

Probability of event A and event B

FOUNDATIONS OF PROBABILITY IN R



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Event A: "Coin is heads"

$A = 1$

$A = 0$



Events A and B: Two Different Coins

A = 1



A = 0



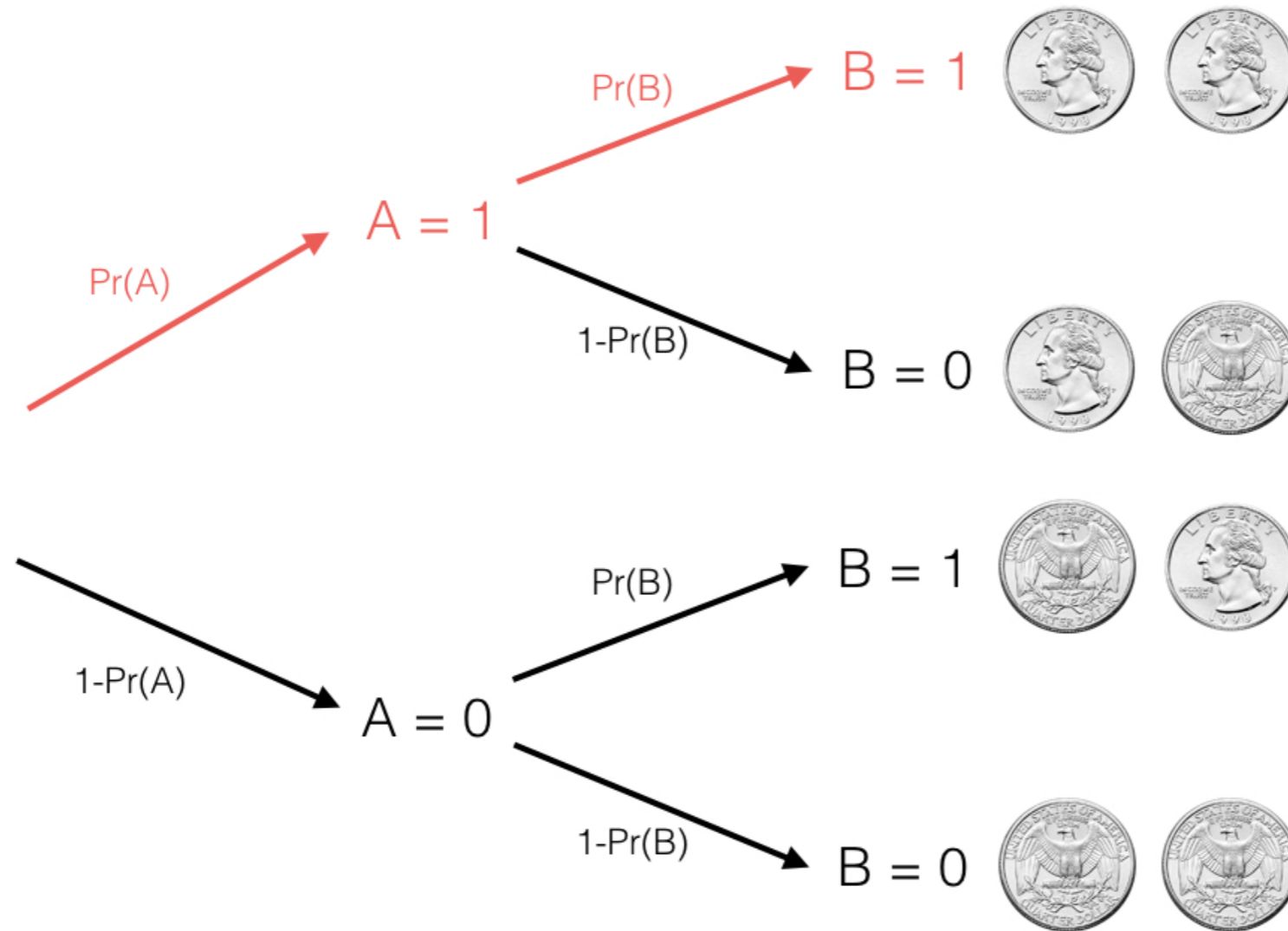
B = 1



B = 0



Probability of A and B



Simulating two coins

```
A <- rbinom(100000, 1, .5)
```

```
B <- rbinom(100000, 1, .5)
```

```
A & B  
# [1] FALSE TRUE FALSE FALSE...
```

```
mean(A & B)  
[1] 0.24959
```

