Određeni integral

MATEMATIKA 2

Damir Horvat

FOI, Varaždin

Sadržaj

Newton-Leibnizova formula

Newton-Leibnizova formula

Teorem

Ako je f neprekidna funkcija na otvorenom intervalu I i F bilo koja primitivna funkcija funkcije f na I, tada za svaki $[a, b] \subseteq I$ vrijedi

$$\int_a^b f(x)\,\mathrm{d}x = F(b) - F(a).$$

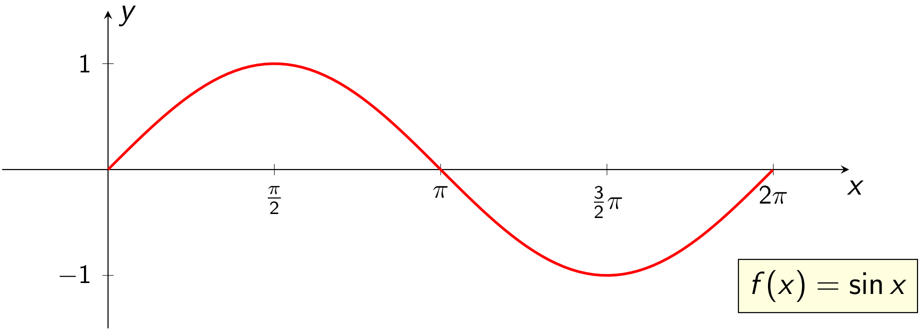
Newton-Leibnizova formula

Teorem

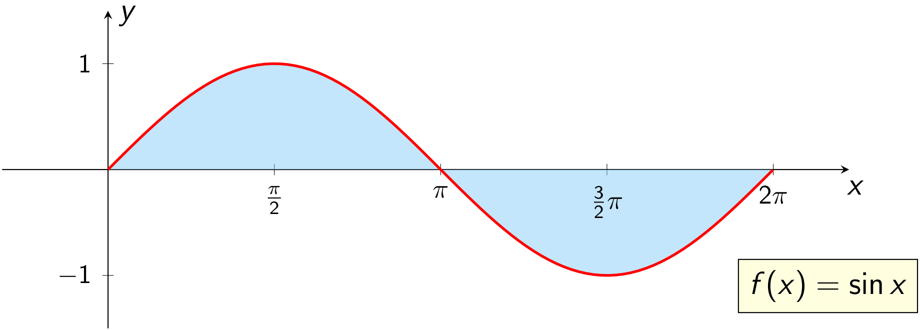
Ako je f neprekidna funkcija na otvorenom intervalu I i F bilo koja primitivna funkcija funkcije f na I, tada za svaki $[a, b] \subseteq I$ vrijedi

$$\int_a^b f(x)\,\mathrm{d}x = F(b) - F(a).$$

$$\boxed{\int_a^b f(x)\,\mathrm{d}x = F(b) - F(a) = F(x)\Big|_a^b \qquad F'(x) = f(x),\ x \in [a, b]}$$



y
1
-1
x
$\frac{\pi}{2}$
π
$\frac{3}{2}\pi$
2π
$f(x) = \sin x$



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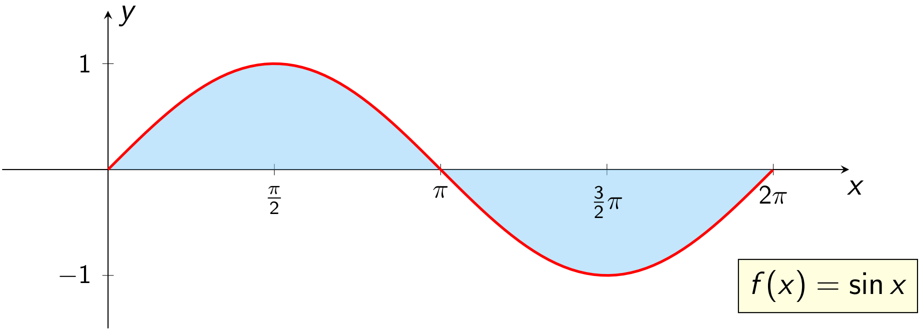
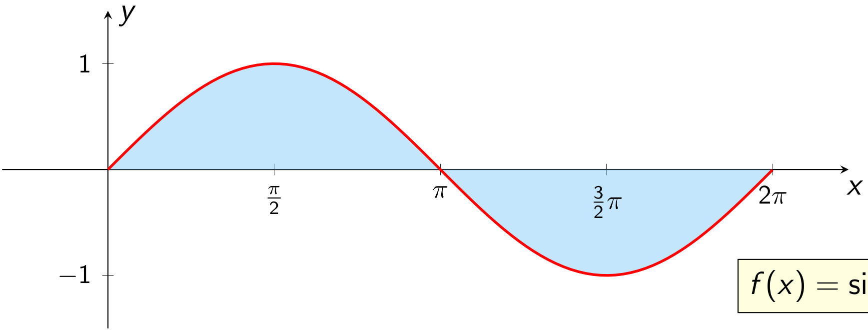


Vrijednost integrala na segmentu $[0, 2\pi]$

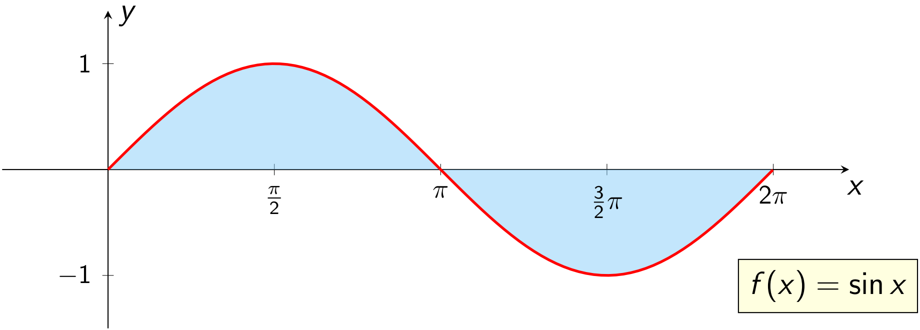
$$\int_0^{2\pi} \sin x \,\mathrm{d}x =$$



$f(x) = \sin x$

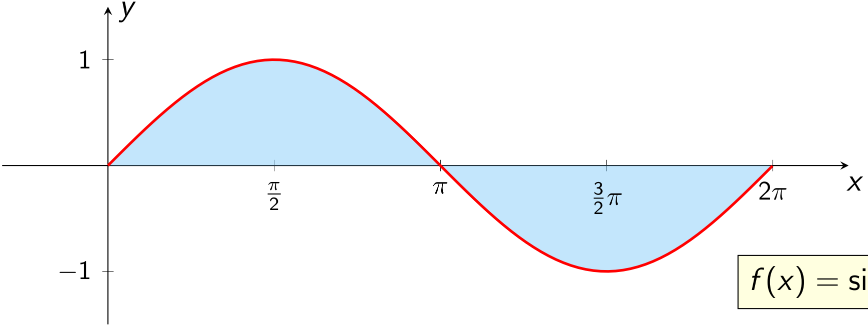
Vrijednost integrala na segmentu $[0, 2\pi]$

$$\int_0^{2\pi} \sin x \, \mathrm{d}x = -\cos x \Big|_0^{2\pi}$$



Vrijednost integrala na segmentu $[0, 2\pi]$

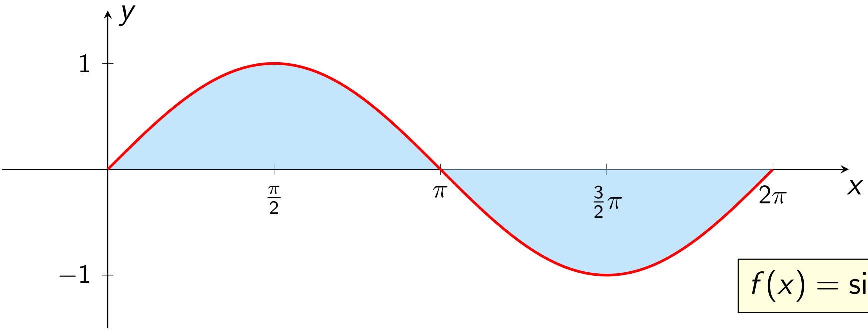
$$\int_0^{2\pi} \sin x \,\mathrm{d}x = -\cos x \Big|_0^{2\pi} = -\cos 2\pi$$



$f(x) = \sin x$

Vrijednost integrala na segmentu $[0, 2\pi]$

$$\int_0^{2\pi} \sin x \, \mathrm{d}x = -\cos x \Big|_0^{2\pi} = -\cos 2\pi -$$



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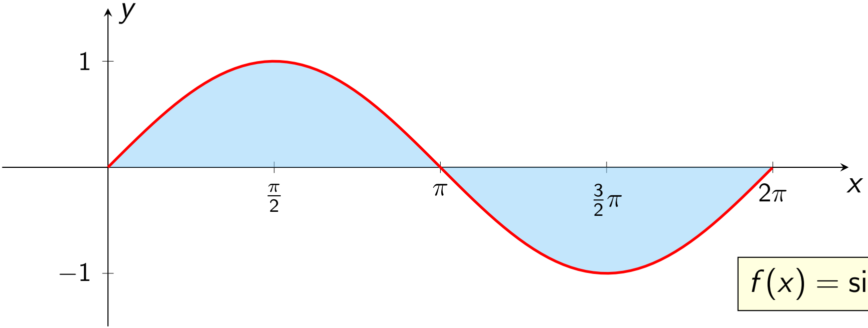
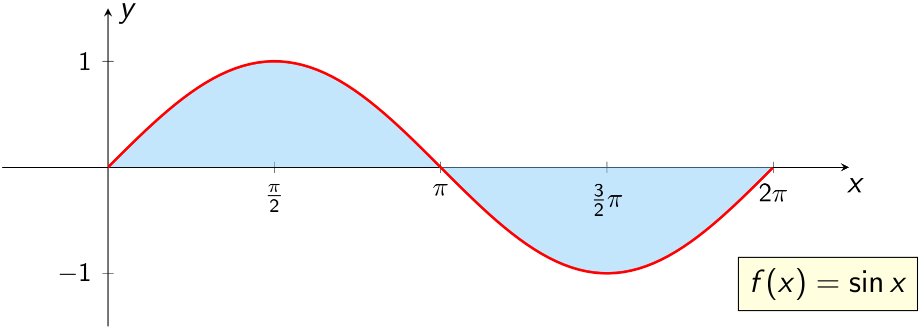


$f(x) = \sin x$

Vrijednost integrala na segmentu $[0, 2\pi]$

$$\int_0^{2\pi} \sin x \,\mathrm{d}x = -\cos x \Big|_0^{2\pi} = -\cos 2\pi - \left(-\cos 0\right) =$$

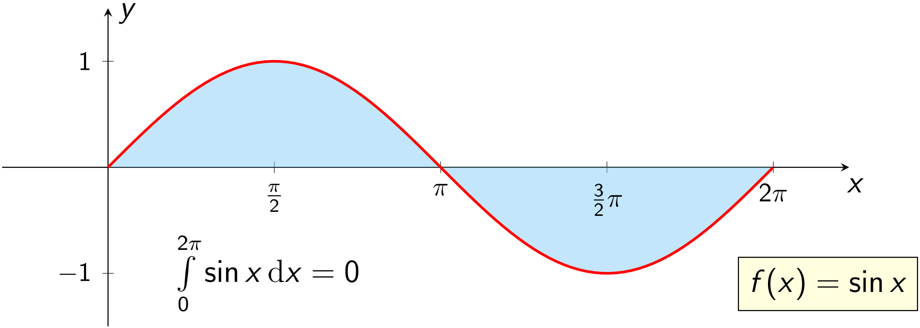
$$= -1 - (-1)$$



Vrijednost integrala na segmentu $[0, 2\pi]$

$$\int_0^{2\pi} \sin x \, \mathrm{d}x = -\cos x \Big|_0^{2\pi} = -\cos 2\pi - \left(-\cos 0\right) =$$

$$= -1 - (-1) = -1 + 1$$



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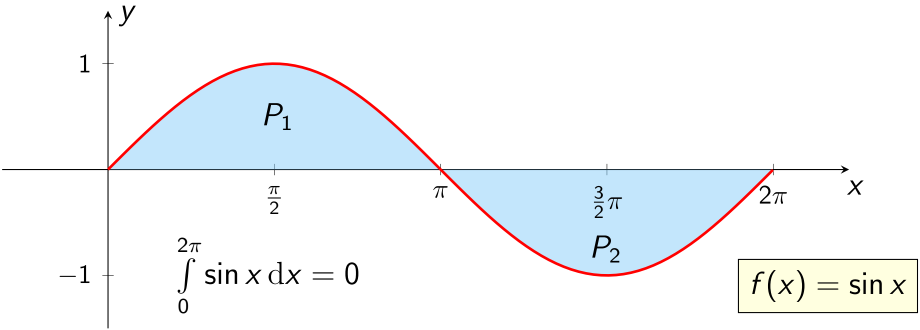
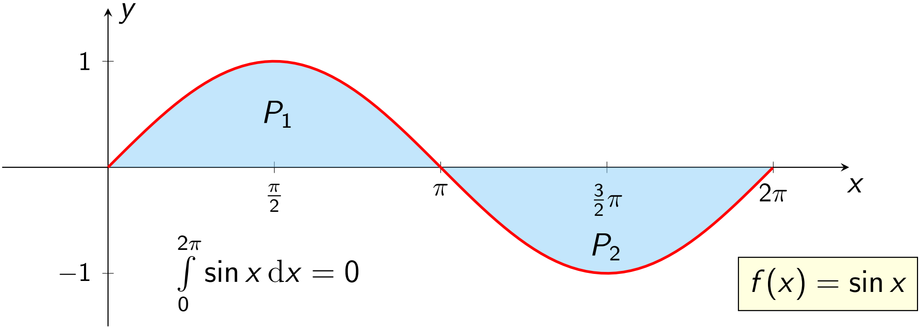
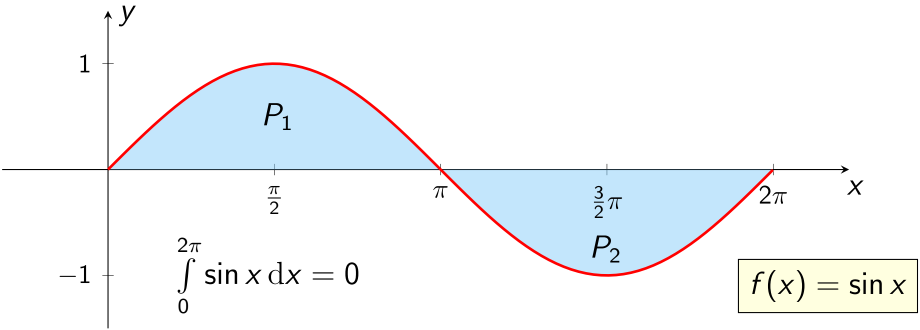


Površina između grafa funkcije i x-osi na segmentu $[0, 2\pi]$



Površina između grafa funkcije i x-osi na segmentu $[0, 2\pi]$

$$P_1 = \int\limits_0^{\pi} \sin x \, \mathrm{d}x =$$



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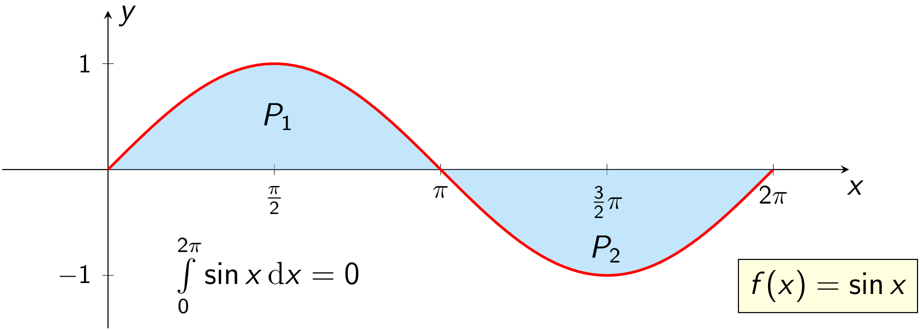

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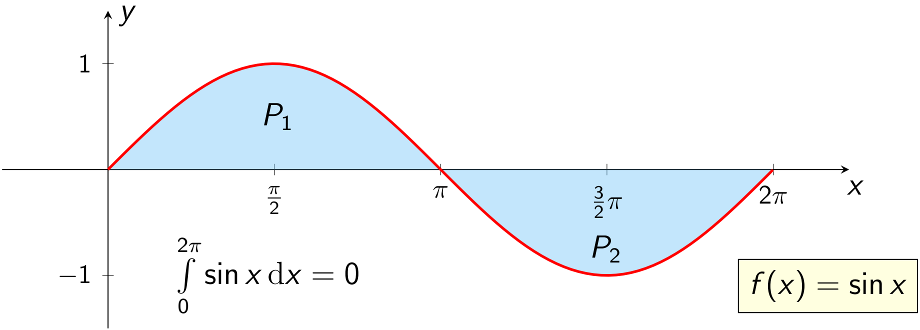

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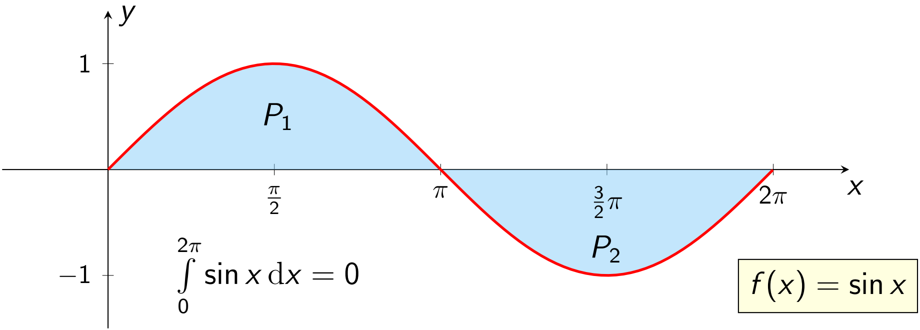

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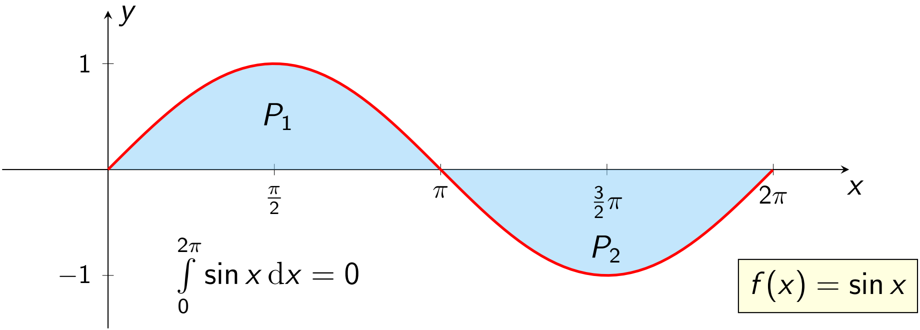

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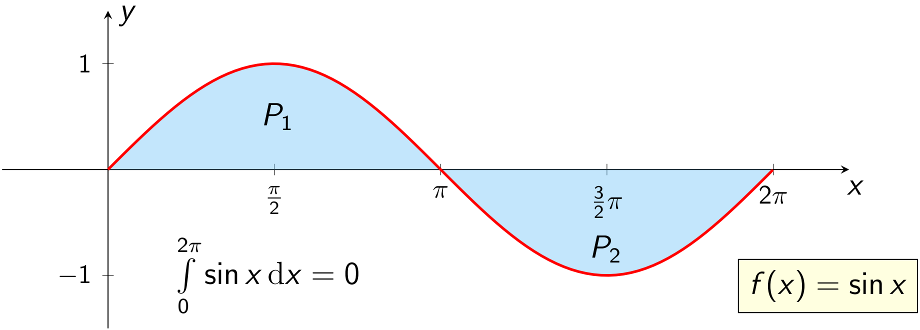
$$= -(-1) - (-1)$$



