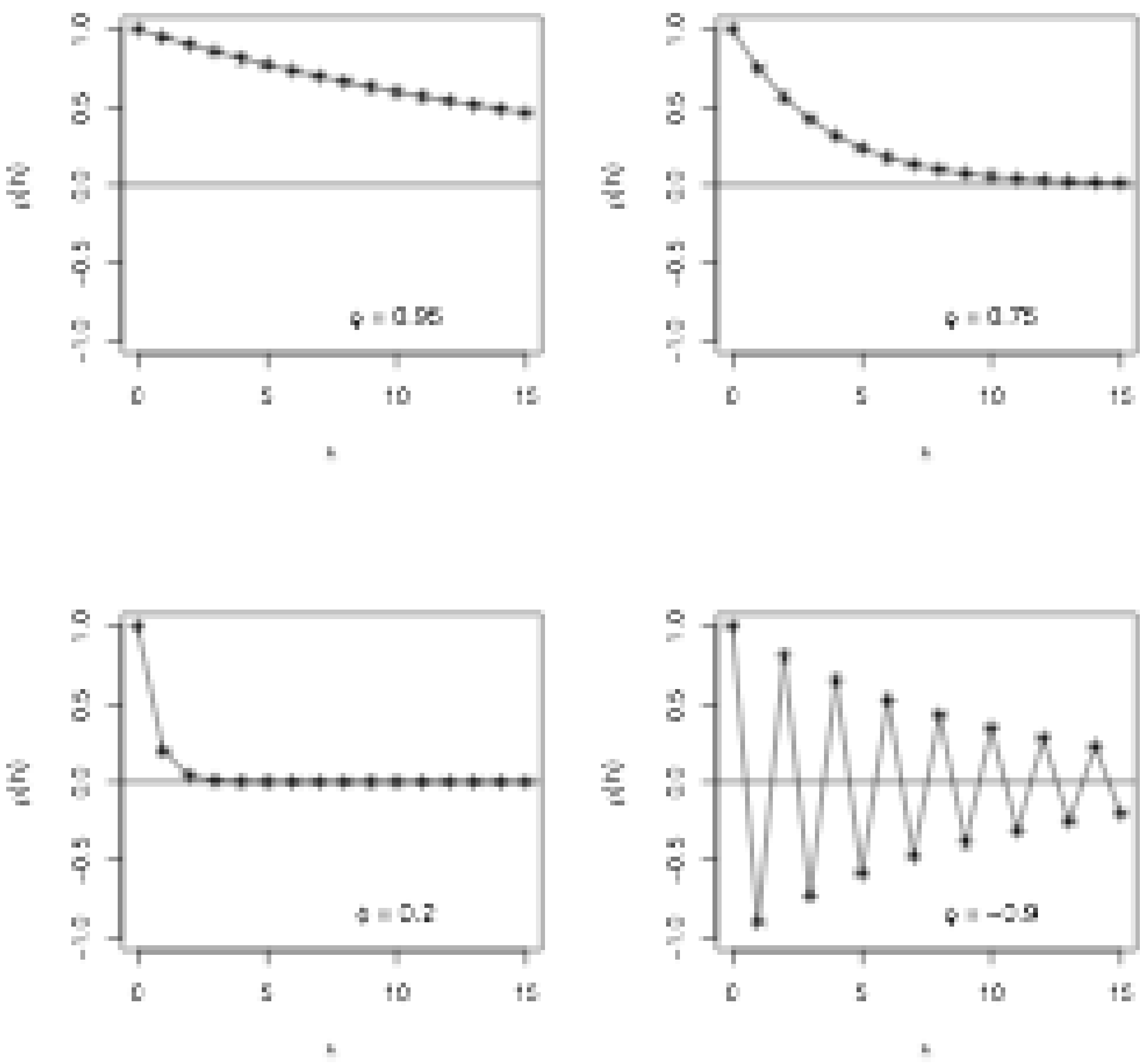
And the process $\{Y_t\}$ is autocorrelated

- Large values of ϕ lead to greater autocorrelation
- Negative values of ϕ result in oscillatory time series

AR examples



Autocorrelations



Random walk

If $\mu = 0$ and slope $\phi = 1$, then:

$$Y_t = Y_{t-1} + \epsilon_t$$

Which is:

$$Today = Yesterday + Noise$$

But this is a **random walk**.

And $\{Y_t\}$ is **not** stationary in this case.

Let's practice!

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AR model estimation and forecasting

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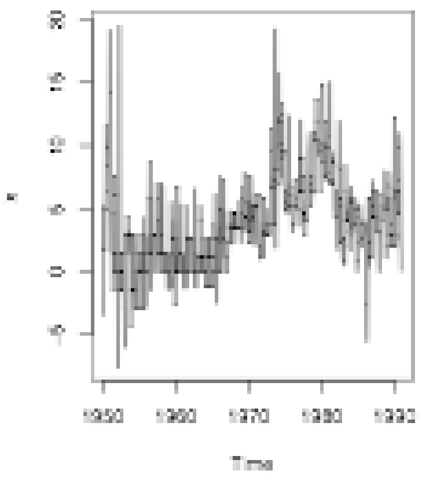
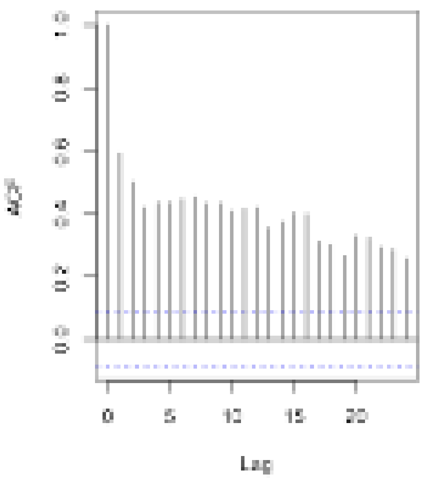
AR processes: inflation rate