$$\Pr(A \text{ and } B) = \Pr(A) \cdot \Pr(B)$$

$$\Pr(A \text{ and } B) = .5 \cdot .5 = .25$$

```
A <- rbinom(100000, 1, .1)
```

```
B <- rbinom(100000, 1, .7)
```

```
A & B  
# [1] FALSE FALSE FALSE FALSE...
```

```
mean(A & B)  
[1] 0.07043
```

$$\Pr(A \text{ and } B) = \Pr(A) \cdot \Pr(B)$$

$$\Pr(A \text{ and } B) = .1 \cdot .7 = .07$$

Let's practice!

FOUNDATIONS OF PROBABILITY IN R

Probability of A or B

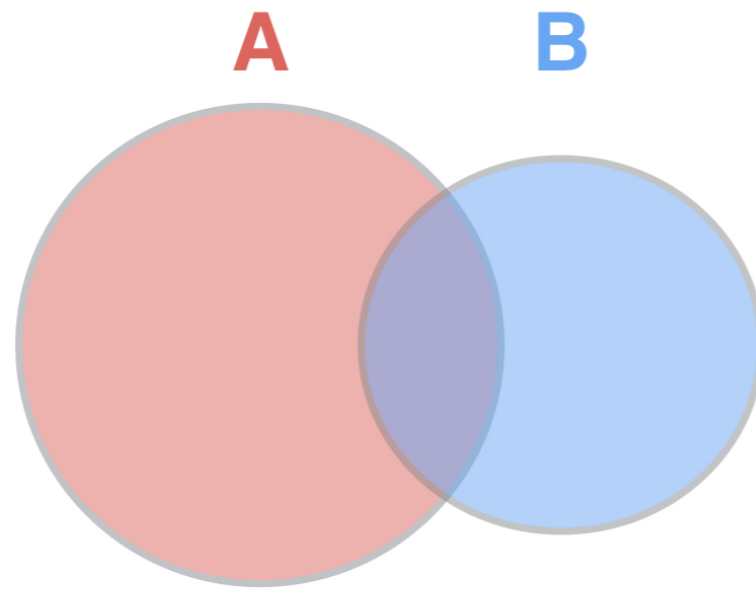
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Probability of A or B



$$\Pr(A \text{ or } B) = \Pr(A) + \Pr(B) - \Pr(A \text{ and } B)$$

$$\Pr(A \text{ or } B) = \Pr(A) + \Pr(B) - \Pr(A) \cdot \Pr(B)$$

$$\Pr(A \text{ or } B) = .5 + .5 - .5 \cdot .5 = .75$$

Simulating two events

```
A <- rbinom(100000, 1, .5)
```

```
A <- rbinom(100000, 1, .2)
```

```
B <- rbinom(100000, 1, .5)
```

```
B <- rbinom(100000, 1, .6)
```

```
mean(A | B)  
[1] 0.75125
```

```
mean(A | B)  
[1] 0.6803
```

$$\Pr(A \text{ or } B) = \Pr(A) + \Pr(B) - \Pr(A \text{ and } B)$$

$$.75 = .5 + .5 - .5 \cdot .5$$

$$\Pr(A \text{ or } B) = \Pr(A) + \Pr(B) - \Pr(A \text{ and } B)$$

$$.68 = .2 + .6 - .2 \cdot .6$$

Three coins

$$\Pr(A \text{ or } B \text{ or } C)$$

$$= \Pr(A) + \Pr(B) + \Pr(C) -$$

$$\Pr(A \text{ and } B) - \Pr(A \text{ and } C) - \Pr(A \text{ and } B) +$$

$$\Pr(A \text{ and } B \text{ and } C)$$

```
mean(A | B | C)
```

Let's practice!