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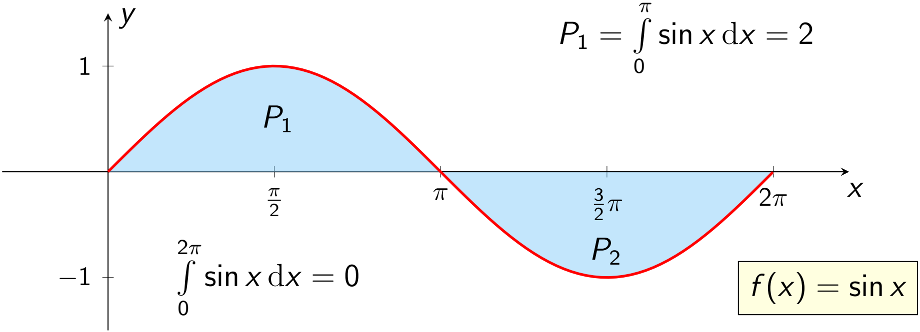
$$P_1 = \int_0^{\pi} \sin x \, \mathrm{d}x = -\cos x \Big|_0^{\pi} = -\cos\pi - \left(-\cos 0\right) =$$

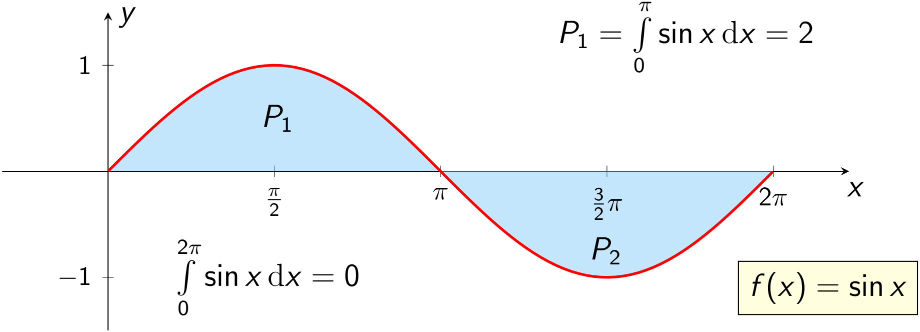
$$= -(-1) - (-1) = 1 + 1$$



Površina između grafa funkcije i x-osi na segmentu $[0, 2\pi]$

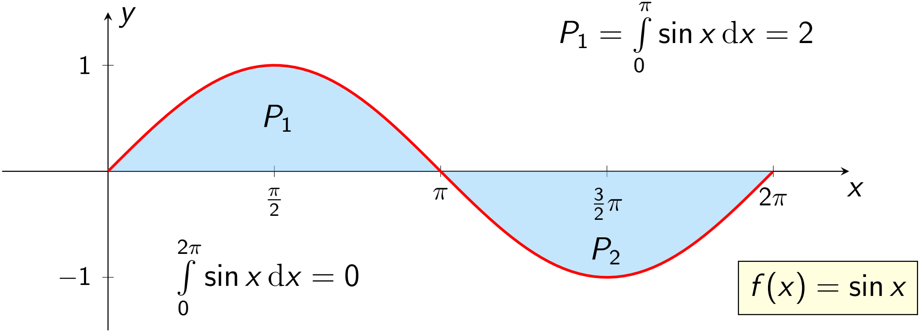
$$P_1 = \int\limits_0^\pi \sin x \, \mathrm{d}x = -\cos x \Big|_0^\pi = -\cos \pi - \big(-\cos 0\big) =$$

$$= -(-1) - (-1) = 1 + 1 = 2$$



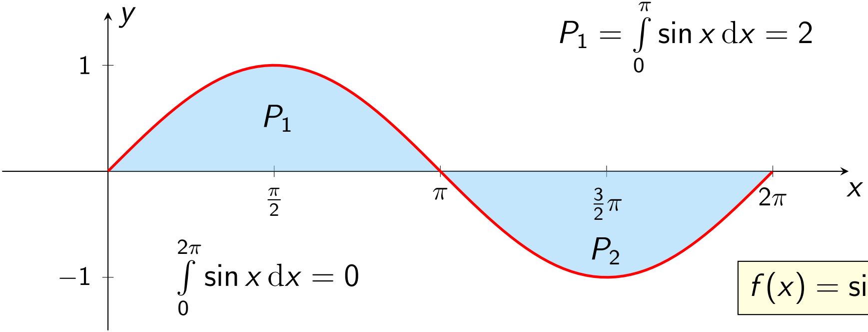
Površina između grafa funkcije i x-osi na segmentu $[0, 2\pi]$

$$P_2 = -\int_{\pi}^{2\pi} \sin x \, \mathrm{d}x =$$



Površina između grafa funkcije i x-osi na segmentu $[0, 2\pi]$

$$P_2 = -\int\limits_{\pi}^{2\pi} \sin x \, \mathrm{d}x = -\big(-\cos x\big)\Big|_{\pi}^{2\pi}$$



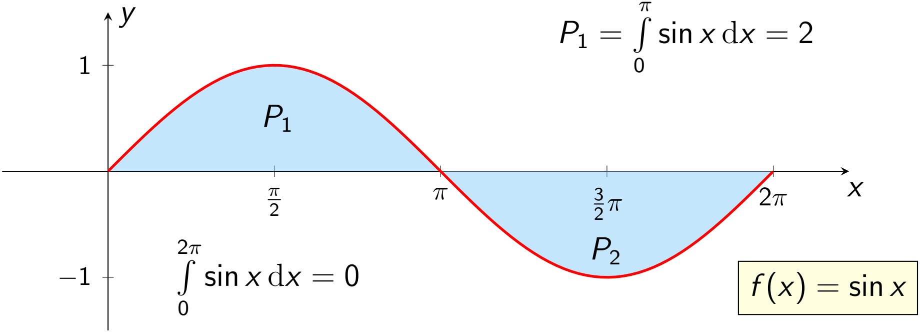
$$P_1 = \int_0^{\pi} \sin x \, \mathrm{d}x = 2$$

$$\int_0^{2\pi} \sin x \, \mathrm{d}x = 0$$

$$\boxed{f(x) = \sin x}$$

Površina između grafa funkcije i x-osi na segmentu $[0, 2\pi]$

$$P_2 = -\int_{\pi}^{2\pi} \sin x \, \mathrm{d}x = -\big(-\cos x\big)\Big|_{\pi}^{2\pi} = \cos x \Big|_{\pi}^{2\pi}$$



Površina između grafa funkcije i x-osi na segmentu $[0, 2\pi]$

$$P_2 = -\int_{\pi}^{2\pi} \sin x \,\mathrm{d}x = -\big(-\cos x\big)\Big|_{\pi}^{2\pi} = \cos x\Big|_{\pi}^{2\pi} =$$

$$= \cos 2\pi$$

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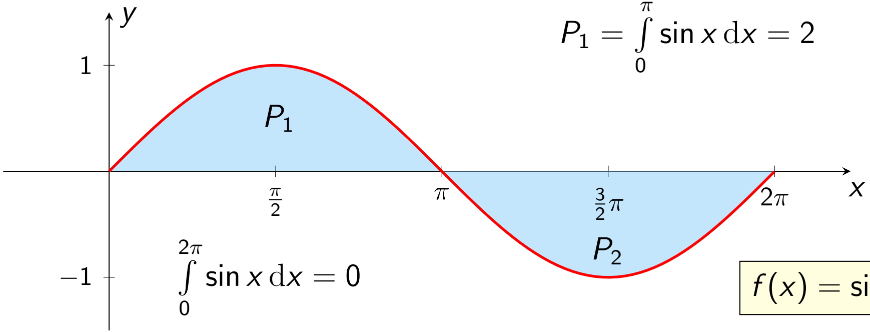


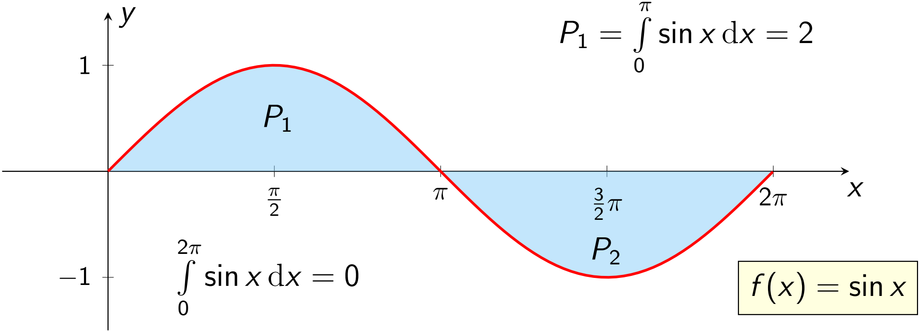
$$\int_0^{2\pi} \sin x \, \mathrm{d}x = 0$$

$$\boxed{f(x) = \sin x}$$

Površina između grafa funkcije i x-osi na segmentu $[0, 2\pi]$

$$P_2 = -\int_{\pi}^{2\pi} \sin x \, \mathrm{d}x = -\big(-\cos x\big)\Big|_{\pi}^{2\pi} = \cos x\Big|_{\pi}^{2\pi} =$$

$$= \cos 2\pi -$$



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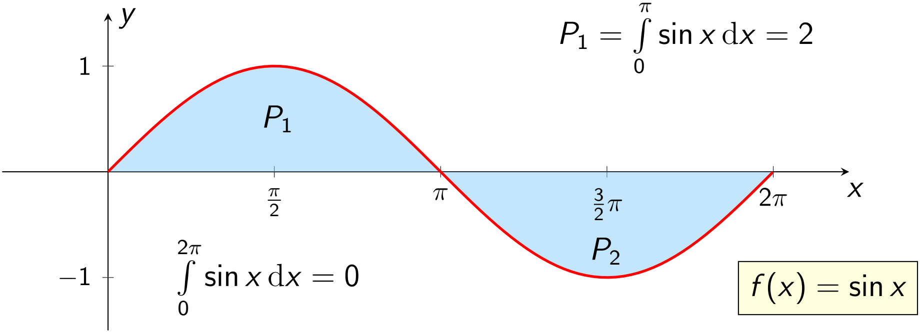
$$P_2 = -\int_{\pi}^{2\pi} \sin x \,\mathrm{d}x = -\big(-\cos x\big)\Big|_{\pi}^{2\pi} = \cos x\Big|_{\pi}^{2\pi} =$$

$$= \cos 2\pi - \cos \pi$$



Površina između grafa funkcije i x-osi na segmentu $[0, 2\pi]$

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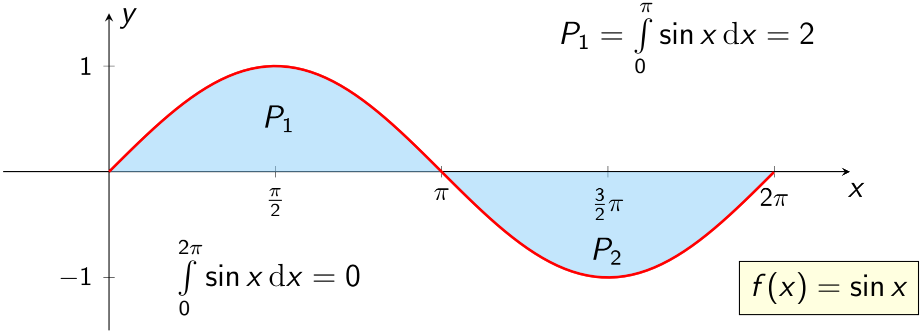
$$= \cos 2\pi - \cos \pi = 1 - (-1)$$



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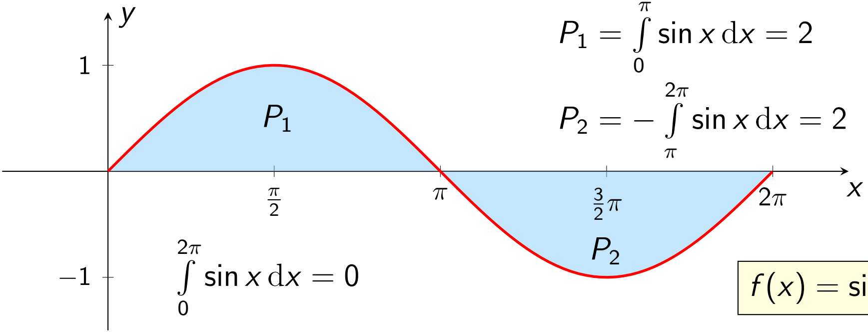
$$= \cos 2\pi - \cos \pi = 1 - (-1) = 1 + 1$$



$$P_1 = \int_0^{\pi} \sin x \,\mathrm{d}x = 2$$

$$P_2 = -\int_{\pi}^{2\pi} \sin x \,\mathrm{d}x = 2$$

$$\int_0^{2\pi} \sin x \,\mathrm{d}x = 0$$

$$\boxed{f(x) = \sin x}$$

Površina između grafa funkcije i x-osi na segmentu $[0, 2\pi]$

$$P_2 = -\int_{\pi}^{2\pi} \sin x \,\mathrm{d}x = -\left(-\cos x\right)\Big|_{\pi}^{2\pi} = \cos x\Big|_{\pi}^{2\pi} =$$

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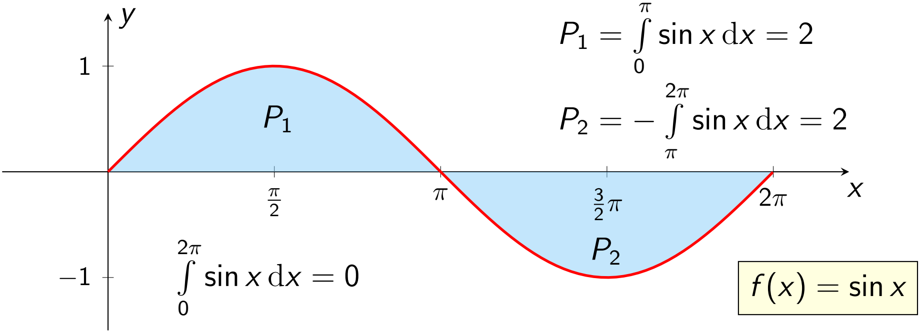
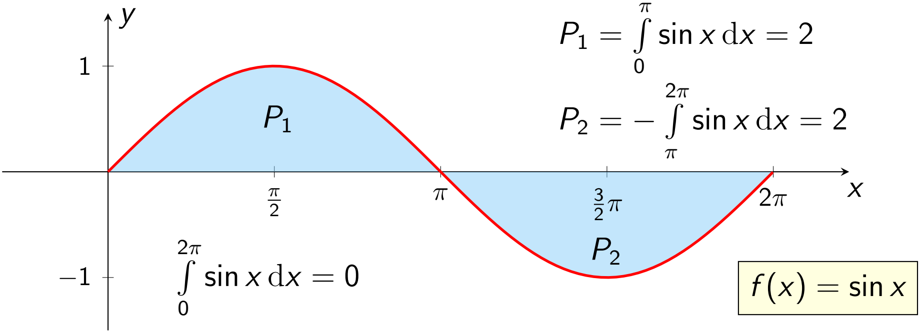


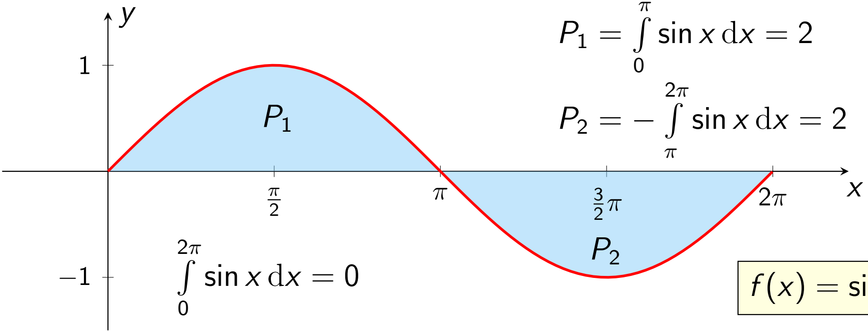
Površina između grafa funkcije i x-osi na segmentu $[0, 2\pi]$

$$P = P_1 + P_2$$



Površina između grafa funkcije i x-osi na segmentu $[0, 2\pi]$

$$P = P_1 + P_2 = 2 + 2$$



$$P_1 = \int\limits_0^{\pi} \sin x \,\mathrm{d}x = 2$$

$$P_2 = -\int\limits_{\pi}^{2\pi} \sin x \,\mathrm{d}x = 2$$

$$\int\limits_0^{2\pi} \sin x \,\mathrm{d}x = 0$$

$$\boxed{f(x) = \sin x}$$

Površina između grafa funkcije i x-osi na segmentu $[0, 2\pi]$

$$P = P_1 + P_2 = 2 + 2 = 4$$

prvi zadatak

Zadatak 1

Zadana je funkcija $f(x) = \dfrac{2}{x^2+1} - 1$.

a) *Izračunajte* $\int_0^{\sqrt{3}} f(x)\,\mathrm{d}x$.

b) *Izračunajte površinu koju graf funkcije* f *zatvara s* x*-osi na segmentu* $\left[0, \sqrt{3}\,\right]$.