- One-month US inflation rate (in percent, annual rate).
- Monthly observations from 1950 through 1990

```
data(Mishkin, package = "Ecdat")
inflation <- as.ts(Mishkin[, 1])
ts.plot(inflation) ; acf(inflation)
```





$$(Today - Mean) = Slope * (Yesterday - Mean) + Noise$$

$$Y_t - \mu = \phi(Y_{t-1} - \mu) + \epsilon_t$$

$$\epsilon_t \; WhiteNoise(0, \sigma_\epsilon^2)$$

```
AR_inflation <- arima(inflation, order = c(1, 0, 0))
print(AR_inflation)
```

```
Coefficients:
         ar1  intercept
      0.5960     3.9745
s.e. 0.0364        0.3471
sigma^2 estimated as 9.713
```

ar1 = $\hat{\phi}$, intercept = $\hat{\mu}$, sigma^2 = $\hat{\sigma}_\epsilon^2$

AR processes: fitted values - I

- AR fitted values:

$$\widehat{Today} = \widehat{Mean} + \widehat{Slope} * (Yesterday - \widehat{Mean})$$

$$\hat{Y}_t = \hat{\mu} + \hat{\phi}(Y_{t-1} - \hat{\mu})$$

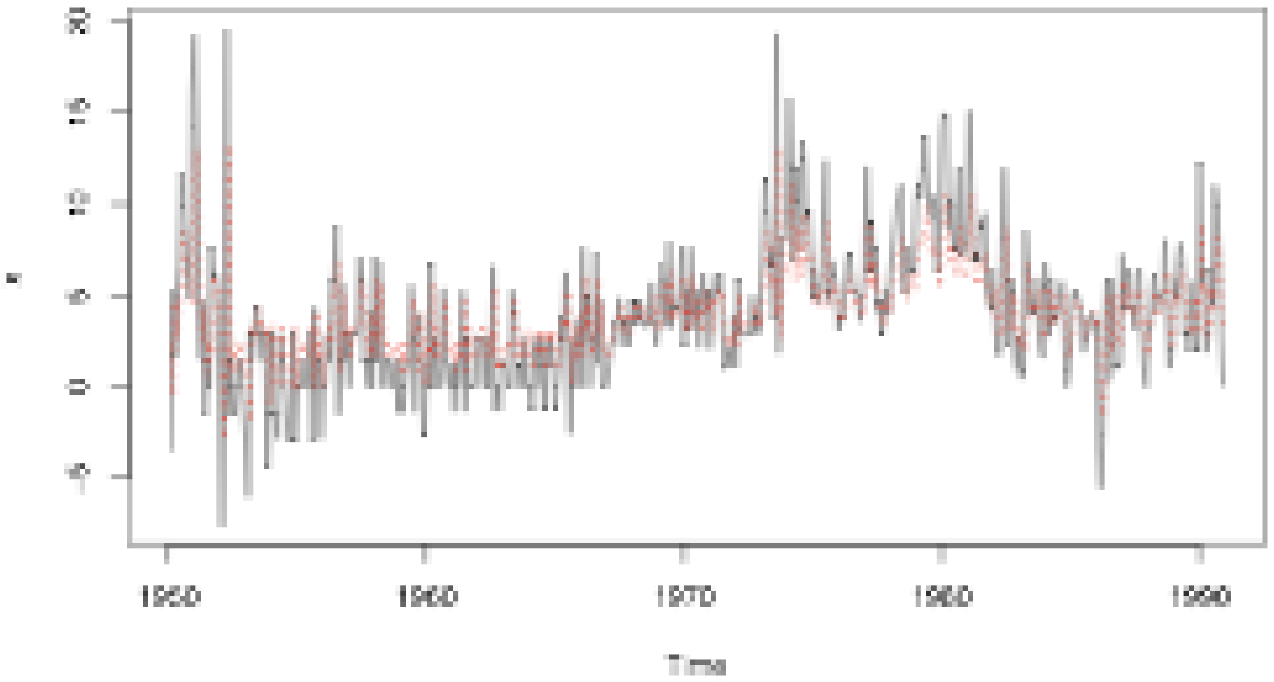
- Residuals =

$$Today - \widehat{Today}$$

$$\hat{\epsilon}_t = Y_t - \hat{Y}_t$$

AR processes: fitted values - II

```
ts.plot(inflation)
AR_inflation_fitted <- inflation - residuals(AR_inflation)
points(AR_inflation_fitted, type = "l"
       col = "red", lty = 2)
```



Forecasting

- 1-step ahead forecasts

```
predict(AR_inflation)$pred
```

```
Jan
1991 1.605797
```

```
predict(AR_inflation)$se
```

```
Jan
1991 3.116526
```

Forecasting (cont.)

- h-step ahead forecasts

```
predict(AR_inflation, n.ahead = 6)$pred
```

```
          Jan      Feb      Mar      Apr      May      Jun
1991 1.605797 2.562810 3.133165 3.473082 3.675664 3.796398
```

```
predict(AR_inflation, n.ahead = 6)$se
```

```
          Jan      Feb      Mar      Apr      May      Jun
1991 3.116526 3.628023 3.793136 3.850077 3.870101 3.877188
```

Let's practice!

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