FOUNDATIONS OF PROBABILITY IN R

Multiplying random variables

FOUNDATIONS OF PROBABILITY IN R



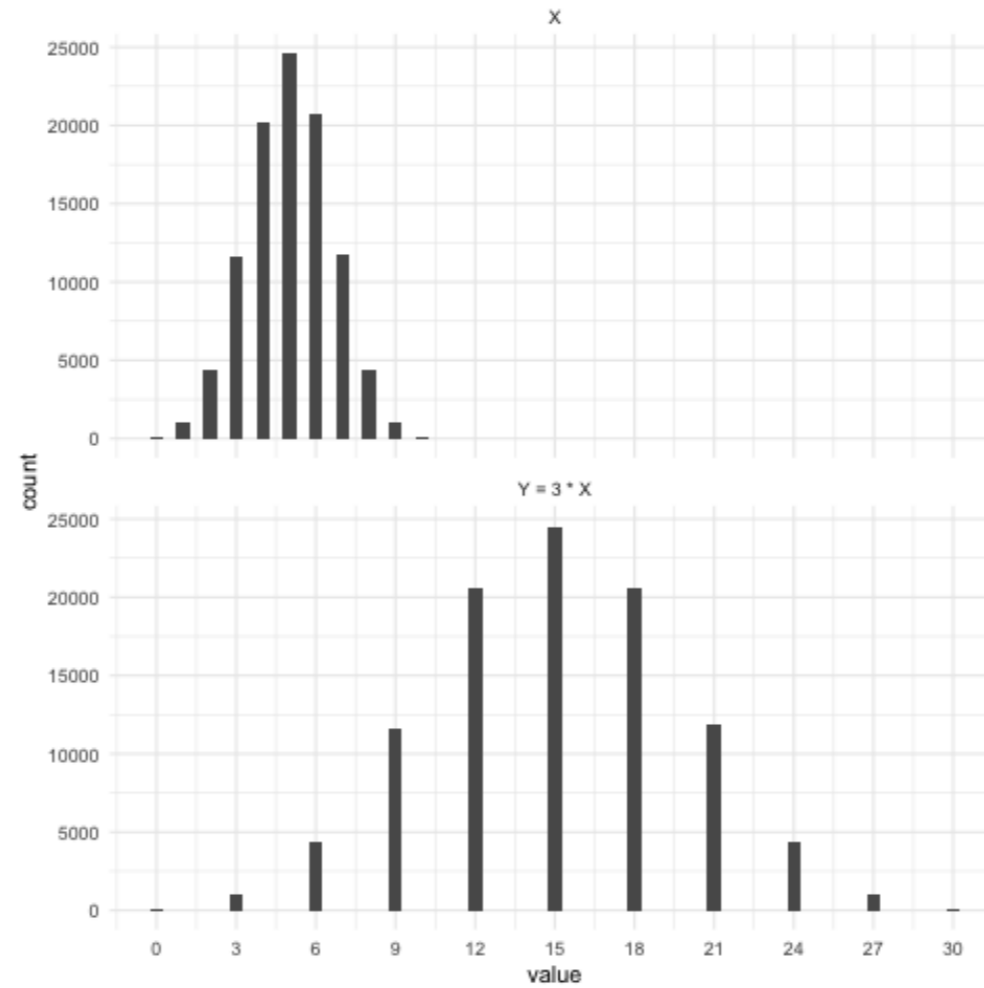
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Multiplying a random variable