Rješenje

a)
$$\int \left(\frac{2}{x^2 + 1} - 1 \right) \mathrm{d}x =$$

Rješenje

a)
$$\int \left(\frac{2}{x^2+1} - 1 \right) \mathrm{d}x = 2 \int \frac{\mathrm{d}x}{x^2+1}$$

Rješenje

a)
$$\int \left(\frac{2}{x^2+1} - 1 \right) \mathrm{d}x = 2 \int \frac{\mathrm{d}x}{x^2+1} -$$

Rješenje

a)
$$\int \left(\frac{2}{x^2+1} - 1 \right) \mathrm{d}x = 2 \int \frac{\mathrm{d}x}{x^2+1} - \int \mathrm{d}x$$

Rješenje

a)
$$\int \left(\frac{2}{x^2+1} - 1 \right) \mathrm{d}x = 2 \int \frac{\mathrm{d}x}{x^2+1} - \int \mathrm{d}x =$$

$$= 2$$

Rješenje

a)

$$\int \left(\frac{2}{x^2+1} - 1\right) \mathrm{d}x = 2\int \frac{\mathrm{d}x}{x^2+1} - \int \mathrm{d}x =$$

$$= 2 \operatorname{arctg} x$$

Rješenje

a)
$$\int \left(\frac{2}{x^2+1} - 1 \right) \mathrm{d}x = 2 \int \frac{\mathrm{d}x}{x^2+1} - \int \mathrm{d}x =$$

$$= 2 \operatorname{arctg} x - x$$

Rješenje

a)
$$\int \left(\frac{2}{x^2+1} - 1 \right) \mathrm{d}x = 2 \int \frac{\mathrm{d}x}{x^2+1} - \int \mathrm{d}x =$$

$$= 2 \operatorname{arctg} x - x + C$$

Rješenje

a)

$$\int \left(\frac{2}{x^2+1} - 1 \right) \mathrm{d}x = 2 \int \frac{\mathrm{d}x}{x^2+1} - \int \mathrm{d}x =$$

$$= 2 \operatorname{arctg} x - x + C, \quad C \in \mathbb{R}$$

Rješenje

a)

$$\int \left(\frac{2}{x^2+1} - 1 \right) \mathrm{d}x = 2 \int \frac{\mathrm{d}x}{x^2+1} - \int \mathrm{d}x =$$

$$= 2 \operatorname{arctg} x - x + C, \quad C \in \mathbb{R}$$

$$\int_0^{\sqrt{3}} \left(\frac{2}{x^2+1} - 1 \right) \mathrm{d}x =$$

Rješenje

a)
$$\int \left(\frac{2}{x^2+1} - 1\right) \mathrm{d}x = 2\int \frac{\mathrm{d}x}{x^2+1} - \int \mathrm{d}x =$$

$$= 2\operatorname{arctg} x - x + C, \quad C \in \mathbb{R}$$

$$\int_0^{\sqrt{3}} \left(\frac{2}{x^2+1} - 1\right) \mathrm{d}x = \left(2\operatorname{arctg} x - x\right)\Big|_0^{\sqrt{3}}$$

Rješenje

a)
$$\int \left(\frac{2}{x^2+1} - 1\right) \mathrm{d}x = 2\int \frac{\mathrm{d}x}{x^2+1} - \int \mathrm{d}x =$$

$$= 2 \operatorname{arctg} x - x + C, \quad C \in \mathbb{R}$$

$$\int_0^{\sqrt{3}} \left(\frac{2}{x^2+1} - 1\right) \mathrm{d}x = \left(2 \operatorname{arctg} x - x\right)\Big|_0^{\sqrt{3}} =$$

$$=$$

Rješenje

a)
$$\int\left(\frac{2}{x^2+1}-1\right)\mathrm{d}x = 2\int\frac{\mathrm{d}x}{x^2+1}-\int\mathrm{d}x =$$

$$= 2\operatorname{arctg} x - x + C, \quad C\in\mathbb{R}$$

$$\int_0^{\sqrt{3}}\left(\frac{2}{x^2+1}-1\right)\mathrm{d}x = \left(2\operatorname{arctg} x - x\right)\Big|_0^{\sqrt{3}} =$$

$$= \left(2\operatorname{arctg}\sqrt{3}-\sqrt{3}\,\right)$$

Rješenje

a)
$$\int \left(\frac{2}{x^2+1} - 1\right) \mathrm{d}x = 2\int \frac{\mathrm{d}x}{x^2+1} - \int \mathrm{d}x =$$

$$= 2\,\mathrm{arctg}\, x - x + C, \quad C \in \mathbb{R}$$

$$\int_0^{\sqrt{3}} \left(\frac{2}{x^2+1} - 1\right) \mathrm{d}x = \left(2\,\mathrm{arctg}\, x - x\right)\Big|_0^{\sqrt{3}} =$$

$$= \left(2\,\mathrm{arctg}\,\sqrt{3} - \sqrt{3}\,\right) -$$

Rješenje

a)

$$\int \left(\frac{2}{x^2+1} - 1\right) \mathrm{d}x = 2\int \frac{\mathrm{d}x}{x^2+1} - \int \mathrm{d}x =$$

$$= 2 \operatorname{arctg} x - x + C, \quad C \in \mathbb{R}$$

$$\int_0^{\sqrt{3}} \left(\frac{2}{x^2+1} - 1\right) \mathrm{d}x = \left(2 \operatorname{arctg} x - x\right)\Big|_0^{\sqrt{3}} =$$

$$= \left(2 \operatorname{arctg} \sqrt{3} - \sqrt{3}\,\right) - \left(2 \operatorname{arctg} 0 - 0\right)$$

Rješenje

a)

$$\int \left(\frac{2}{x^2+1} - 1 \right) \mathrm{d}x = 2 \int \frac{\mathrm{d}x}{x^2+1} - \int \mathrm{d}x =$$

$$= 2 \operatorname{arctg} x - x + C, \quad C \in \mathbb{R}$$

$$\int_0^{\sqrt{3}} \left(\frac{2}{x^2+1} - 1 \right) \mathrm{d}x = \left(2 \operatorname{arctg} x - x\right)\Big|_0^{\sqrt{3}} =$$

$$= \left(2 \operatorname{arctg} \sqrt{3} - \sqrt{3}\,\right) - \left(2 \operatorname{arctg} 0 - 0\right) =$$

$$= 2 \cdot$$

Rješenje

a)

$$\int \left(\frac{2}{x^2+1} - 1\right) \mathrm{d}x = 2\int \frac{\mathrm{d}x}{x^2+1} - \int \mathrm{d}x =$$

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$$= 2 \cdot \frac{\pi}{3}$$

Rješenje

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$$= \left(2 \operatorname{arctg} \sqrt{3} - \sqrt{3}\,\right) - \left(2 \operatorname{arctg} 0 - 0\right) =$$

$$= 2 \cdot \frac{\pi}{3} - \sqrt{3}$$

Rješenje

a)
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Rješenje

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Rješenje

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Rješenje

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$$= 2 \cdot \frac{\pi}{3} - \sqrt{3} - 2 \cdot 0 + 0 = \frac{2}{3}\pi - \sqrt{3} \approx 0.36234$$

Rješenje

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egzaktna vrijednost

Rješenje

a)
$$\int \left(\frac{2}{x^2+1} - 1\right) \mathrm{d}x = 2\int \frac{\mathrm{d}x}{x^2+1} - \int \mathrm{d}x =$$

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egzaktna vrijednost

aproksimacija na pet decimala

b)

$$\boxed{f(x) = \frac{2}{x^2+1} - 1}$$

b) $\lim_{x \to \pm\infty} f(x) =$

$$\boxed{f(x) = \frac{2}{x^2 + 1} - 1}$$

b) $$\lim_{x \to \pm\infty} f(x) = \frac{2}{+\infty}$$

$$\boxed{f(x) = \frac{2}{x^2+1} - 1}$$

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$$\lim_{x\to\pm\infty} f(x) = \frac{2}{+\infty} - 1 = 0 - 1 = -1$$

$$y = -1$$

$$f(x) = \frac{2}{x^2+1} - 1$$

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$$f(x) = \frac{2}{x^2+1} - 1$$

$\boxed{y = -1}$ horizontalna asimptota

b) $$\lim_{x\to\pm\infty} f(x) = \frac{2}{+\infty} - 1 = 0 - 1 = -1$$

$$f(x) = \frac{2}{x^2+1} - 1$$

$$\boxed{y = -1} \leftarrow \text{horizontalna asimptota}$$

$$\frac{2}{x^2+1} - 1 = 0$$

b) $$\lim_{x\to\pm\infty} f(x) = \frac{2}{+\infty} - 1 = 0 - 1 = -1$$

$$f(x) = \frac{2}{x^2+1} - 1$$

$\boxed{y = -1}$ horizontalna asimptota

$$\frac{2}{x^2+1} - 1 = 0 \,\Big/ \cdot (x^2+1)$$

b) $$\lim_{x\to\pm\infty} f(x) = \frac{2}{+\infty} - 1 = 0 - 1 = -1$$

$$f(x) = \frac{2}{x^2+1} - 1$$

$$\boxed{y = -1} \quad \text{horizontalna asimptota}$$

$$\frac{2}{x^2+1} - 1 = 0 \Big/ \cdot (x^2+1)$$

$$2 = x^2 + 1$$

b) $$\lim_{x\to\pm\infty} f(x) = \frac{2}{+\infty} - 1 = 0 - 1 = -1$$

$$\boxed{f(x) = \frac{2}{x^2+1} - 1}$$

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$\boxed{y = -1}$ horizontalna asimptota

$$\frac{2}{x^2+1} - 1 = 0 \,\Big/ \cdot (x^2+1)$$

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$\boxed{y = -1}$ horizontalna asimptota

$$\frac{2}{x^2+1} - 1 = 0 \quad / \cdot (x^2+1)$$

$$2 = x^2 + 1$$

$$x^2 = 1$$

$x_1 = -1 \qquad x_2 = 1$

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$$\boxed{y = -1}$$ horizontalna asimptota

$$\frac{2}{x^2+1} - 1 = 0 \quad / \cdot (x^2+1)$$

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$\boxed{y = -1}$ horizontalna asimptota

$$\frac{2}{x^2+1} - 1 = 0 \,\Big/ \cdot (x^2+1)$$

$$2 = x^2 + 1$$

$$x^2 = 1$$

$\boxed{x_1 = -1}$ $\boxed{x_2 = 1}$

nultočke

b)

$$\lim_{x\to\pm\infty} f(x) = \frac{2}{+\infty} - 1 = 0 - 1 = -1$$

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$\boxed{y = -1}$ horizontalna asimptota

$$\frac{2}{x^2+1} - 1 = 0 \quad / \cdot (x^2+1)$$

$$2 = x^2 + 1$$

$$x^2 = 1$$

$\boxed{x_1 = -1}$ $\boxed{x_2 = 1}$

nultočke

$$f'(x) =$$

b)

$$\boxed{f(x) = \frac{2}{x^2+1} - 1}$$

$$\lim_{x\to\pm\infty} f(x) = \frac{2}{+\infty} - 1 = 0 - 1 = -1$$

$\boxed{y = -1}$ horizontalna asimptota

$$\frac{2}{x^2+1} - 1 = 0 \;\Big/ \cdot (x^2+1)$$

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$\boxed{x_1 = -1}$ $\boxed{x_2 = 1}$

nultočke

$$f'(x) = \rule{10cm}{0.4pt}$$

b) $\lim\limits_{x\to\pm\infty} f(x) = \frac{2}{+\infty} - 1 = 0 - 1 = -1$

$$\boxed{f(x) = \frac{2}{x^2+1} - 1}$$

$\boxed{y = -1}$ horizontalna asimptota

$$\frac{2}{x^2+1} - 1 = 0 \Big/ \cdot (x^2+1)$$

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$\boxed{x_1 = -1}$ $\boxed{x_2 = 1}$

nultočke

$$f'(x) = \frac{\qquad\qquad}{(x^2+1)^2}$$

b)

$$\lim_{x\to\pm\infty} f(x) = \frac{2}{+\infty} - 1 = 0 - 1 = -1$$

$$f(x) = \frac{2}{x^2+1} - 1$$

$\boxed{y = -1}$ horizontalna asimptota

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nultočke

$$f'(x) = \frac{(2)'}{(x^2+1)^2}$$

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$\boxed{y = -1}$ horizontalna asimptota

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nultočke

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$\boxed{y = -1}$ horizontalna asimptota

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nultočke

$$f'(x) = \frac{(2)' \cdot (x^2+1)}{(x^2+1)^2}$$

b) $$\lim_{x\to\pm\infty} f(x) = \frac{2}{+\infty} - 1 = 0 - 1 = -1$$

$$f(x) = \frac{2}{x^2+1} - 1$$

$\boxed{y = -1}$ horizontalna asimptota

$$\frac{2}{x^2+1} - 1 = 0 \quad / \cdot (x^2+1)$$

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$\boxed{x_1 = -1}$ $\boxed{x_2 = 1}$

nultočke

$$f'(x) = \frac{(2)' \cdot (x^2+1) - }{(x^2+1)^2}$$

b) $$\lim_{x\to\pm\infty} f(x) = \frac{2}{+\infty} - 1 = 0 - 1 = -1$$

$$f(x) = \frac{2}{x^2+1} - 1$$

$\boxed{y = -1}$ horizontalna asimptota

$$\frac{2}{x^2+1} - 1 = 0 \Big/ \cdot (x^2+1)$$

$$2 = x^2 + 1$$

$$x^2 = 1$$

$\boxed{x_1 = -1}$ $\boxed{x_2 = 1}$

nultočke

$$f'(x) = \frac{(2)' \cdot (x^2+1) - 2}{(x^2+1)^2}$$

b) $$\lim_{x\to\pm\infty} f(x) = \frac{2}{+\infty} - 1 = 0 - 1 = -1$$

$$\boxed{f(x) = \frac{2}{x^2+1} - 1}$$

$\boxed{y = -1}$ horizontalna asimptota

$$\frac{2}{x^2+1} - 1 = 0 \quad / \cdot (x^2+1)$$

$$2 = x^2 + 1$$

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nultočke

$$f'(x) = \frac{(2)' \cdot (x^2+1) - 2 \cdot}{(x^2+1)^2}$$

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$\boxed{y = -1}$ horizontalna asimptota

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nultočke

$$f'(x) = \frac{(2)' \cdot (x^2+1) - 2 \cdot (x^2+1)'}{(x^2+1)^2}$$

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$\boxed{y = -1}$ horizontalna asimptota

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nultočke

$$f'(x) = \frac{(2)' \cdot (x^2+1) - 2 \cdot (x^2+1)'}{(x^2+1)^2} - 0$$

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$\boxed{y = -1}$ horizontalna asimptota

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nultočke

$$f'(x) = \frac{(2)' \cdot (x^2+1) - 2 \cdot (x^2+1)'}{(x^2+1)^2} - 0 = \underline{\qquad\qquad}$$

b) $$\lim_{x\to\pm\infty} f(x) = \frac{2}{+\infty} - 1 = 0 - 1 = -1$$

$$f(x) = \frac{2}{x^2+1} - 1$$

$\boxed{y = -1}$ horizontalna asimptota

$$\frac{2}{x^2+1} - 1 = 0 \Big/ \cdot (x^2+1)$$

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nultočke

$$f'(x) = \frac{(2)' \cdot (x^2+1) - 2 \cdot (x^2+1)'}{(x^2+1)^2} - 0 = \frac{}{(x^2+1)^2}$$

b) $$\lim_{x\to\pm\infty} f(x) = \frac{2}{+\infty} - 1 = 0 - 1 = -1$$

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$\boxed{y = -1}$ horizontalna asimptota

$$\frac{2}{x^2+1} - 1 = 0 \quad / \cdot (x^2+1)$$

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nultočke

$$f'(x) = \frac{\underbrace{(2)'}_{=0} \cdot (x^2+1) - 2 \cdot (x^2+1)'}{(x^2+1)^2} - 0 = \frac{}{(x^2+1)^2}$$

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$\boxed{y = -1}$ horizontalna asimptota

$$\frac{2}{x^2+1} - 1 = 0 \Big/ \cdot (x^2+1)$$

$$2 = x^2 + 1$$

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$\boxed{x_1 = -1}$ $\boxed{x_2 = 1}$

nultočke

$$f'(x) = \frac{\overbrace{(2)'}^{=0} \cdot (x^2+1) - 2 \cdot \overbrace{(x^2+1)'}^{=2x}}{(x^2+1)^2} - 0 = \frac{}{(x^2+1)^2}$$

b) $$\lim_{x\to\pm\infty} f(x) = \frac{2}{+\infty} - 1 = 0 - 1 = -1$$

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$\boxed{y = -1}$ horizontalna asimptota

$$\frac{2}{x^2+1} - 1 = 0 \Big/ \cdot (x^2+1)$$

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$$x^2 = 1$$

$\boxed{x_1 = -1}$ $\boxed{x_2 = 1}$

nultočke

$$f'(x) = \frac{\overbrace{(2)'}^{=0} \cdot (x^2+1) - 2 \cdot \overbrace{(x^2+1)'}^{=2x}}{(x^2+1)^2} - 0 = \frac{-4x}{(x^2+1)^2}$$

b) $$\lim_{x\to\pm\infty} f(x) = \frac{2}{+\infty} - 1 = 0 - 1 = -1$$

$$\boxed{f(x) = \frac{2}{x^2+1} - 1}$$

$$\boxed{y = -1}$$ horizontalna asimptota

$$\frac{2}{x^2+1} - 1 = 0 \Big/ \cdot (x^2+1)$$

$$2 = x^2 + 1$$

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nultočke

$$f'(x) = \frac{\underbrace{(2)'}_{=0} \cdot (x^2+1) - 2 \cdot \underbrace{(x^2+1)'}_{=2x}}{(x^2+1)^2} - 0 = \frac{-4x}{(x^2+1)^2}$$

$$\frac{-4x}{(x^2+1)^2} = 0$$

b)

$$\lim_{x\to\pm\infty} f(x) = \frac{2}{+\infty} - 1 = 0 - 1 = -1$$

$$f(x) = \frac{2}{x^2+1} - 1$$

$\boxed{y = -1}$ horizontalna asimptota

$$\frac{2}{x^2+1} - 1 = 0 \quad / \cdot (x^2+1)$$

$$2 = x^2 + 1$$

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$\boxed{x_1 = -1}$ $\boxed{x_2 = 1}$

nultočke

$$f'(x) = \frac{\underbrace{(2)'}_{=0} \cdot (x^2+1) - 2 \cdot \overbrace{(x^2+1)'}^{=2x}}{(x^2+1)^2} - 0 = \frac{-4x}{(x^2+1)^2}$$

$$\frac{-4x}{(x^2+1)^2} = 0$$

$$-4x = 0$$

b)
$$\lim_{x\to\pm\infty} f(x) = \frac{2}{+\infty} - 1 = 0 - 1 = -1$$

$$\boxed{f(x) = \frac{2}{x^2+1} - 1}$$

$\boxed{y = -1}$ horizontalna asimptota

$$\frac{2}{x^2+1} - 1 = 0 \quad / \cdot (x^2+1)$$

$$2 = x^2 + 1$$

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$\boxed{x_1 = -1}$ $\boxed{x_2 = 1}$

nultočke

$$f'(x) = \frac{\underbrace{(2)'}_{=0} \cdot (x^2+1) - 2 \cdot \underbrace{(x^2+1)'}_{=2x}}{(x^2+1)^2} - 0 = \frac{-4x}{(x^2+1)^2}$$

$$\frac{-4x}{(x^2+1)^2} = 0$$

$$-4x = 0$$

$$x = 0$$

$$f(x) = \frac{2}{x^2+1} - 1$$

b) $$\lim_{x\to\pm\infty} f(x) = \frac{2}{+\infty} - 1 = 0 - 1 = -1$$

$\boxed{y = -1}$ horizontalna asimptota

$$\frac{2}{x^2+1} - 1 = 0 \;\Big/ \cdot (x^2+1)$$

$$2 = x^2 + 1$$

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$\boxed{x_1 = -1}$ $\boxed{x_2 = 1}$

nultočke

$$f'(x) = \frac{\underbrace{(2)'}_{=0} \cdot (x^2+1) - 2 \cdot \underbrace{(x^2+1)'}_{=2x}}{(x^2+1)^2} - 0 = \frac{-4x}{(x^2+1)^2}$$

$$\frac{-4x}{(x^2+1)^2} = 0$$

$$-4x = 0$$

$$\boxed{x = 0}$$

b) $$\lim_{x\to\pm\infty} f(x) = \frac{2}{+\infty} - 1 = 0 - 1 = -1$$

$$\boxed{f(x) = \frac{2}{x^2+1} - 1}$$

$\boxed{y = -1}$ horizontalna asimptota

$$\frac{2}{x^2+1} - 1 = 0 \Big/ \cdot (x^2+1)$$

$$2 = x^2 + 1$$

$$x^2 = 1$$

$\boxed{x_1 = -1}$ $\boxed{x_2 = 1}$

nultočke

$$f'(x) = \frac{\underbrace{(2)'}_{=0} \cdot (x^2+1) - 2 \cdot \underbrace{(x^2+1)'}_{=2x}}{(x^2+1)^2} - 0 = \frac{-4x}{(x^2+1)^2}$$

$$\frac{-4x}{(x^2+1)^2} = 0$$

$$-4x = 0$$

$\boxed{x = 0}$ stacionarna točka

$$f(x) = \frac{2}{x^2+1} - 1$$

b)

$$\lim_{x\to\pm\infty} f(x) = \frac{2}{+\infty} - 1 = 0 - 1 = -1$$

$\boxed{y = -1}$ ⟵ horizontalna asimptota

$$\frac{2}{x^2+1} - 1 = 0 \Big/ \cdot (x^2+1)$$

$$2 = x^2 + 1$$

$$x^2 = 1$$

$\boxed{x_1 = -1}$ $\boxed{x_2 = 1}$

nultočke

$$f'(x) = \frac{\underbrace{(2)'}_{=0} \cdot (x^2+1) - 2 \cdot \underbrace{(x^2+1)'}_{=2x}}{(x^2+1)^2} - 0 = \frac{-4x}{(x^2+1)^2}$$

$$\frac{-4x}{(x^2+1)^2} = 0$$

$$-4x = 0$$

$\boxed{x = 0}$ ⟵ stacionarna točka

	$-\infty$		0		$+\infty$
f'					
f					

b)

$$\lim_{x\to\pm\infty} f(x) = \frac{2}{+\infty} - 1 = 0 - 1 = -1$$

$$\boxed{f(x) = \frac{2}{x^2+1} - 1}$$

$\boxed{y = -1}$ ⟵ horizontalna asimptota

$$\frac{2}{x^2+1} - 1 = 0 \Big/ \cdot (x^2+1)$$

$$2 = x^2 + 1$$

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$\boxed{x_1 = -1}$ $\boxed{x_2 = 1}$

nultočke

$$f'(x) = \frac{\underbrace{(2)'}_{=0} \cdot (x^2+1) - 2 \cdot \underbrace{(x^2+1)'}_{=2x}}{(x^2+1)^2} - 0 = \frac{-4x}{(x^2+1)^2}$$

$$\frac{-4x}{(x^2+1)^2} = 0$$

$$-4x = 0$$

$\boxed{x = 0}$ ⟵ stacionarna točka

	$-\infty$		0		$+\infty$
f'		$+$			
f					

b) $$\lim_{x\to\pm\infty} f(x) = \frac{2}{+\infty} - 1 = 0 - 1 = -1$$

$$\boxed{f(x) = \frac{2}{x^2+1} - 1}$$

$\boxed{y = -1}$ horizontalna asimptota

$$\frac{2}{x^2+1} - 1 = 0 \Big/ \cdot (x^2+1)$$

$$2 = x^2 + 1$$

$$x^2 = 1$$

$\boxed{x_1 = -1}$ $\boxed{x_2 = 1}$

nultočke

$$f'(x) = \frac{\underbrace{(2)'}_{=0} \cdot (x^2+1) - 2 \cdot \underbrace{(x^2+1)'}_{=2x}}{(x^2+1)^2} - 0 = \frac{-4x}{(x^2+1)^2}$$

$$\frac{-4x}{(x^2+1)^2} = 0$$

$$-4x = 0$$

$\boxed{x = 0}$ stacionarna točka

	$-\infty$		0		$+\infty$
f'		$+$		$-$	
f					

b) $$\lim_{x\to\pm\infty} f(x) = \frac{2}{+\infty} - 1 = 0 - 1 = -1$$

$$f(x) = \frac{2}{x^2+1} - 1$$

$\boxed{y = -1}$ horizontalna asimptota

$$\frac{2}{x^2+1} - 1 = 0 \quad / \cdot (x^2+1)$$

$$2 = x^2 + 1$$

$$x^2 = 1$$

$\boxed{x_1 = -1}$ $\boxed{x_2 = 1}$

nultočke

$$f'(x) = \frac{\underbrace{(2)'}_{=0} \cdot (x^2+1) - 2 \cdot \underbrace{(x^2+1)'}_{=2x}}{(x^2+1)^2} - 0 = \frac{-4x}{(x^2+1)^2}$$

$$\frac{-4x}{(x^2+1)^2} = 0$$

$$-4x = 0$$

$\boxed{x = 0}$ stacionarna točka

	$-\infty$		0		$+\infty$
f'		$+$		$-$	
f		$\nearrow$			

b) $$\lim_{x\to\pm\infty} f(x) = \frac{2}{+\infty} - 1 = 0 - 1 = -1$$

$$f(x) = \frac{2}{x^2+1} - 1$$

$\boxed{y = -1}$ horizontalna asimptota

$$\frac{2}{x^2+1} - 1 = 0 \quad / \cdot (x^2+1)$$

$$2 = x^2 + 1$$

$$x^2 = 1$$

$\boxed{x_1 = -1}$ $\boxed{x_2 = 1}$

nultočke

$$f'(x) = \frac{\underbrace{(2)'}_{=0} \cdot (x^2+1) - 2 \cdot \underbrace{(x^2+1)'}_{=2x}}{(x^2+1)^2} - 0 = \frac{-4x}{(x^2+1)^2}$$

$$\frac{-4x}{(x^2+1)^2} = 0$$

$$-4x = 0$$

$\boxed{x = 0}$ stacionarna točka

	$-\infty$		0		$+\infty$
f'		$+$		$-$	
f		$\nearrow$		$\searrow$	

b) $\lim\limits_{x\to\pm\infty} f(x) = \dfrac{2}{+\infty} - 1 = 0 - 1 = -1$

$$\boxed{f(x) = \frac{2}{x^2+1} - 1}$$

$\boxed{y = -1}$ horizontalna asimptota

$$\frac{2}{x^2+1} - 1 = 0 \Big/ \cdot (x^2+1)$$

$$2 = x^2 + 1$$

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$\boxed{x_1 = -1}$ $\boxed{x_2 = 1}$

nultočke

$$f'(x) = \frac{\underbrace{(2)'}_{=0} \cdot (x^2+1) - 2 \cdot \underbrace{(x^2+1)'}_{=2x}}{(x^2+1)^2} - 0 = \frac{-4x}{(x^2+1)^2}$$

$$\frac{-4x}{(x^2+1)^2} = 0$$

$$-4x = 0$$

$\boxed{x = 0}$ stacionarna točka

$f(0) = 1$

	$-\infty$	0	$+\infty$
f'	$+$	$-$	
f	$\nearrow$	$\searrow$	

b) $\lim\limits_{x\to\pm\infty} f(x) = \dfrac{2}{+\infty} - 1 = 0 - 1 = -1$

$$\boxed{f(x) = \frac{2}{x^2+1} - 1}$$

$\boxed{y = -1}$ horizontalna asimptota

$$\frac{2}{x^2+1} - 1 = 0 \quad / \cdot (x^2+1)$$

$$2 = x^2 + 1$$

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$\boxed{x_1 = -1}$ $\boxed{x_2 = 1}$

nultočke

$$f'(x) = \frac{\underbrace{(2)'}_{=0} \cdot (x^2+1) - 2 \cdot \underbrace{(x^2+1)'}_{=2x}}{(x^2+1)^2} - 0 = \frac{-4x}{(x^2+1)^2}$$

$$\frac{-4x}{(x^2+1)^2} = 0$$

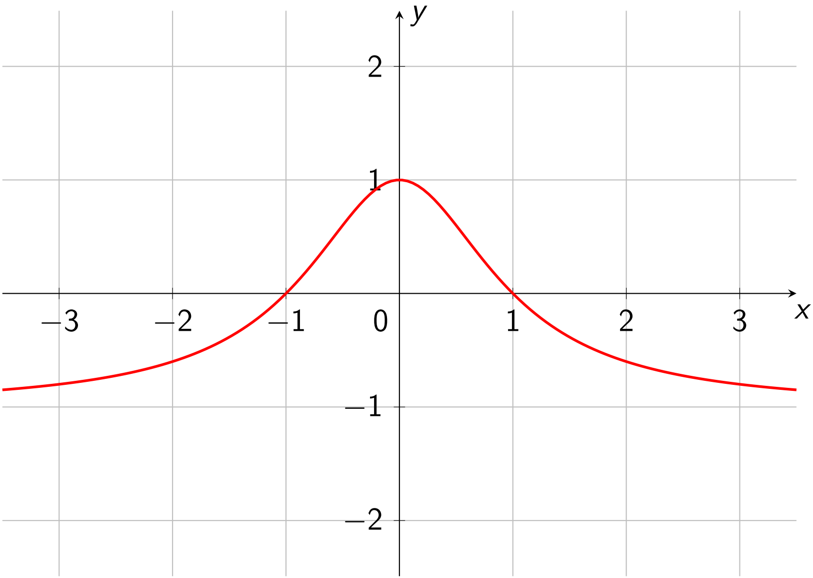
$$-4x = 0$$

$\boxed{x = 0}$ stacionarna točka

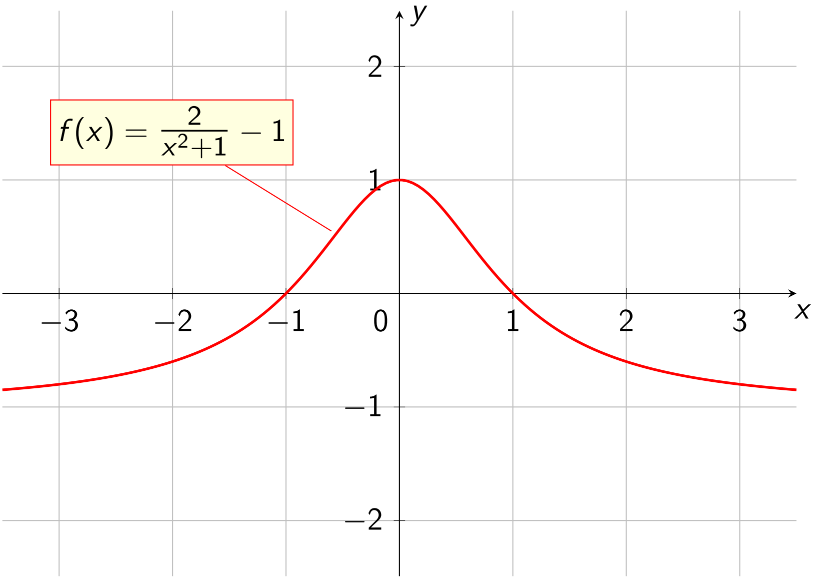
$f(0) = 1$

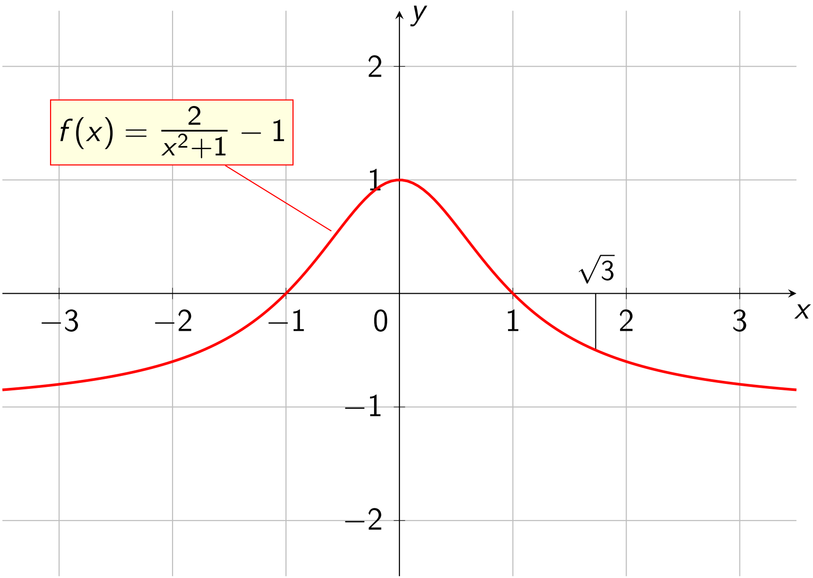
globalni maksimum

	$-\infty$		0		$+\infty$
f'		$+$		$-$	
f		$\nearrow$		$\searrow$	

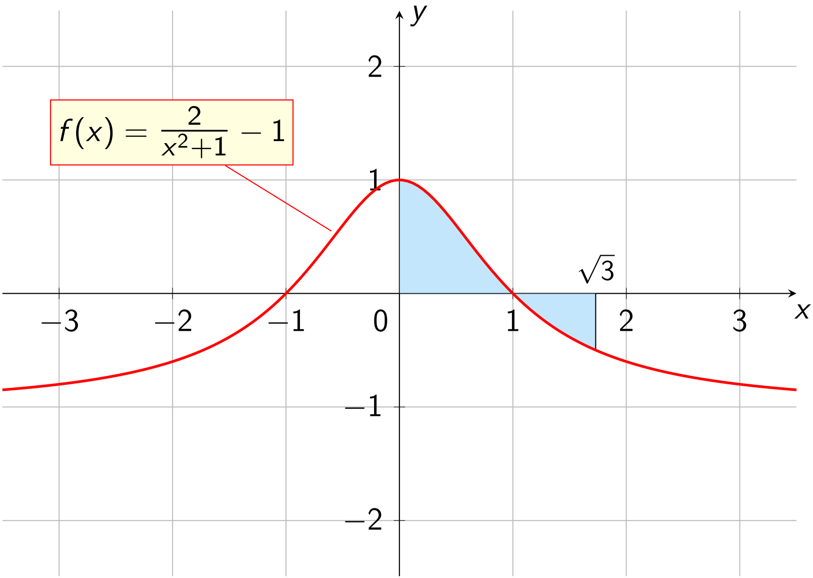


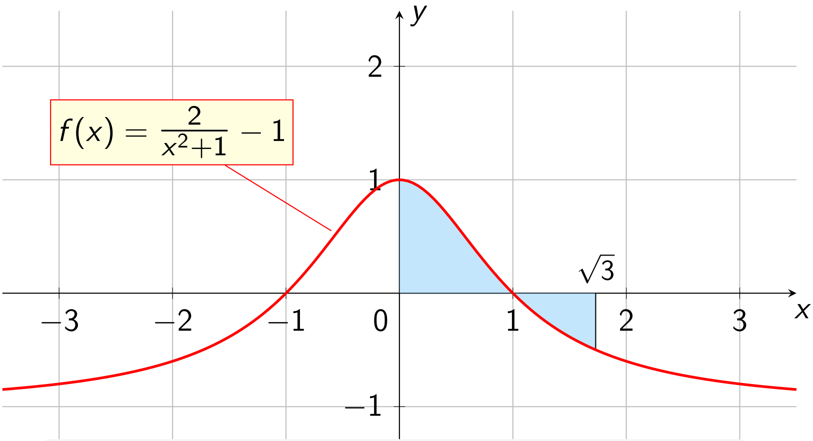
y
2
1
−3
−2
−1
0
1
2
3
x
−1
−2


$f(x) = \frac{2}{x^2+1} - 1$
y
2
1
−3
−2
−1
0
1
2
3
x
−1
−2

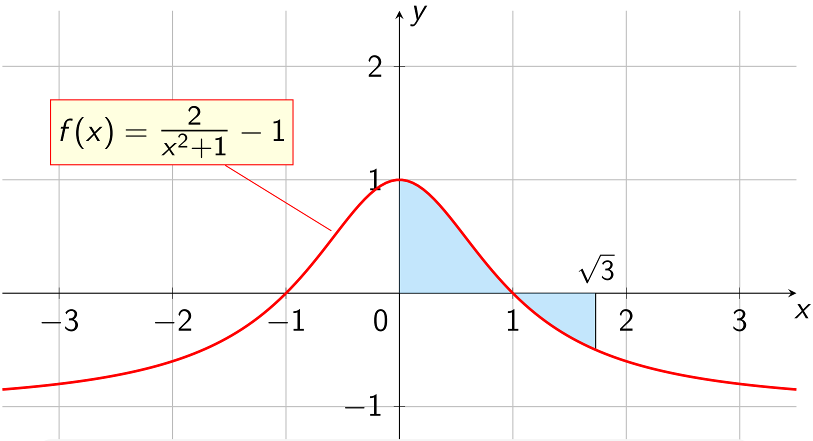


y
2
$f(x) = \frac{2}{x^2+1} - 1$
1
$\sqrt{3}$
-3
-2
-1
0
1
2
3
x
-1
-2

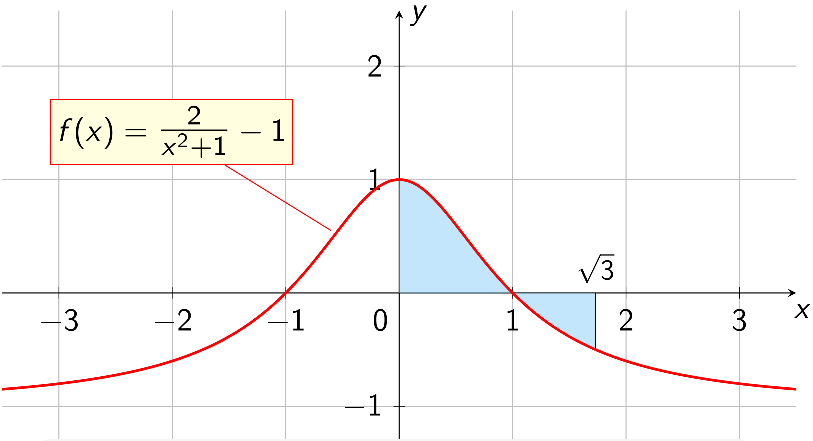

$f(x) = \frac{2}{x^2+1} - 1$
y
x
2
1
-1
-2
-3
-2
-1
0
1
2
3
$\sqrt{3}$



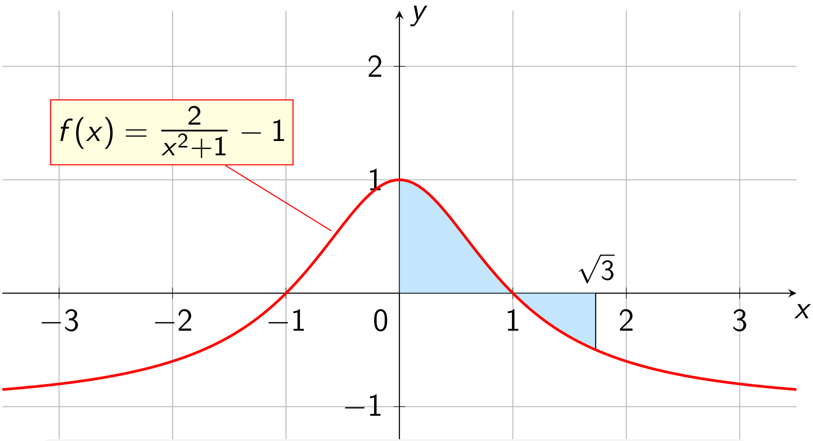
$P =$



$$P = \int_0^1 \left(\frac{2}{x^2+1} - 1 \right) \mathrm{d}x$$



$$P = \int_0^1 \left(\frac{2}{x^2+1} - 1 \right) \mathrm{d}x -$$



$$P = \int_0^1 \left(\frac{2}{x^2+1} - 1\right) \mathrm{d}x - \int_1^{\sqrt{3}} \left(\frac{2}{x^2+1} - 1\right) \mathrm{d}x$$

$$P = \int_0^1 \left(\frac{2}{x^2+1} - 1 \right) \mathrm{d}x - \int_1^{\sqrt{3}} \left(\frac{2}{x^2+1} - 1 \right) \mathrm{d}x$$

$$P = \int_0^1 \left(\frac{2}{x^2+1} - 1 \right) \mathrm{d}x - \int_1^{\sqrt{3}} \left(\frac{2}{x^2+1} - 1 \right) \mathrm{d}x =$$

$$=$$

$$P = \int_0^1 \left(\frac{2}{x^2+1} - 1 \right) \mathrm{d}x - \int_1^{\sqrt{3}} \left(\frac{2}{x^2+1} - 1 \right) \mathrm{d}x =$$

$$= \big(2 \operatorname{arctg} x - x\big)\Big|_0^1$$

$$P = \int_0^1 \left(\frac{2}{x^2+1} - 1 \right) \mathrm{d}x - \int_1^{\sqrt{3}} \left(\frac{2}{x^2+1} - 1 \right) \mathrm{d}x =$$

$$= \left(2 \operatorname{arctg} x - x\right)\Big|_0^1 -$$

$$P = \int_0^1 \left(\frac{2}{x^2+1} - 1 \right) \mathrm{d}x - \int_1^{\sqrt{3}} \left(\frac{2}{x^2+1} - 1 \right) \mathrm{d}x =$$

$$= \big(2 \operatorname{arctg} x - x\big)\Big|_0^1 - \big(2 \operatorname{arctg} x - x\big)\Big|_1^{\sqrt{3}}$$

$$P=\int_0^1\left(\frac{2}{x^2+1}-1\right)\mathrm{d}x-\int_1^{\sqrt{3}}\left(\frac{2}{x^2+1}-1\right)\mathrm{d}x=$$