$$X \sim \text{Binomial}(10, .5)$$

$$Y \sim 3 \cdot X$$



Simulation: Effect of multiplying on expected value

$X \sim \text{Binom}(10, .5)$

$Y = 3 \cdot X$

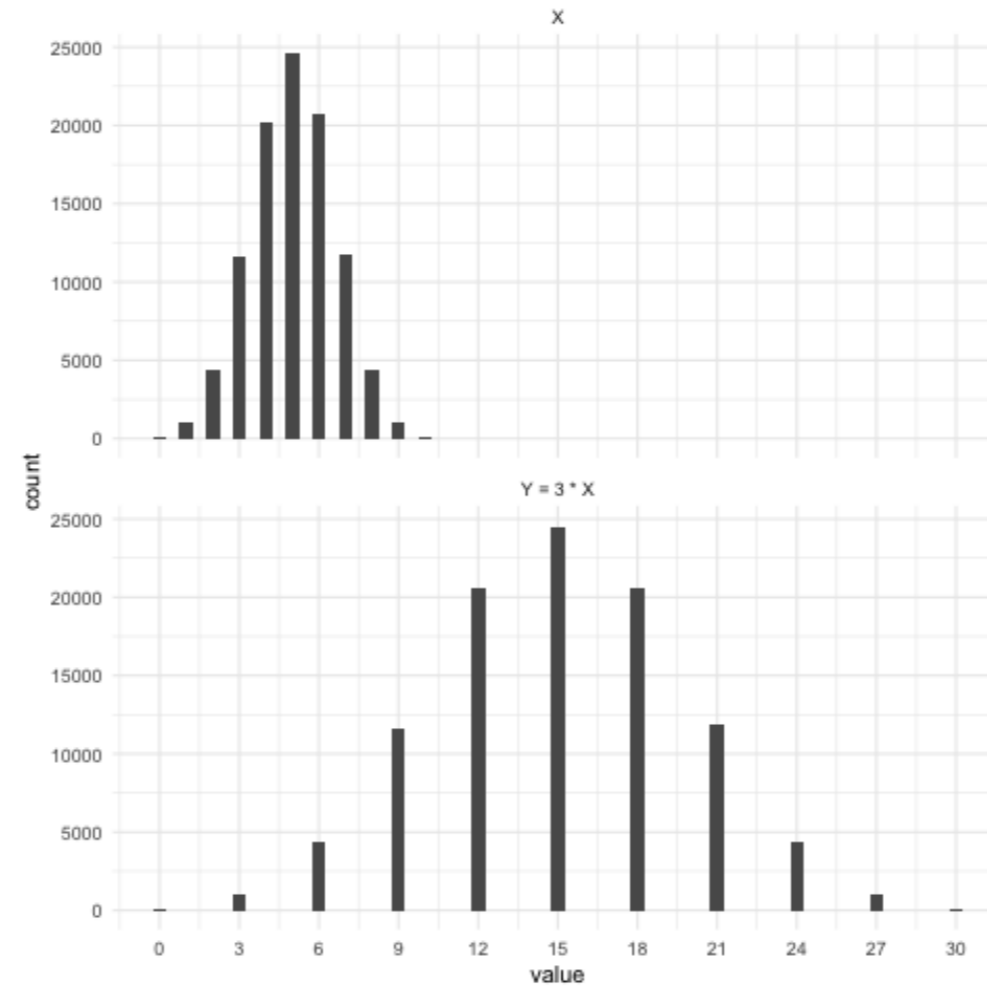
```
X <- rbinom(100000, 10, .5)
```

```
mean(X)  
# [1] 5.006753
```

```
Y <- 3 * X
```

```
mean(Y)  
# [1] 15.02026
```