$$=\left(2\operatorname{arctg}x-x\right)\Big|_0^1-\left(2\operatorname{arctg}x-x\right)\Big|_1^{\sqrt{3}}=$$

$$P = \int_0^1 \left(\frac{2}{x^2+1} - 1\right) \mathrm{d}x - \int_1^{\sqrt{3}} \left(\frac{2}{x^2+1} - 1\right) \mathrm{d}x =$$

$$= \big(2 \operatorname{arctg} x - x\big)\Big|_0^1 - \big(2 \operatorname{arctg} x - x\big)\Big|_1^{\sqrt{3}} =$$

$$= \Big[\qquad\qquad\qquad\qquad\qquad\qquad \Big]$$

$$P = \int_0^1 \left(\frac{2}{x^2+1} - 1 \right) \mathrm{d}x - \int_1^{\sqrt{3}} \left(\frac{2}{x^2+1} - 1 \right) \mathrm{d}x =$$

$$= \big(2 \operatorname{arctg} x - x\big)\Big|_0^1 - \big(2 \operatorname{arctg} x - x\big)\Big|_1^{\sqrt{3}} =$$

$$= \Big[\big(2 \operatorname{arctg} 1 - 1\big) \qquad\qquad\qquad\qquad \Big]$$

$$P=\int_0^1\left(\frac{2}{x^2+1}-1\right)\mathrm{d}x-\int_1^{\sqrt{3}}\left(\frac{2}{x^2+1}-1\right)\mathrm{d}x=$$

$$=\left(2\operatorname{arctg}x-x\right)\Big|_0^1-\left(2\operatorname{arctg}x-x\right)\Big|_1^{\sqrt{3}}=$$

$$=\Big[\left(2\operatorname{arctg}1-1\right)-\qquad\qquad\qquad\qquad\Big]$$

$$P = \int_0^1 \left(\frac{2}{x^2+1} - 1 \right) \mathrm{d}x - \int_1^{\sqrt{3}} \left(\frac{2}{x^2+1} - 1 \right) \mathrm{d}x =$$

$$= \left(2 \operatorname{arctg} x - x\right)\Big|_0^1 - \left(2 \operatorname{arctg} x - x\right)\Big|_1^{\sqrt{3}} =$$

$$= \Big[\left(2 \operatorname{arctg} 1 - 1\right) - \left(2 \operatorname{arctg} 0 - 0\right) \Big]$$

$$P = \int_0^1 \left(\frac{2}{x^2+1} - 1 \right) \mathrm{d}x - \int_1^{\sqrt{3}} \left(\frac{2}{x^2+1} - 1 \right) \mathrm{d}x =$$

$$= \big(2 \operatorname{arctg} x - x\big)\Big|_0^1 - \big(2 \operatorname{arctg} x - x\big)\Big|_1^{\sqrt{3}} =$$

$$= \Big[\big(2 \operatorname{arctg} 1 - 1\big) - \big(2 \operatorname{arctg} 0 - 0\big)\Big] -$$

$$- \Big[\qquad\qquad\qquad\qquad\qquad\qquad \Big]$$

$$P = \int_0^1 \left(\frac{2}{x^2+1} - 1\right) \mathrm{d}x - \int_1^{\sqrt{3}} \left(\frac{2}{x^2+1} - 1\right) \mathrm{d}x =$$

$$= \big(2 \operatorname{arctg} x - x\big)\Big|_0^1 - \big(2 \operatorname{arctg} x - x\big)\Big|_1^{\sqrt{3}} =$$

$$= \Big[\big(2 \operatorname{arctg} 1 - 1\big) - \big(2 \operatorname{arctg} 0 - 0\big)\Big] -$$

$$- \Big[\big(2 \operatorname{arctg} \sqrt{3} - \sqrt{3}\,\big) \qquad\qquad\qquad\qquad \Big]$$

$$P = \int_0^1 \left(\frac{2}{x^2+1} - 1 \right) \mathrm{d}x - \int_1^{\sqrt{3}} \left(\frac{2}{x^2+1} - 1 \right) \mathrm{d}x =$$

$$= \left(2 \operatorname{arctg} x - x\right)\Big|_0^1 - \left(2 \operatorname{arctg} x - x\right)\Big|_1^{\sqrt{3}} =$$

$$= \Big[\left(2 \operatorname{arctg} 1 - 1\right) - \left(2 \operatorname{arctg} 0 - 0\right) \Big] -$$

$$- \Big[\left(2 \operatorname{arctg} \sqrt{3} - \sqrt{3}\,\right) - \qquad\qquad\qquad \Big]$$

$$P = \int_0^1 \left(\frac{2}{x^2+1} - 1 \right) \mathrm{d}x - \int_1^{\sqrt{3}} \left(\frac{2}{x^2+1} - 1 \right) \mathrm{d}x =$$

$$= \big(2 \operatorname{arctg} x - x\big)\Big|_0^1 - \big(2 \operatorname{arctg} x - x\big)\Big|_1^{\sqrt{3}} =$$

$$= \Big[\big(2 \operatorname{arctg} 1 - 1\big) - \big(2 \operatorname{arctg} 0 - 0\big)\Big] -$$

$$- \Big[\big(2 \operatorname{arctg} \sqrt{3} - \sqrt{3}\,\big) - \big(2 \operatorname{arctg} 1 - 1\big)\Big]$$

$$P = \int_0^1 \left(\frac{2}{x^2+1} - 1 \right) \mathrm{d}x - \int_1^{\sqrt{3}} \left(\frac{2}{x^2+1} - 1 \right) \mathrm{d}x =$$

$$= \big(2 \operatorname{arctg} x - x\big)\Big|_0^1 - \big(2 \operatorname{arctg} x - x\big)\Big|_1^{\sqrt{3}} =$$

$$= \Big[\big(2 \operatorname{arctg} 1 - 1\big) - \big(2 \operatorname{arctg} 0 - 0\big)\Big] -$$

$$- \Big[\big(2 \operatorname{arctg} \sqrt{3} - \sqrt{3}\,\big) - \big(2 \operatorname{arctg} 1 - 1\big)\Big] =$$

$$=$$

$$P = \int_0^1 \left(\frac{2}{x^2+1} - 1\right) \mathrm{d}x - \int_1^{\sqrt{3}} \left(\frac{2}{x^2+1} - 1\right) \mathrm{d}x =$$

$$= \big(2 \operatorname{arctg} x - x\big)\Big|_0^1 - \big(2 \operatorname{arctg} x - x\big)\Big|_1^{\sqrt{3}} =$$

$$= \Big[\big(2 \operatorname{arctg} 1 - 1\big) - \big(2 \operatorname{arctg} 0 - 0\big)\Big] -$$

$$- \Big[\big(2 \operatorname{arctg} \sqrt{3} - \sqrt{3}\,\big) - \big(2 \operatorname{arctg} 1 - 1\big)\Big] =$$

$$= \left(2 \cdot \frac{\pi}{4} - 1\right)$$

$$P = \int_0^1 \left(\frac{2}{x^2+1} - 1 \right) \mathrm{d}x - \int_1^{\sqrt{3}} \left(\frac{2}{x^2+1} - 1 \right) \mathrm{d}x =$$

$$= \big(2 \operatorname{arctg} x - x\big)\Big|_0^1 - \big(2 \operatorname{arctg} x - x\big)\Big|_1^{\sqrt{3}} =$$

$$= \Big[\big(2 \operatorname{arctg} 1 - 1\big) - \big(2 \operatorname{arctg} 0 - 0\big)\Big] -$$

$$- \Big[\big(2 \operatorname{arctg} \sqrt{3} - \sqrt{3}\,\big) - \big(2 \operatorname{arctg} 1 - 1\big)\Big] =$$

$$= \left(2 \cdot \frac{\pi}{4} - 1\right) - \big(2 \cdot 0 - 0\big)$$

$$P = \int_0^1 \left(\frac{2}{x^2+1} - 1 \right) \mathrm{d}x - \int_1^{\sqrt{3}} \left(\frac{2}{x^2+1} - 1 \right) \mathrm{d}x =$$

$$= \big(2 \operatorname{arctg} x - x\big)\Big|_0^1 - \big(2 \operatorname{arctg} x - x\big)\Big|_1^{\sqrt{3}} =$$

$$= \Big[\big(2 \operatorname{arctg} 1 - 1\big) - \big(2 \operatorname{arctg} 0 - 0\big)\Big] -$$

$$- \Big[\big(2 \operatorname{arctg} \sqrt{3} - \sqrt{3}\,\big) - \big(2 \operatorname{arctg} 1 - 1\big)\Big] =$$

$$= \left(2 \cdot \frac{\pi}{4} - 1\right) - \big(2 \cdot 0 - 0\big) - \left(2 \cdot \frac{\pi}{3} - \sqrt{3}\right)$$

$$P = \int_0^1 \left(\frac{2}{x^2+1} - 1 \right) \mathrm{d}x - \int_1^{\sqrt{3}} \left(\frac{2}{x^2+1} - 1 \right) \mathrm{d}x =$$

$$= \left(2 \operatorname{arctg} x - x\right)\Big|_0^1 - \left(2 \operatorname{arctg} x - x\right)\Big|_1^{\sqrt{3}} =$$

$$= \Big[\left(2 \operatorname{arctg} 1 - 1\right) - \left(2 \operatorname{arctg} 0 - 0\right)\Big] -$$

$$- \Big[\left(2 \operatorname{arctg} \sqrt{3} - \sqrt{3}\,\right) - \left(2 \operatorname{arctg} 1 - 1\right)\Big] =$$

$$= \left(2 \cdot \frac{\pi}{4} - 1\right) - (2 \cdot 0 - 0) - \left(2 \cdot \frac{\pi}{3} - \sqrt{3}\right) + \left(2 \cdot \frac{\pi}{4} - 1\right)$$

$$P = \int_0^1 \left(\frac{2}{x^2+1} - 1 \right) \mathrm{d}x - \int_1^{\sqrt{3}} \left(\frac{2}{x^2+1} - 1 \right) \mathrm{d}x =$$

$$= \left(2 \operatorname{arctg} x - x\right)\Big|_0^1 - \left(2 \operatorname{arctg} x - x\right)\Big|_1^{\sqrt{3}} =$$

$$= \Big[\left(2 \operatorname{arctg} 1 - 1\right) - \left(2 \operatorname{arctg} 0 - 0\right)\Big] -$$

$$- \Big[\left(2 \operatorname{arctg} \sqrt{3} - \sqrt{3}\,\right) - \left(2 \operatorname{arctg} 1 - 1\right)\Big] =$$

$$= \left(2 \cdot \frac{\pi}{4} - 1\right) - (2 \cdot 0 - 0) - \left(2 \cdot \frac{\pi}{3} - \sqrt{3}\right) + \left(2 \cdot \frac{\pi}{4} - 1\right) =$$

$$=$$

$$P = \int_0^1 \left(\frac{2}{x^2+1} - 1 \right) \mathrm{d}x - \int_1^{\sqrt{3}} \left(\frac{2}{x^2+1} - 1 \right) \mathrm{d}x =$$

$$= \left(2 \operatorname{arctg} x - x\right)\Big|_0^1 - \left(2 \operatorname{arctg} x - x\right)\Big|_1^{\sqrt{3}} =$$

$$= \Big[\left(2 \operatorname{arctg} 1 - 1\right) - \left(2 \operatorname{arctg} 0 - 0\right)\Big] -$$

$$- \Big[\left(2 \operatorname{arctg} \sqrt{3} - \sqrt{3}\,\right) - \left(2 \operatorname{arctg} 1 - 1\right)\Big] =$$

$$= \left(2 \cdot \frac{\pi}{4} - 1\right) - \left(2 \cdot 0 - 0\right) - \left(2 \cdot \frac{\pi}{3} - \sqrt{3}\right) + \left(2 \cdot \frac{\pi}{4} - 1\right) =$$

$$= \frac{\pi}{2} - 1$$

$$P = \int_0^1 \left(\frac{2}{x^2+1} - 1 \right) \mathrm{d}x - \int_1^{\sqrt{3}} \left(\frac{2}{x^2+1} - 1 \right) \mathrm{d}x =$$

$$= \big(2 \operatorname{arctg} x - x\big)\Big|_0^1 - \big(2 \operatorname{arctg} x - x\big)\Big|_1^{\sqrt{3}} =$$

$$= \Big[\big(2 \operatorname{arctg} 1 - 1\big) - \big(2 \operatorname{arctg} 0 - 0\big)\Big] -$$

$$- \Big[\big(2 \operatorname{arctg} \sqrt{3} - \sqrt{3}\,\big) - \big(2 \operatorname{arctg} 1 - 1\big)\Big] =$$

$$= \left(2 \cdot \frac{\pi}{4} - 1\right) - (2 \cdot 0 - 0) - \left(2 \cdot \frac{\pi}{3} - \sqrt{3}\right) + \left(2 \cdot \frac{\pi}{4} - 1\right) =$$

$$= \frac{\pi}{2} - 1 - 0$$

$$
\begin{aligned}
P &= \int_0^1 \left(\frac{2}{x^2+1} - 1\right) \mathrm{d}x - \int_1^{\sqrt{3}} \left(\frac{2}{x^2+1} - 1\right) \mathrm{d}x = \\
&= \big(2 \operatorname{arctg} x - x\big)\Big|_0^1 - \big(2 \operatorname{arctg} x - x\big)\Big|_1^{\sqrt{3}} = \\
&= \Big[\big(2 \operatorname{arctg} 1 - 1\big) - \big(2 \operatorname{arctg} 0 - 0\big)\Big] - \\
&\qquad - \Big[\big(2 \operatorname{arctg} \sqrt{3} - \sqrt{3}\,\big) - \big(2 \operatorname{arctg} 1 - 1\big)\Big] = \\
&= \left(2 \cdot \frac{\pi}{4} - 1\right) - \big(2 \cdot 0 - 0\big) - \left(2 \cdot \frac{\pi}{3} - \sqrt{3}\right) + \left(2 \cdot \frac{\pi}{4} - 1\right) = \\
&= \frac{\pi}{2} - 1 - 0 - \frac{2}{3}\pi + \sqrt{3}
\end{aligned}
$$

$$P = \int_0^1 \left(\frac{2}{x^2+1} - 1\right) \mathrm{d}x - \int_1^{\sqrt{3}} \left(\frac{2}{x^2+1} - 1\right) \mathrm{d}x =$$

$$= \big(2 \operatorname{arctg} x - x\big)\Big|_0^1 - \big(2 \operatorname{arctg} x - x\big)\Big|_1^{\sqrt{3}} =$$

$$= \Big[\big(2 \operatorname{arctg} 1 - 1\big) - \big(2 \operatorname{arctg} 0 - 0\big)\Big] -$$

$$- \Big[\big(2 \operatorname{arctg} \sqrt{3} - \sqrt{3}\,\big) - \big(2 \operatorname{arctg} 1 - 1\big)\Big] =$$

$$= \left(2 \cdot \frac{\pi}{4} - 1\right) - \big(2 \cdot 0 - 0\big) - \left(2 \cdot \frac{\pi}{3} - \sqrt{3}\right) + \left(2 \cdot \frac{\pi}{4} - 1\right) =$$

$$= \frac{\pi}{2} - 1 - 0 - \frac{2}{3}\pi + \sqrt{3} + \frac{\pi}{2} - 1$$

$$P = \int_0^1 \left(\frac{2}{x^2+1} - 1\right) \mathrm{d}x - \int_1^{\sqrt{3}} \left(\frac{2}{x^2+1} - 1\right) \mathrm{d}x =$$

$$= \left(2 \operatorname{arctg} x - x\right)\Big|_0^1 - \left(2 \operatorname{arctg} x - x\right)\Big|_1^{\sqrt{3}} =$$

$$= \Big[\left(2 \operatorname{arctg} 1 - 1\right) - \left(2 \operatorname{arctg} 0 - 0\right)\Big] -$$

$$- \Big[\left(2 \operatorname{arctg} \sqrt{3} - \sqrt{3}\,\right) - \left(2 \operatorname{arctg} 1 - 1\right)\Big] =$$

$$= \left(2 \cdot \frac{\pi}{4} - 1\right) - \left(2 \cdot 0 - 0\right) - \left(2 \cdot \frac{\pi}{3} - \sqrt{3}\right) + \left(2 \cdot \frac{\pi}{4} - 1\right) =$$

$$= \frac{\pi}{2} - 1 - 0 - \frac{2}{3}\pi + \sqrt{3} + \frac{\pi}{2} - 1 =$$

$$P = \int_0^1 \left(\frac{2}{x^2+1} - 1 \right) \mathrm{d}x - \int_1^{\sqrt{3}} \left(\frac{2}{x^2+1} - 1 \right) \mathrm{d}x =$$

$$= \big(2 \operatorname{arctg} x - x\big)\Big|_0^1 - \big(2 \operatorname{arctg} x - x\big)\Big|_1^{\sqrt{3}} =$$

$$= \Big[\big(2 \operatorname{arctg} 1 - 1\big) - \big(2 \operatorname{arctg} 0 - 0\big)\Big] -$$

$$- \Big[\big(2 \operatorname{arctg} \sqrt{3} - \sqrt{3}\,\big) - \big(2 \operatorname{arctg} 1 - 1\big)\Big] =$$

$$= \left(2 \cdot \frac{\pi}{4} - 1\right) - (2 \cdot 0 - 0) - \left(2 \cdot \frac{\pi}{3} - \sqrt{3}\right) + \left(2 \cdot \frac{\pi}{4} - 1\right) =$$

$$= \frac{\pi}{2} - 1 - 0 - \frac{2}{3}\pi + \sqrt{3} + \frac{\pi}{2} - 1 = \frac{\pi}{3}$$

$$P = \int_0^1 \left(\frac{2}{x^2+1} - 1\right) \mathrm{d}x - \int_1^{\sqrt{3}} \left(\frac{2}{x^2+1} - 1\right) \mathrm{d}x =$$

$$= \left(2 \operatorname{arctg} x - x\right)\Big|_0^1 - \left(2 \operatorname{arctg} x - x\right)\Big|_1^{\sqrt{3}} =$$

$$= \Big[\left(2 \operatorname{arctg} 1 - 1\right) - \left(2 \operatorname{arctg} 0 - 0\right)\Big] -$$

$$- \Big[\left(2 \operatorname{arctg} \sqrt{3} - \sqrt{3}\,\right) - \left(2 \operatorname{arctg} 1 - 1\right)\Big] =$$

$$= \left(2 \cdot \frac{\pi}{4} - 1\right) - \left(2 \cdot 0 - 0\right) - \left(2 \cdot \frac{\pi}{3} - \sqrt{3}\right) + \left(2 \cdot \frac{\pi}{4} - 1\right) =$$

$$= \frac{\pi}{2} - 1 - 0 - \frac{2}{3}\pi + \sqrt{3} + \frac{\pi}{2} - 1 = \frac{\pi}{3} + \sqrt{3}$$

$$P=\int_0^1\left(\frac{2}{x^2+1}-1\right)\mathrm{d}x-\int_1^{\sqrt{3}}\left(\frac{2}{x^2+1}-1\right)\mathrm{d}x=$$

$$=\big(2\operatorname{arctg}x-x\big)\Big|_0^1-\big(2\operatorname{arctg}x-x\big)\Big|_1^{\sqrt{3}}=$$

$$=\Big[\big(2\operatorname{arctg}1-1\big)-\big(2\operatorname{arctg}0-0\big)\Big]-$$

$$-\Big[\big(2\operatorname{arctg}\sqrt{3}-\sqrt{3}\,\big)-\big(2\operatorname{arctg}1-1\big)\Big]=$$

$$=\left(2\cdot\frac{\pi}{4}-1\right)-\big(2\cdot 0-0\big)-\left(2\cdot\frac{\pi}{3}-\sqrt{3}\right)+\left(2\cdot\frac{\pi}{4}-1\right)=$$

$$=\frac{\pi}{2}-1-0-\frac{2}{3}\pi+\sqrt{3}+\frac{\pi}{2}-1=\frac{\pi}{3}+\sqrt{3}-2$$

$$P = \int_0^1 \left(\frac{2}{x^2+1} - 1\right) \mathrm{d}x - \int_1^{\sqrt{3}} \left(\frac{2}{x^2+1} - 1\right) \mathrm{d}x =$$

$$= \big(2 \operatorname{arctg} x - x\big)\Big|_0^1 - \big(2 \operatorname{arctg} x - x\big)\Big|_1^{\sqrt{3}} =$$

$$= \Big[\big(2 \operatorname{arctg} 1 - 1\big) - \big(2 \operatorname{arctg} 0 - 0\big)\Big] -$$

$$- \Big[\big(2 \operatorname{arctg} \sqrt{3} - \sqrt{3}\,\big) - \big(2 \operatorname{arctg} 1 - 1\big)\Big] =$$

$$= \left(2 \cdot \frac{\pi}{4} - 1\right) - (2 \cdot 0 - 0) - \left(2 \cdot \frac{\pi}{3} - \sqrt{3}\right) + \left(2 \cdot \frac{\pi}{4} - 1\right) =$$

$$= \frac{\pi}{2} - 1 - 0 - \frac{2}{3}\pi + \sqrt{3} + \frac{\pi}{2} - 1 = \frac{\pi}{3} + \sqrt{3} - 2 \approx 0.77925$$

$$P = \int_0^1 \left(\frac{2}{x^2+1} - 1\right) \mathrm{d}x - \int_1^{\sqrt{3}} \left(\frac{2}{x^2+1} - 1\right) \mathrm{d}x =$$

$$= \left(2 \operatorname{arctg} x - x\right)\Big|_0^1 - \left(2 \operatorname{arctg} x - x\right)\Big|_1^{\sqrt{3}} =$$

$$= \Big[\big(2 \operatorname{arctg} 1 - 1\big) - \big(2 \operatorname{arctg} 0 - 0\big)\Big] -$$

$$- \Big[\big(2 \operatorname{arctg} \sqrt{3} - \sqrt{3}\,\big) - \big(2 \operatorname{arctg} 1 - 1\big)\Big] =$$

$$= \left(2 \cdot \frac{\pi}{4} - 1\right) - (2 \cdot 0 - 0) - \left(2 \cdot \frac{\pi}{3} - \sqrt{3}\right) + \left(2 \cdot \frac{\pi}{4} - 1\right) =$$

$$= \frac{\pi}{2} - 1 - 0 - \frac{2}{3}\pi + \sqrt{3} + \frac{\pi}{2} - 1 = \boxed{\frac{\pi}{3} + \sqrt{3} - 2} \approx 0.77925$$

$$P = \int_0^1 \left(\frac{2}{x^2+1} - 1\right) \mathrm{d}x - \int_1^{\sqrt{3}} \left(\frac{2}{x^2+1} - 1\right) \mathrm{d}x =$$

$$= \left(2 \operatorname{arctg} x - x\right)\Big|_0^1 - \left(2 \operatorname{arctg} x - x\right)\Big|_1^{\sqrt{3}} =$$

$$= \Big[\left(2 \operatorname{arctg} 1 - 1\right) - \left(2 \operatorname{arctg} 0 - 0\right)\Big] -$$

$$- \Big[\left(2 \operatorname{arctg} \sqrt{3} - \sqrt{3}\,\right) - \left(2 \operatorname{arctg} 1 - 1\right)\Big] =$$

$$= \left(2 \cdot \frac{\pi}{4} - 1\right) - \left(2 \cdot 0 - 0\right) - \left(2 \cdot \frac{\pi}{3} - \sqrt{3}\right) + \left(2 \cdot \frac{\pi}{4} - 1\right) =$$

$$= \frac{\pi}{2} - 1 - 0 - \frac{2}{3}\pi + \sqrt{3} + \frac{\pi}{2} - 1 = \boxed{\frac{\pi}{3} + \sqrt{3} - 2} \approx 0.77925$$

egzaktna vrijednost

$$P=\int_0^1\left(\frac{2}{x^2+1}-1\right)\mathrm{d}x-\int_1^{\sqrt{3}}\left(\frac{2}{x^2+1}-1\right)\mathrm{d}x=$$

$$=\left(2\operatorname{arctg}x-x\right)\Big|_0^1-\left(2\operatorname{arctg}x-x\right)\Big|_1^{\sqrt{3}}=$$

$$=\Big[\left(2\operatorname{arctg}1-1\right)-\left(2\operatorname{arctg}0-0\right)\Big]-$$

$$-\Big[\left(2\operatorname{arctg}\sqrt{3}-\sqrt{3}\,\right)-\left(2\operatorname{arctg}1-1\right)\Big]=$$

$$=\left(2\cdot\frac{\pi}{4}-1\right)-\left(2\cdot 0-0\right)-\left(2\cdot\frac{\pi}{3}-\sqrt{3}\right)+\left(2\cdot\frac{\pi}{4}-1\right)=$$

$$=\frac{\pi}{2}-1-0-\frac{2}{3}\pi+\sqrt{3}+\frac{\pi}{2}-1=\boxed{\frac{\pi}{3}+\sqrt{3}-2}\approx 0.77925$$

egzaktna vrijednost

aproksimacija na pet decimala

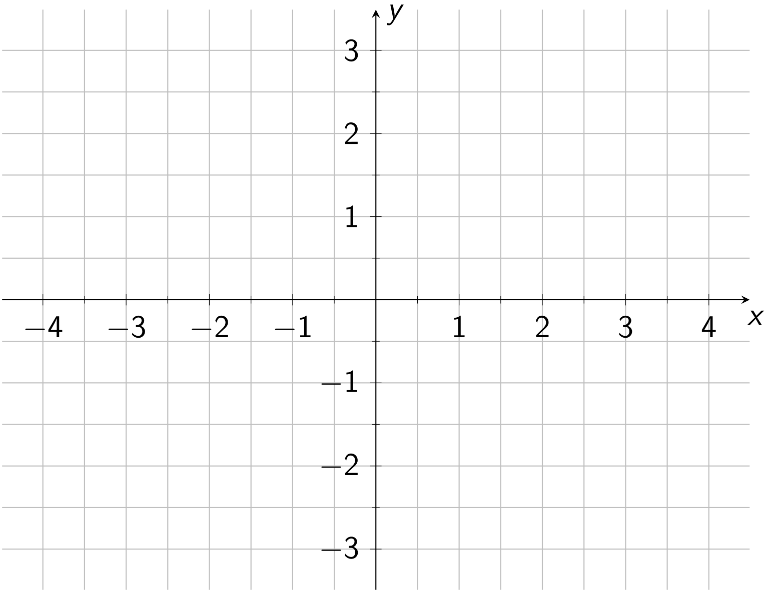
drugi zadatak

Zadatak 2

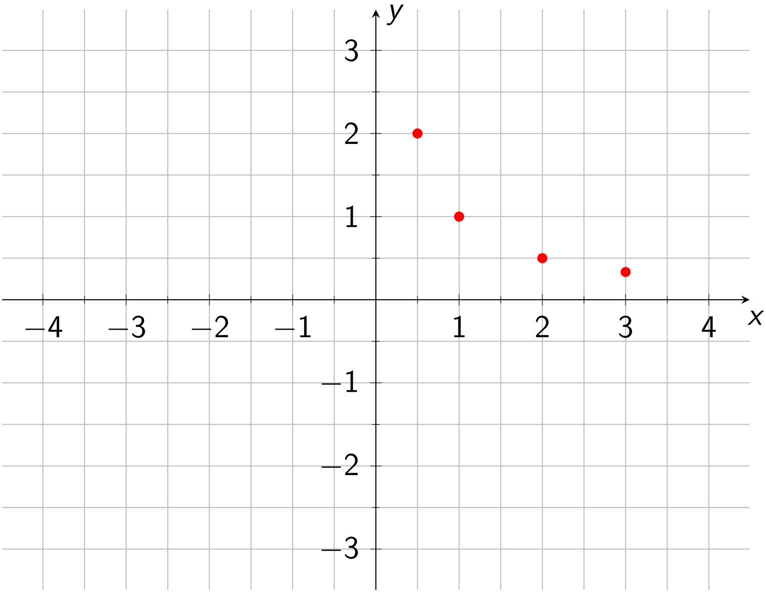
Izračunajte površinu lika kojeg omeđuju krivulje

$$y = \frac{1}{x}, \quad y = x, \quad y = 0, \quad x = 3.$$

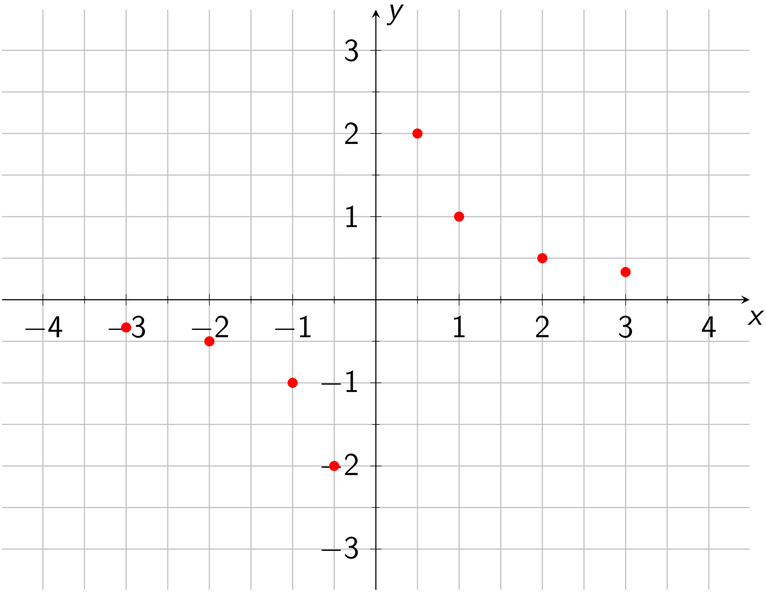
Rješenje



Rješenje



Rješenje



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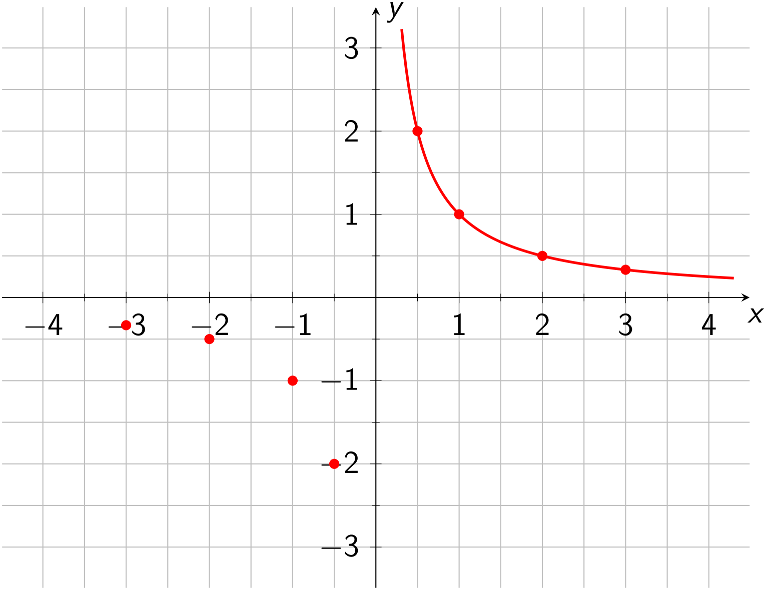
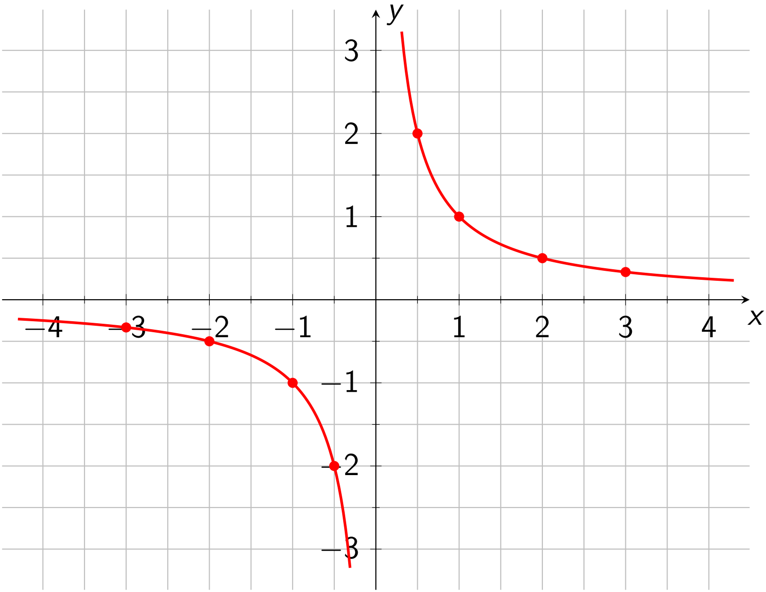
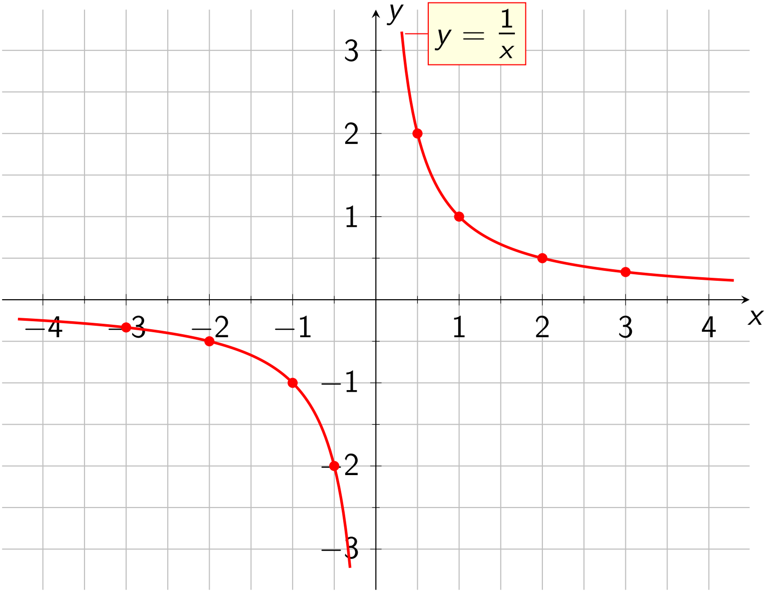