$$E[k \cdot X] = k \cdot E[X]$$



Simulation: Effect of multiplying on variance

$$X \sim \text{Binom}(10, .5)$$

$$Y = 3 \cdot X$$

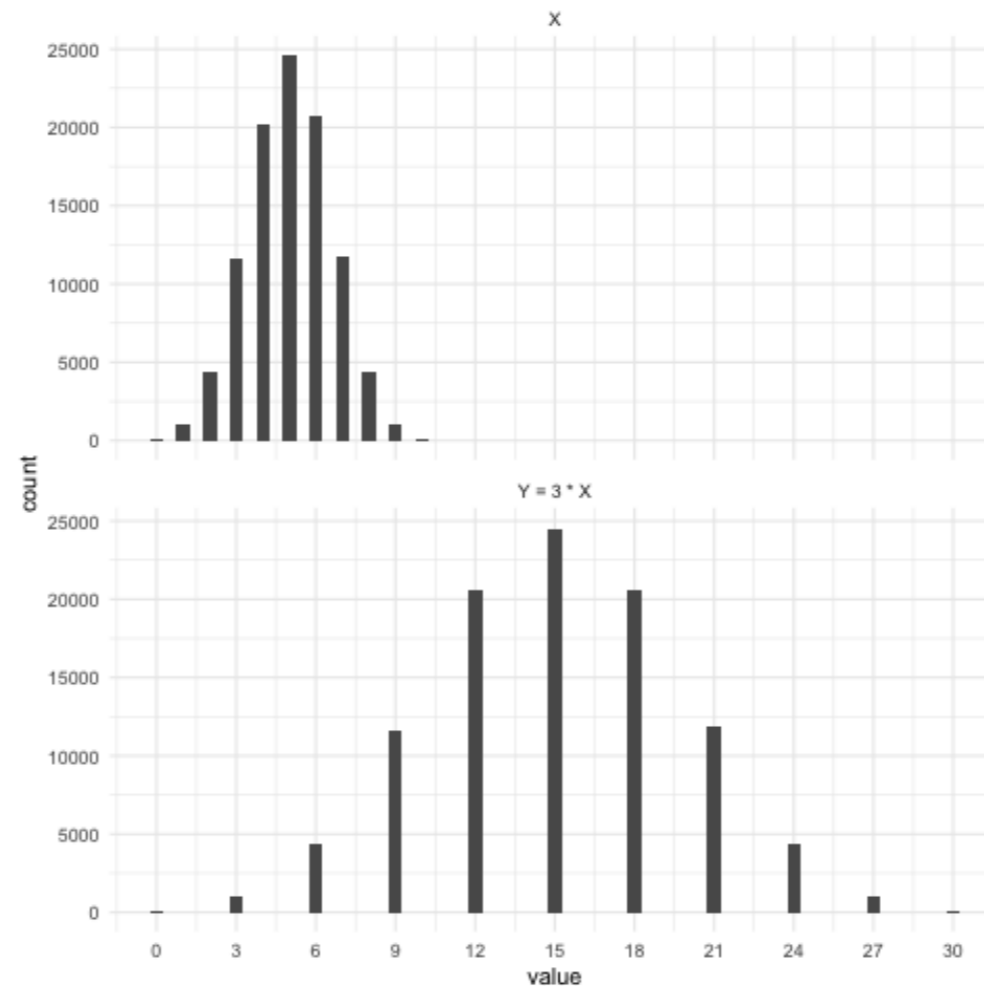
```
X <- rbinom(100000, 10, .5)
```

```
var(X)  
# [1] 2.500388
```

```
Y <- 3 * X
```

```
var(Y)  
# [1] 22.50349
```

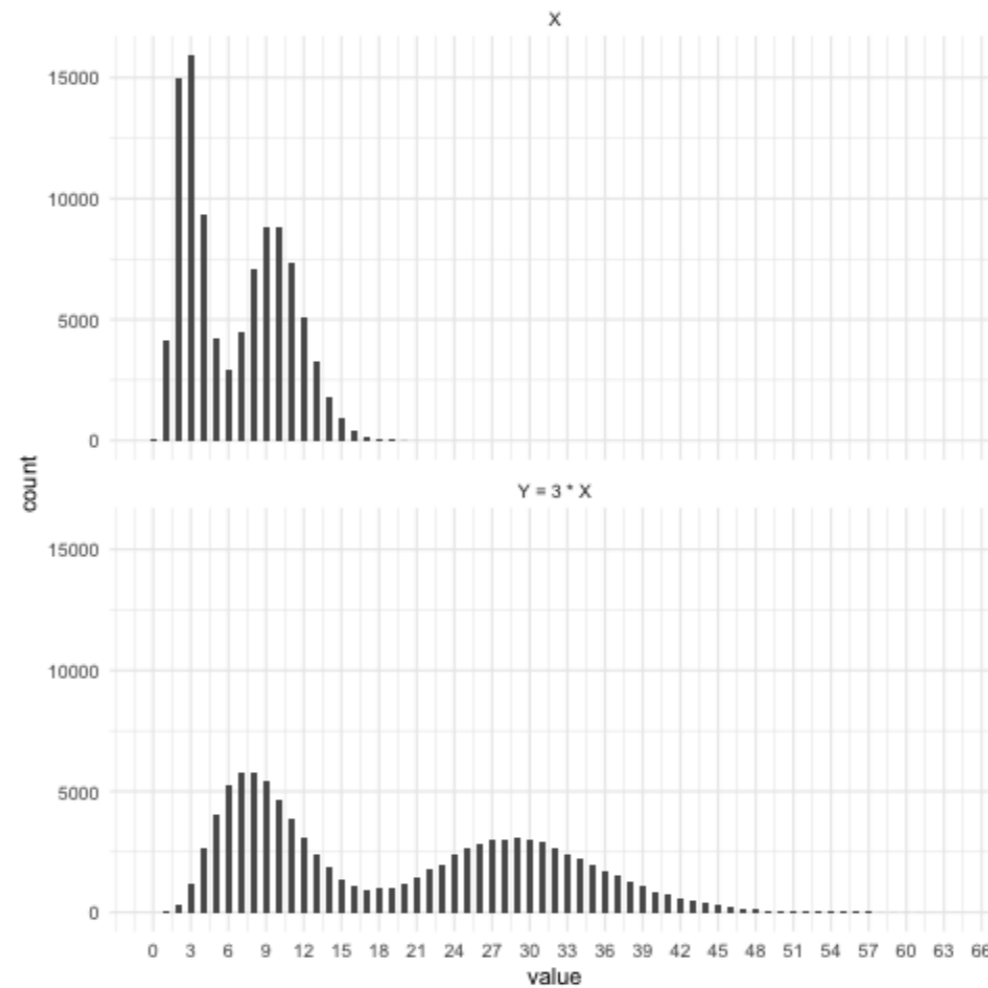
$$\text{Var}[k \cdot X] = k^2 \cdot \text{Var}[X]$$