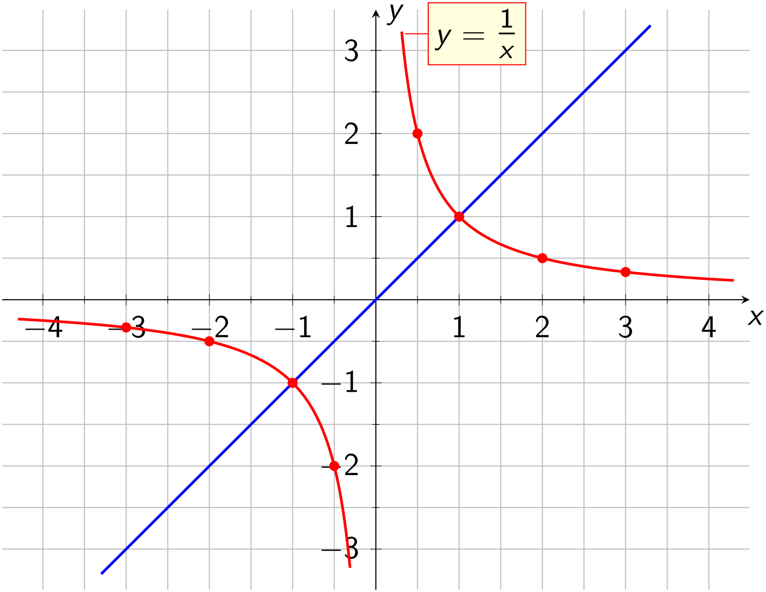
Rješenje



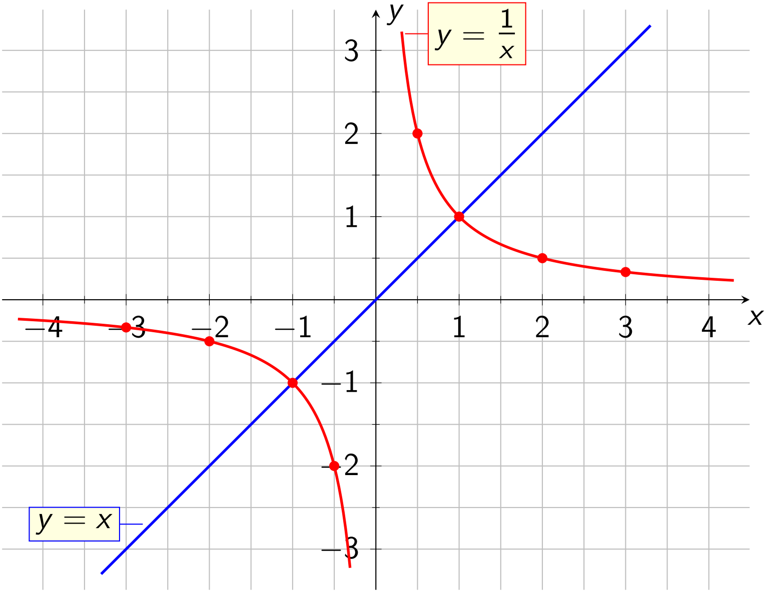
Rješenje



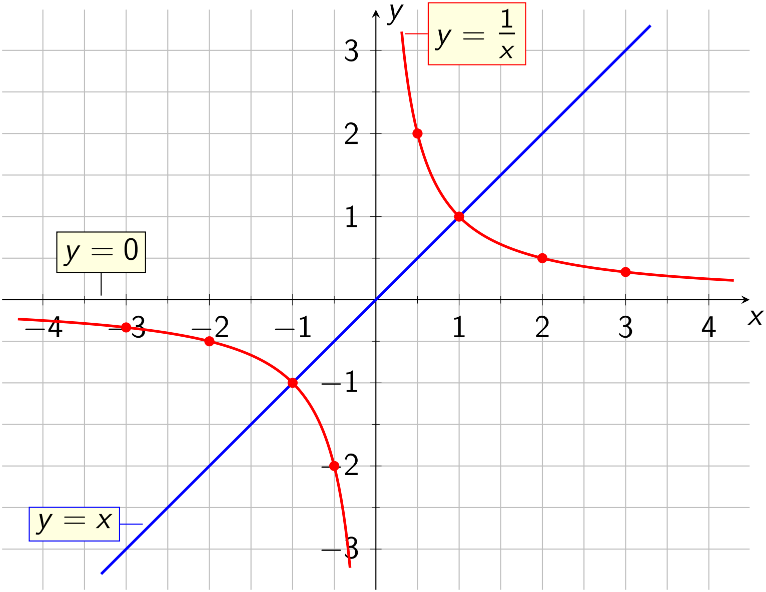
Rješenje



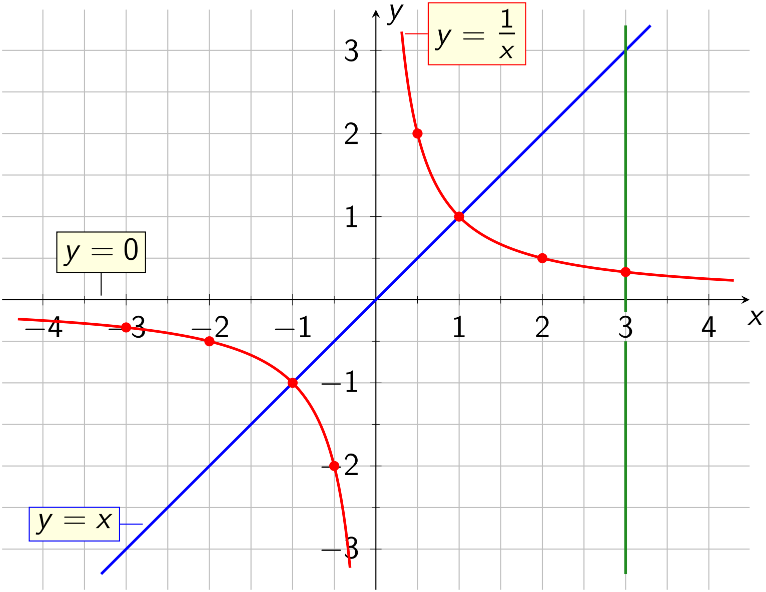
Rješenje



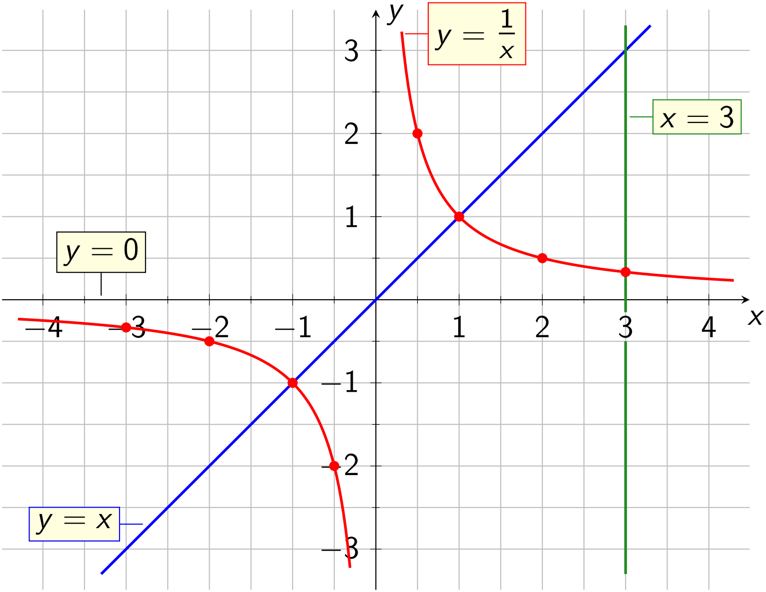
Rješenje



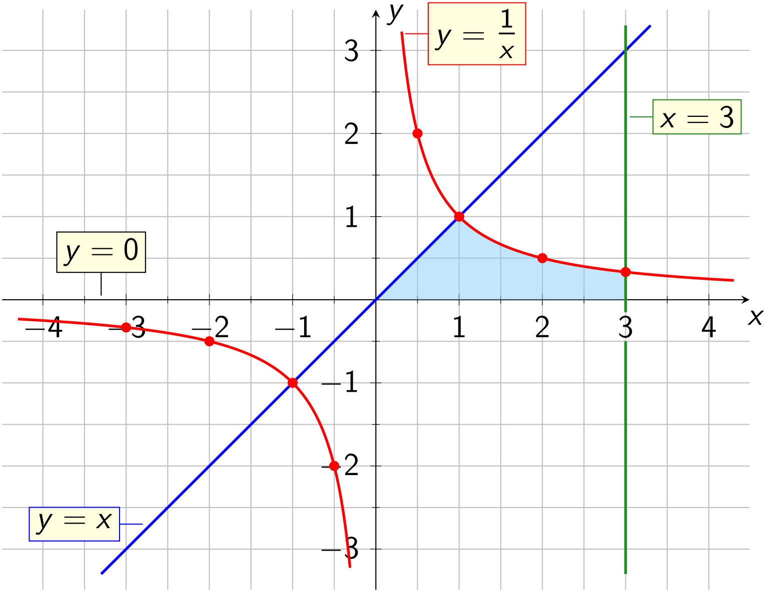
Rješenje



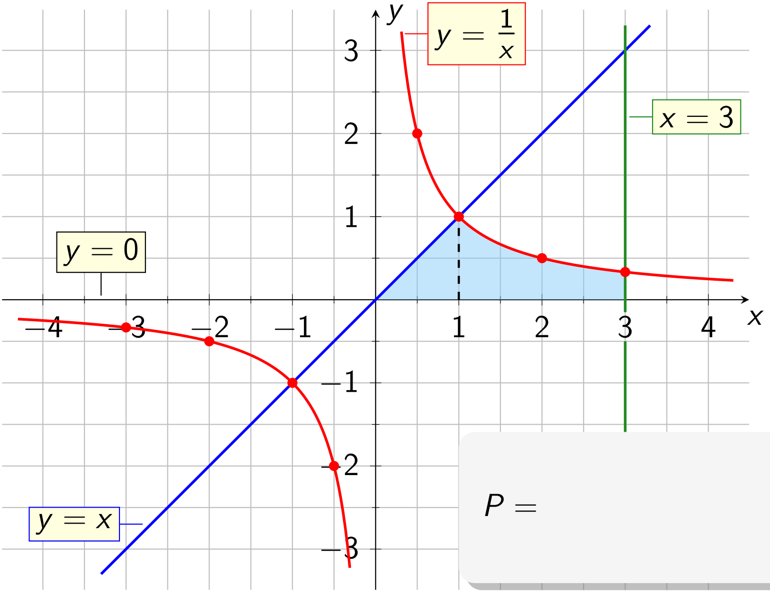
Rješenje



Rješenje

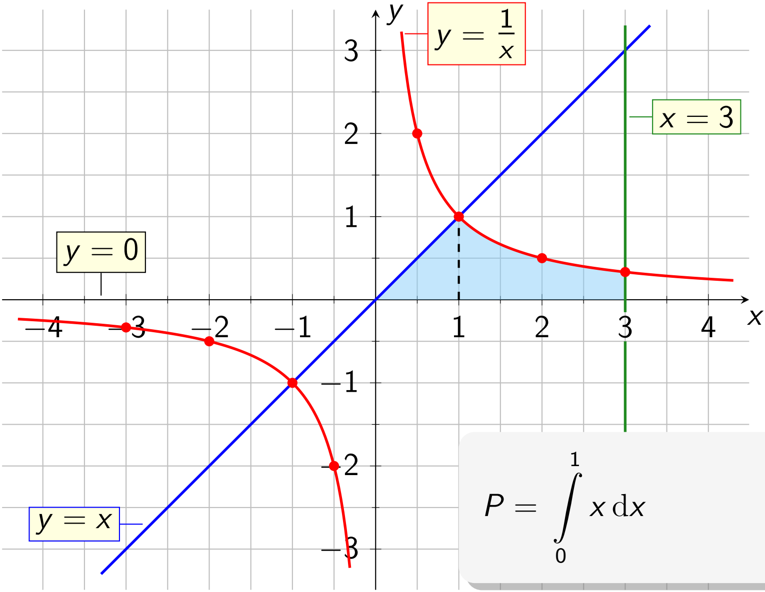


Rješenje



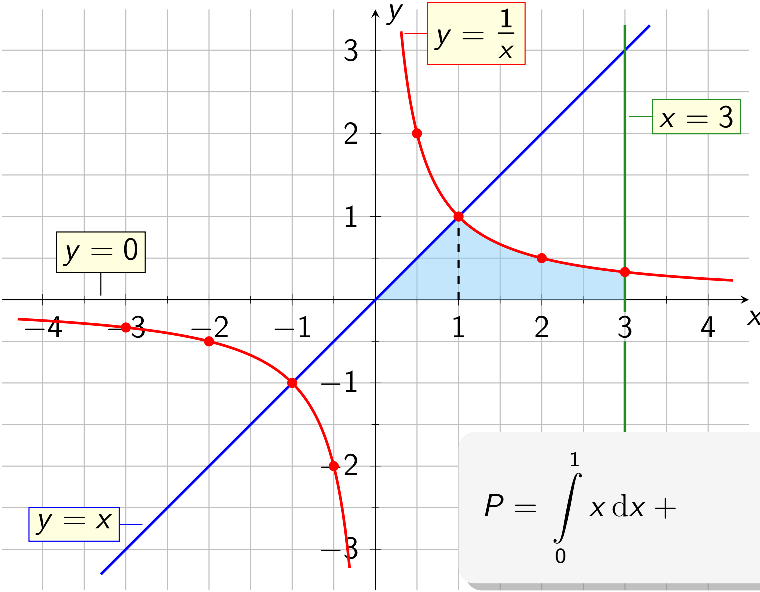
$P =$

Rješenje



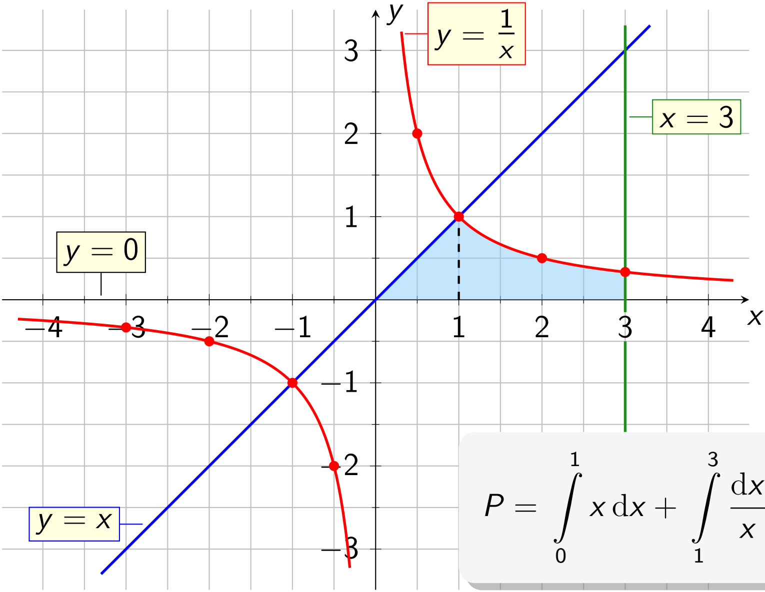
$$P = \int_0^1 x \, \mathrm{d}x$$

Rješenje



$$P = \int_0^1 x \,\mathrm{d}x +$$

Rješenje



$$P = \int_0^1 x\,\mathrm{d}x + \int_1^3 \frac{\mathrm{d}x}{x}$$

$$P = \int_0^1 x \mathrm{d}x + \int_1^3 \frac{\mathrm{d}x}{x}$$

$$P = \int\limits_0^1 x \mathrm{d}x + \int\limits_1^3 \frac{\mathrm{d}x}{x} = \frac{x^2}{2}\bigg|_0^1$$

$$P = \int_0^1 x \mathrm{d}x + \int_1^3 \frac{\mathrm{d}x}{x} = \frac{x^2}{2}\bigg|_0^1 +$$

$$P = \int_0^1 x\mathrm{d}x + \int_1^3 \frac{\mathrm{d}x}{x} = \frac{x^2}{2}\Big|_0^1 + \ln|x|\Big|_1^3$$

$$P = \int_0^1 x\mathrm{d}x + \int_1^3 \frac{\mathrm{d}x}{x} = \frac{x^2}{2}\bigg|_0^1 + \ln|x|\bigg|_1^3 =$$

$$= \left(\qquad \right)$$

$$P = \int\limits_0^1 x \mathrm{d}x + \int\limits_1^3 \frac{\mathrm{d}x}{x} = \frac{x^2}{2}\Big|_0^1 + \ln|x|\Big|_1^3 =$$

$$= \left(\frac{1}{2} \qquad \right)$$

$$P = \int_0^1 x\mathrm{d}x + \int_1^3 \frac{\mathrm{d}x}{x} = \frac{x^2}{2}\Big|_0^1 + \ \ln|x|\Big|_1^3 =$$

$$= \left(\frac{1}{2} - \quad\right)$$

$$P = \int\limits_0^1 x\mathrm{d}x + \int\limits_1^3 \frac{\mathrm{d}x}{x} = \frac{x^2}{2}\bigg|_0^1 + \ln|x|\bigg|_1^3 =$$

$$= \left(\frac{1}{2} - 0\right)$$

$$P = \int_0^1 x\mathrm{d}x + \int_1^3 \frac{\mathrm{d}x}{x} = \left.\frac{x^2}{2}\right|_0^1 + \left.\ln|x|\right|_1^3 =$$

$$= \left(\frac{1}{2} - 0\right) +$$

$$P = \int_0^1 x \mathrm{d}x + \int_1^3 \frac{\mathrm{d}x}{x} = \left.\frac{x^2}{2}\right|_0^1 + \left.\ln|x|\right|_1^3 =$$

$$= \left(\frac{1}{2} - 0\right) + (\qquad\qquad\qquad)$$

$$P = \int_0^1 x\mathrm{d}x + \int_1^3 \frac{\mathrm{d}x}{x} = \frac{x^2}{2}\Big|_0^1 + \ln|x|\Big|_1^3 =$$

$$= \left(\frac{1}{2} - 0\right) + (\ln 3 \qquad)$$

$$P = \int_0^1 x\mathrm{d}x + \int_1^3 \frac{\mathrm{d}x}{x} = \frac{x^2}{2}\bigg|_0^1 + \ln|x|\bigg|_1^3 =$$

$$= \left(\frac{1}{2} - 0\right) + (\ln 3 - \qquad)$$

$$P = \int_0^1 x\mathrm{d}x + \int_1^3 \frac{\mathrm{d}x}{x} = \left.\frac{x^2}{2}\right|_0^1 + \left. \ln|x| \right|_1^3 =$$

$$= \left(\frac{1}{2} - 0\right) + (\ln 3 - \ln 1)$$

$$P = \int_0^1 x\mathrm{d}x + \int_1^3 \frac{\mathrm{d}x}{x} = \frac{x^2}{2}\Big|_0^1 + \ln|x|\Big|_1^3 =$$

$$= \left(\frac{1}{2} - 0\right) + (\ln 3 - \ln 1) = \frac{1}{2} + \ln 3$$

treći zadatak

Zadatak 3

Izračunajte površinu lika omeđenog grafovima funkcija $f(x) = x^2$ *i* $g(x) = x + 2$.

Zadatak 3

Izračunajte površinu lika omeđenog grafovima funkcija $f(x) = x^2$ *i* $g(x) = x + 2$.

Rješenje

- Presjek pravca i parabole

Zadatak 3

Izračunajte površinu lika omeđenog grafovima funkcija $f(x) = x^2$ *i* $g(x) = x + 2$.

Rješenje

- Presjek pravca i parabole

$$x^2 = x + 2$$

Zadatak 3

Izračunajte površinu lika omeđenog grafovima funkcija $f(x) = x^2$ *i* $g(x) = x + 2$.

Rješenje

- Presjek pravca i parabole

$$x^2 = x + 2$$
$$x^2 - x - 2 = 0$$

Zadatak 3

Izračunajte površinu lika omeđenog grafovima funkcija $f(x) = x^2$ i $g(x) = x + 2$.

Rješenje

$$ax^2 + bx + c = 0$$
$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- Presjek pravca i parabole

$$x^2 = x + 2$$
$$x^2 - x - 2 = 0$$
$$x_{1,2} = \frac{-(-1) \pm \sqrt{(-1)^2 - 4 \cdot 1 \cdot (-2)}}{2 \cdot 1}$$

Zadatak 3

Izračunajte površinu lika omeđenog grafovima funkcija $f(x) = x^2$ *i* $g(x) = x + 2$.

$$ax^2 + bx + c = 0$$
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Rješenje

- Presjek pravca i parabole

$$x^2 = x + 2$$
$$x^2 - x - 2 = 0$$
$$x_{1,2} = \frac{-(-1) \pm \sqrt{(-1)^2 - 4 \cdot 1 \cdot (-2)}}{2 \cdot 1}$$
$$x_{1,2} = \frac{1 \pm 3}{2}$$

Zadatak 3

Izračunajte površinu lika omeđenog grafovima funkcija $f(x) = x^2$ i $g(x) = x + 2$.

Rješenje

$$ax^2 + bx + c = 0$$
$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- Presjek pravca i parabole

$$x^2 = x + 2$$
$$x^2 - x - 2 = 0$$
$$x_{1,2} = \frac{-(-1) \pm \sqrt{(-1)^2 - 4 \cdot 1 \cdot (-2)}}{2 \cdot 1}$$
$$x_{1,2} = \frac{1 \pm 3}{2}$$
$$x_1 = 2, \quad x_2 = -1$$

Zadatak 3

Izračunajte površinu lika omeđenog grafovima funkcija $f(x) = x^2$ *i* $g(x) = x + 2$.

$$ax^2 + bx + c = 0$$
$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Rješenje

- Presjek pravca i parabole

$$x^2 = x + 2$$
$$x^2 - x - 2 = 0$$
$$x_{1,2} = \frac{-(-1) \pm \sqrt{(-1)^2 - 4 \cdot 1 \cdot (-2)}}{2 \cdot 1}$$
$$x_{1,2} = \frac{1 \pm 3}{2}$$
$$x_1 = 2, \quad x_2 = -1$$
$$y_1 = 4, \quad y_2 = 1$$

Zadatak 3

Izračunajte površinu lika omeđenog grafovima funkcija $f(x) = x^2$ *i* $g(x) = x + 2$.

$$ax^2 + bx + c = 0$$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Rješenje

- Presjek pravca i parabole

$$x^2 = x + 2$$

$$x^2 - x - 2 = 0$$

$$x_{1,2} = \frac{-(-1) \pm \sqrt{(-1)^2 - 4 \cdot 1 \cdot (-2)}}{2 \cdot 1}$$

$$x_{1,2} = \frac{1 \pm 3}{2}$$

$$x_1 = 2, \quad x_2 = -1$$

$$y_1 = 4, \quad y_2 = 1$$

$$T_1(2, 4)$$

$$T_2(-1, 1)$$

Zadatak 3

Izračunajte površinu lika omeđenog grafovima funkcija $f(x) = x^2$ i $g(x) = x + 2$.