Rules of manipulating random variables

$$E[k \cdot X] = k \cdot E[X]$$

$$\text{Var}(k \cdot Y) = k^2 \cdot \text{Var}(X)$$



Let's practice!

FOUNDATIONS OF PROBABILITY IN R

Adding two random variables together

FOUNDATIONS OF PROBABILITY IN R



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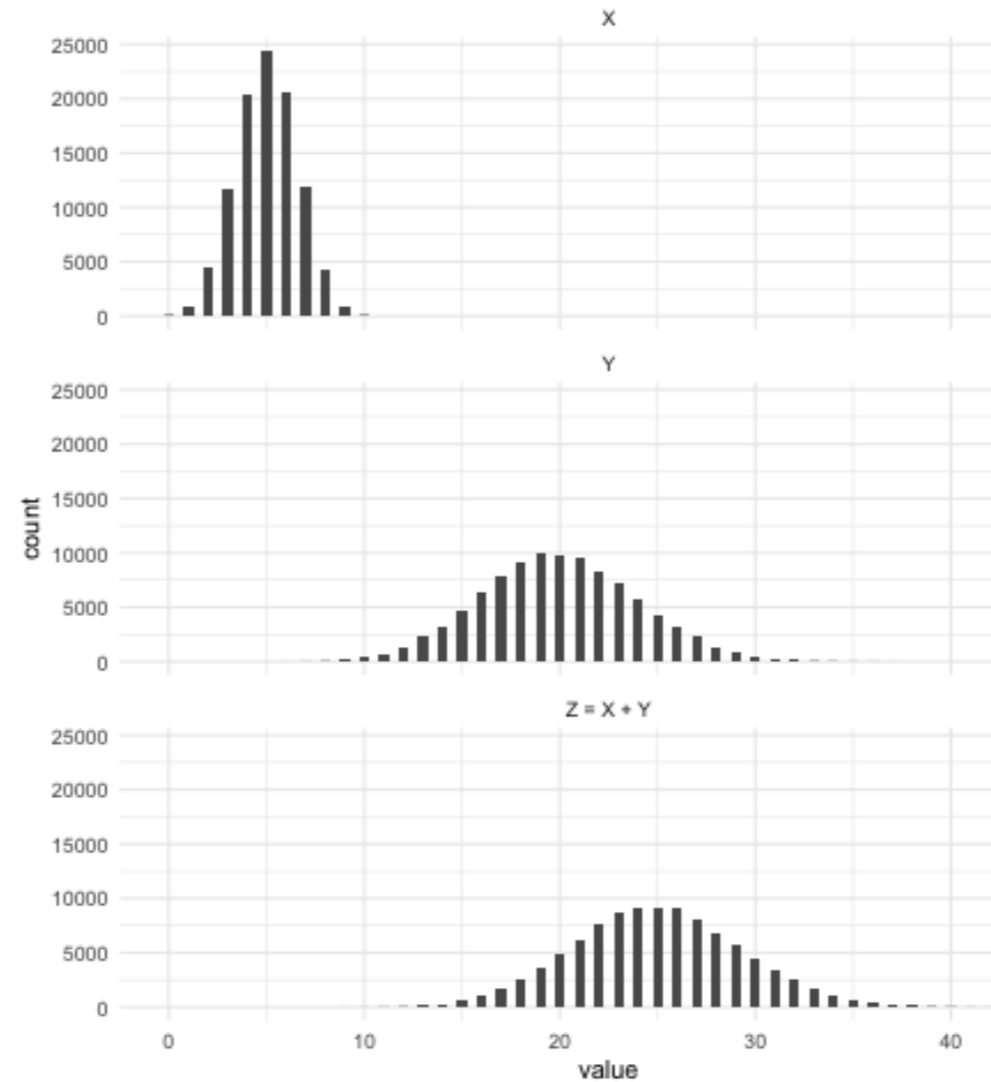
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Adding two random variables

$$X \sim \text{Binom}(10, .5)$$

$$Y \sim \text{Binom}(100, .2)$$

$$Z \sim X + Y$$



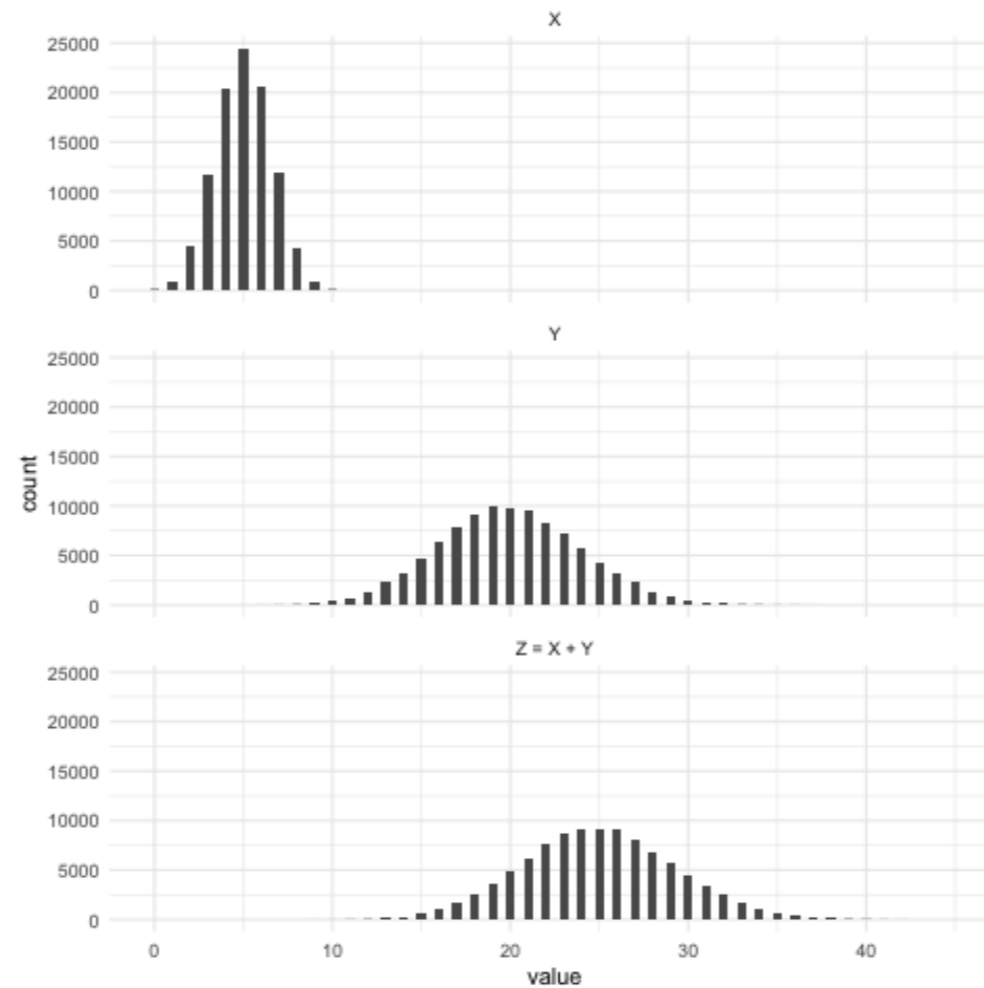
Simulation: expected value of $X + Y$

```
X <- rbinom(100000, 10, .5)
mean(X)
# [1] 5.00938
```

```
Y <- rbinom(100000, 100, .2)
mean(Y)
# [1] 19.99422
```

```
Z <- X + Y
mean(Z)
# [1] 25.0036
```

$$E[X + Y] = E[X] + E[Y]$$



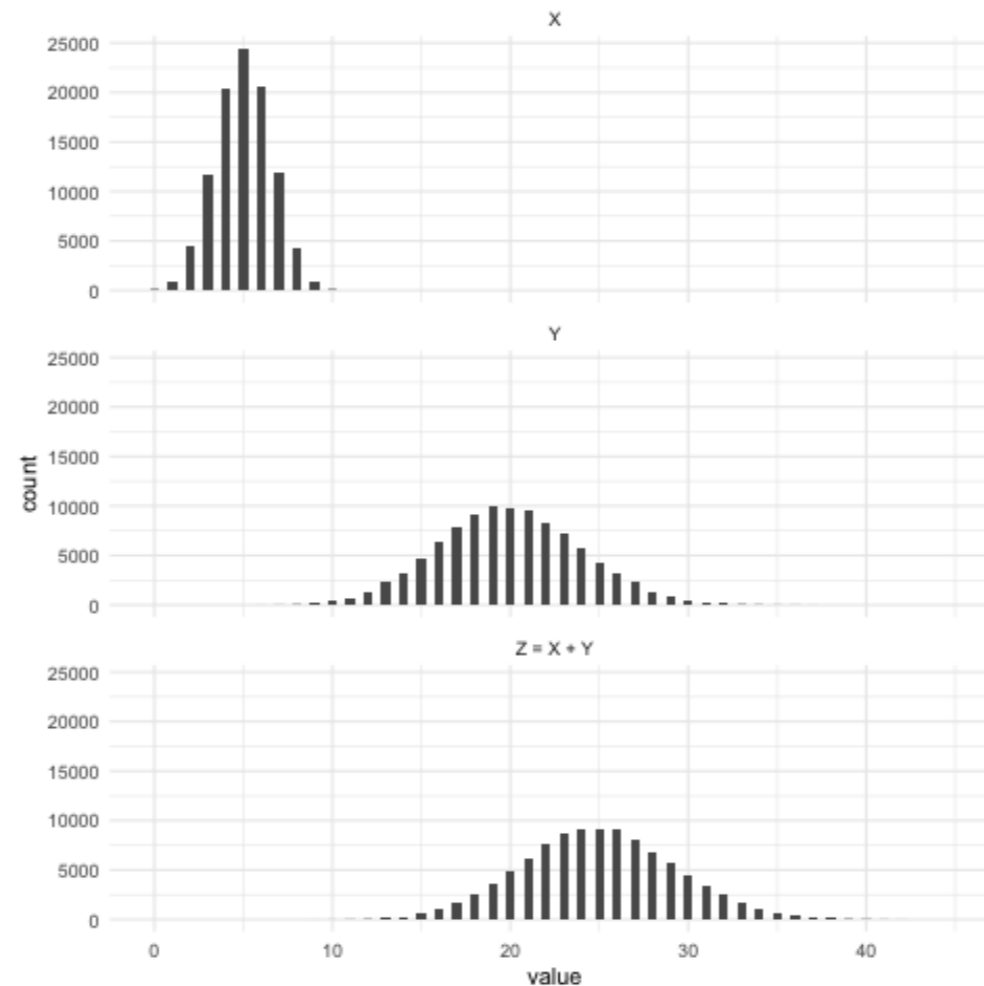
Simulation: variance of $X + Y$

```
X <- rbinom(100000, 10, .5)
var(X)
# [1] 2.500895
```

```
Y <- rbinom(100000, 100, .2)
var(Y)
# [1] 16.06289
```

```
Z <- X + Y
var(Z)
# [1] 18.58055
```

$$\text{Var}[X + Y] = \text{Var}[X] + \text{Var}[Y]$$



Rules for combining random variables

$$E[X + Y] = E[X] + E[Y]$$

(Even if X and Y aren't independent)

$$\text{Var}[X + Y] = \text{Var}[X] + \text{Var}[Y]$$

(Only if X and Y are independent)

Let's practice!

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