Rješenje

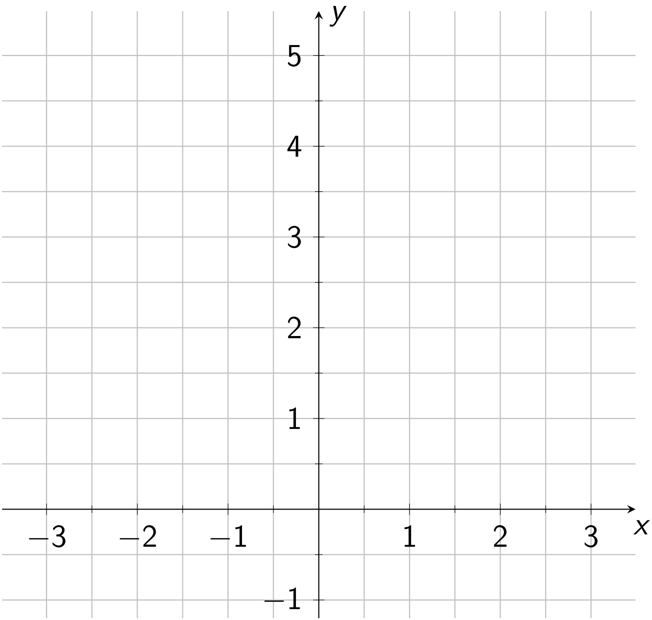
- Presjek pravca i parabole

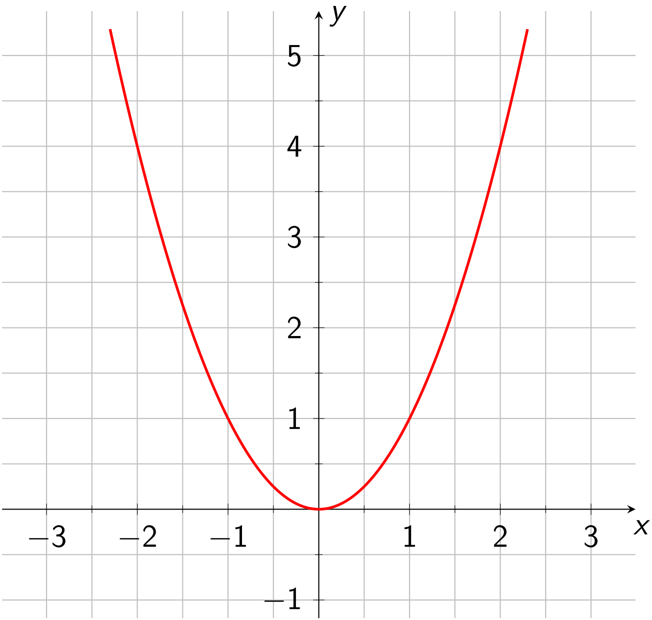
$$ax^2 + bx + c = 0$$
$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

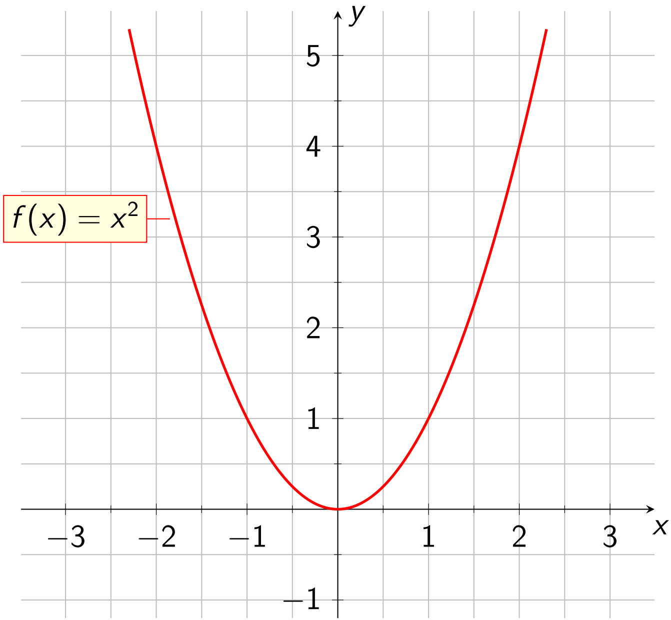
$$x^2 = x + 2$$
$$x^2 - x - 2 = 0$$
$$x_{1,2} = \frac{-(-1) \pm \sqrt{(-1)^2 - 4 \cdot 1 \cdot (-2)}}{2 \cdot 1}$$
$$x_{1,2} = \frac{1 \pm 3}{2}$$
$$x_1 = 2, \quad x_2 = -1$$
$$y_1 = 4, \quad y_2 = 1$$

točke presjeka

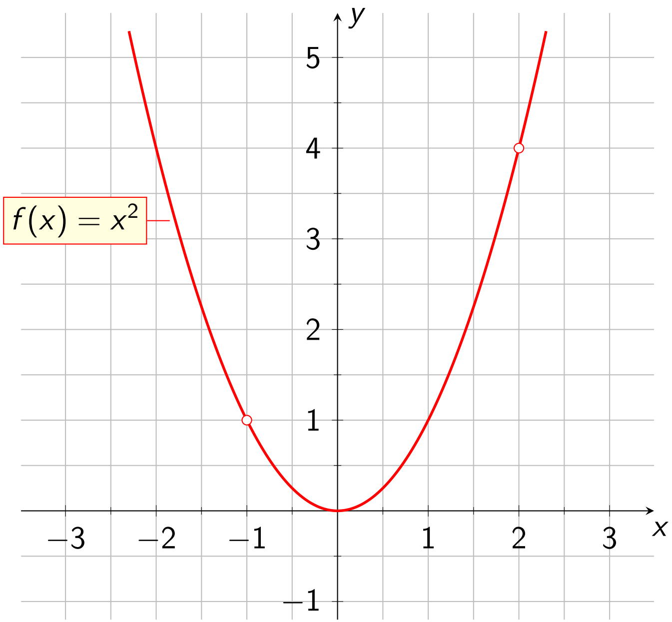
$$\boxed{\begin{array}{l} T_1(2,4) \\ T_2(-1,1) \end{array}}$$


y
5
4
3
2
1
−1
−3
−2
−1
1
2
3
x

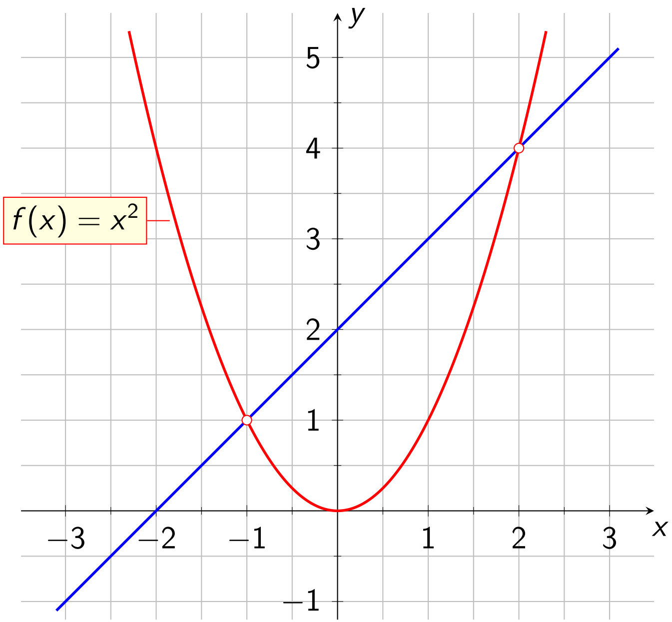

y
x
5
4
3
2
1
−1
−3
−2
−1
1
2
3

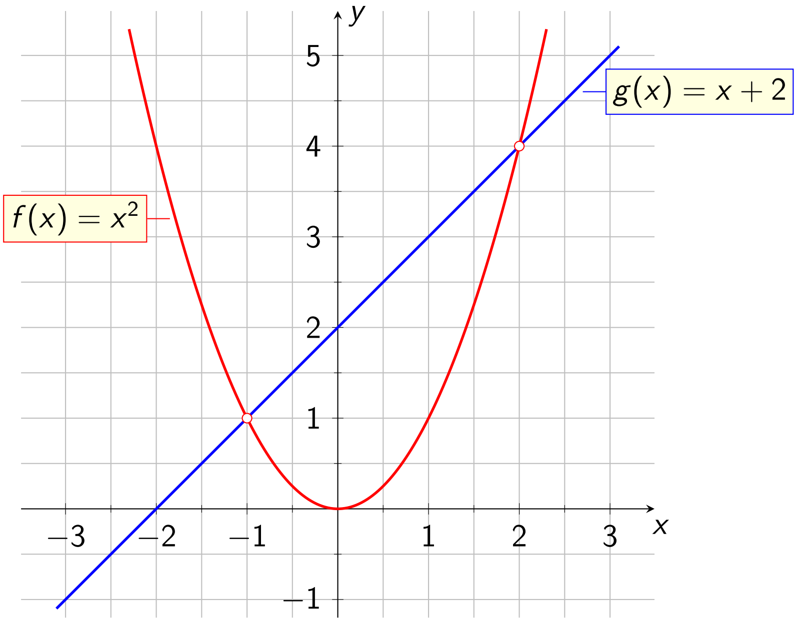


y
5
4
3
2
1
−1
−3
−2
−1
1
2
3
x
$f(x) = x^2$

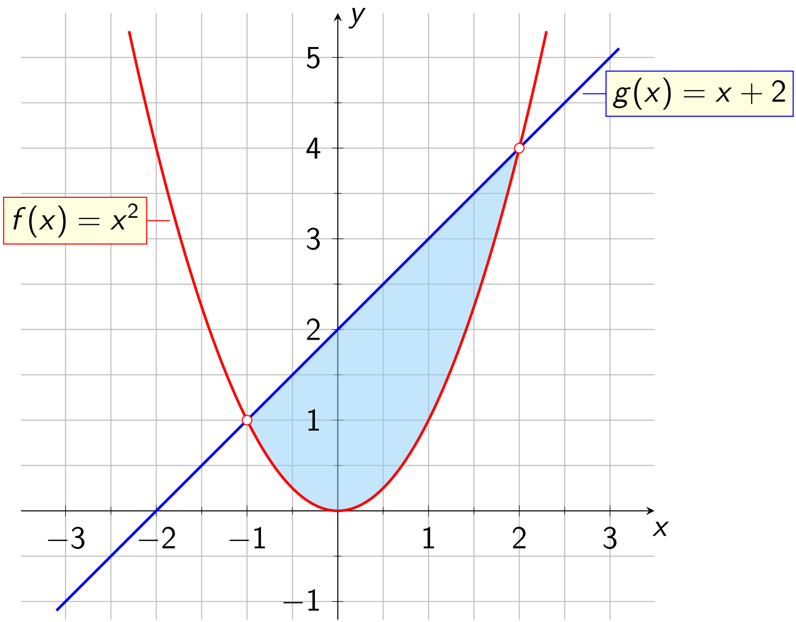


y
5
4
3
2
1
−1
$f(x) = x^2$
−3
−2
−1
1
2
3
x

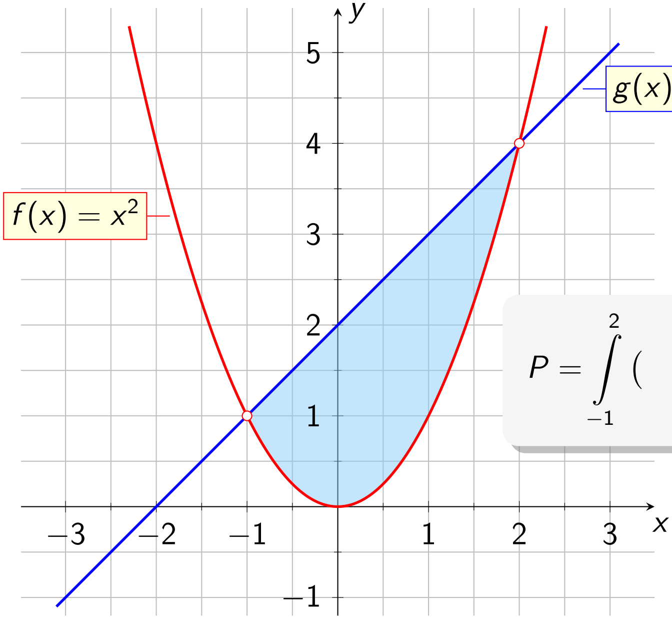

y
5
4
3
2
1
−1
−3
−2
−1
1
2
3
x
$f(x) = x^2$



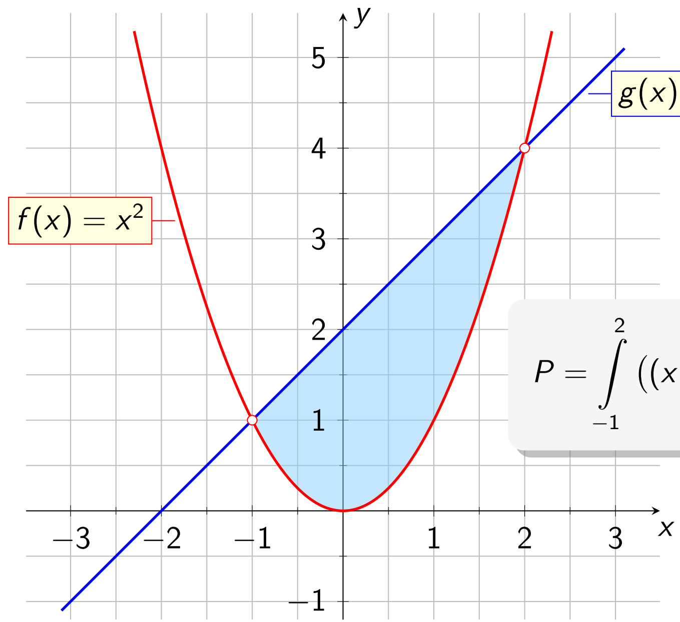
y
5
4
3
2
1
−1
x
−3
−2
−1
1
2
3
$g(x) = x + 2$
$f(x) = x^2$



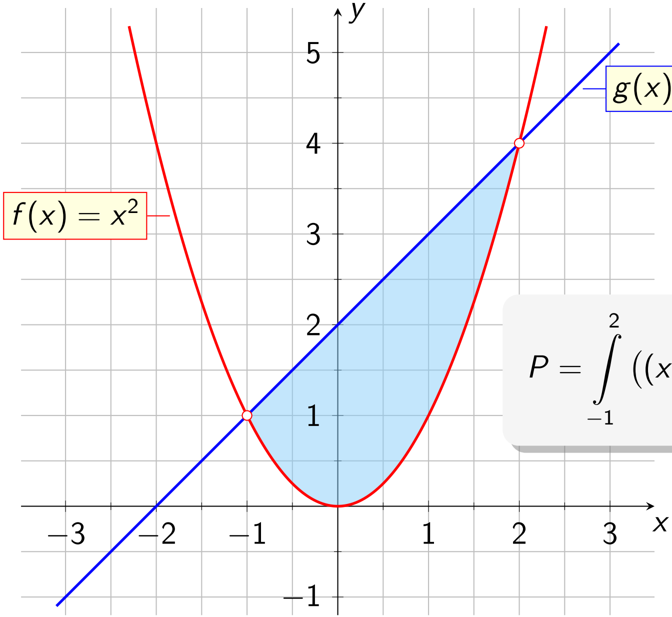
y
x
$f(x) = x^2$
$g(x) = x + 2$
5
4
3
2
1
−1
−3
−2
−1
1
2
3



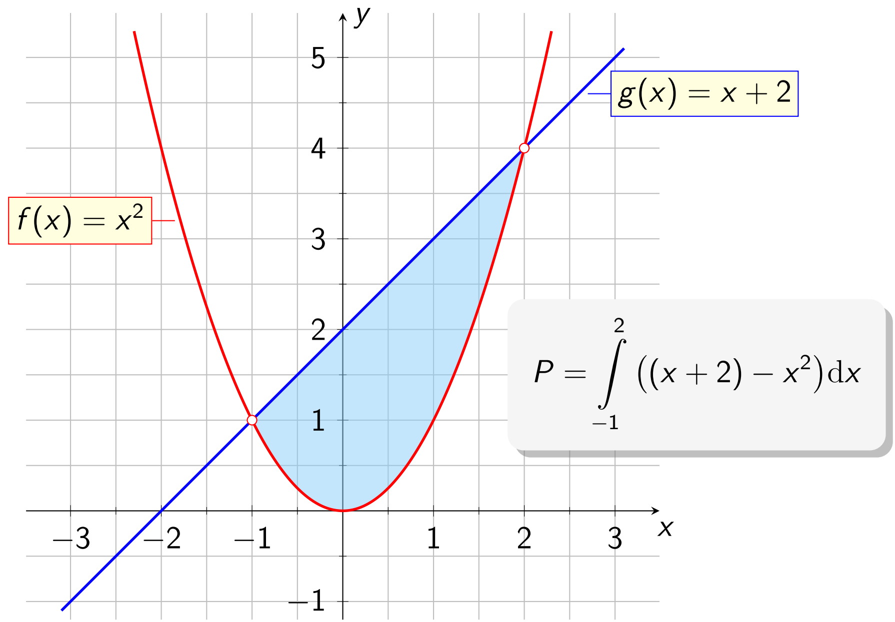
$$P = \int_{-1}^{2} \Big(\qquad\qquad\qquad \Big)\,\mathrm{d}x$$



$$P = \int_{-1}^{2} \left((x+2) \qquad \right) \mathrm{d}x$$



$$P = \int_{-1}^{2} \left((x+2) - \quad\right) \mathrm{d}x$$



y
x
5
4
3
2
1
−1
−3
−2
−1
1
2
3
$f(x) = x^2$
$g(x) = x + 2$
$P = \int_{-1}^{2} \left((x+2) - x^2\right) \mathrm{d}x$

$$P = \int_{-1}^{2} \left((x+2) - x^2\right)\mathrm{d}x$$

$$P = \int_{-1}^{2} \left((x+2) - x^2\right)\mathrm{d}x = \int_{-1}^{2} \left(-x^2 + x + 2\right)\mathrm{d}x$$

$$P = \int_{-1}^{2} \left((x+2) - x^2\right)\mathrm{d}x = \int_{-1}^{2} \left(-x^2 + x + 2\right)\mathrm{d}x =$$

$$= \Big($$

$$P = \int_{-1}^{2} \left((x+2) - x^2\right)\mathrm{d}x = \int_{-1}^{2} \left(-x^2 + x + 2\right)\mathrm{d}x =$$

$$= \left(-\frac{x^3}{3}\right.$$

$$P = \int_{-1}^{2} \left((x+2) - x^2\right)\mathrm{d}x = \int_{-1}^{2} \left(-x^2 + x + 2\right)\mathrm{d}x =$$

$$= \left(-\frac{x^3}{3} + \right.$$

$$P = \int_{-1}^{2} \left((x+2) - x^2\right) \mathrm{d}x = \int_{-1}^{2} \left(-x^2 + x + 2\right) \mathrm{d}x =$$

$$= \left(-\frac{x^3}{3} + \frac{x^2}{2}\right.$$

$$P = \int_{-1}^{2} \left((x+2) - x^2\right)\mathrm{d}x = \int_{-1}^{2} \left(-x^2 + x + 2\right)\mathrm{d}x =$$

$$= \left(-\frac{x^3}{3} + \frac{x^2}{2} +\right.$$

$$P = \int_{-1}^{2} \left((x+2) - x^2\right) \mathrm{d}x = \int_{-1}^{2} \left(-x^2 + x + 2\right) \mathrm{d}x =$$

$$= \left(-\frac{x^3}{3} + \frac{x^2}{2} + 2x\right.$$

$$P = \int_{-1}^{2} \left((x+2) - x^2\right)\mathrm{d}x = \int_{-1}^{2} \left(-x^2 + x + 2\right)\mathrm{d}x =$$

$$= \left(-\frac{x^3}{3} + \frac{x^2}{2} + 2x\right)$$

$$P = \int_{-1}^{2} \left((x+2) - x^2\right)\mathrm{d}x = \int_{-1}^{2} \left(-x^2 + x + 2\right)\mathrm{d}x =$$

$$= \left(-\frac{x^3}{3} + \frac{x^2}{2} + 2x\right)\Big|_{-1}^{2}$$

$$P = \int_{-1}^{2} \left((x+2) - x^2\right)\mathrm{d}x = \int_{-1}^{2} \left(-x^2 + x + 2\right)\mathrm{d}x =$$

$$= \left(-\frac{x^3}{3} + \frac{x^2}{2} + 2x\right)\Big|_{-1}^{2} =$$

$$= \Bigg($$

$$P = \int_{-1}^{2} \left((x+2) - x^2\right) \mathrm{d}x = \int_{-1}^{2} \left(-x^2 + x + 2\right) \mathrm{d}x =$$

$$= \left(-\frac{x^3}{3} + \frac{x^2}{2} + 2x\right) \Big|_{-1}^{2} =$$

$$= \Bigg(-\frac{2^3}{3}$$

$$P = \int_{-1}^{2} \left((x+2) - x^2\right)\mathrm{d}x = \int_{-1}^{2} \left(-x^2 + x + 2\right)\mathrm{d}x =$$

$$= \left(-\frac{x^3}{3} + \frac{x^2}{2} + 2x\right)\Bigg|_{-1}^{2} =$$

$$= \Bigg(-\frac{2^3}{3} +$$

$$P = \int_{-1}^{2} \left((x+2) - x^2\right)\mathrm{d}x = \int_{-1}^{2} \left(-x^2 + x + 2\right)\mathrm{d}x =$$

$$= \left(-\frac{x^3}{3} + \frac{x^2}{2} + 2x\right)\Big|_{-1}^{2} =$$

$$= \left(-\frac{2^3}{3} + \frac{2^2}{2}\right.$$

$$P = \int_{-1}^{2} \left((x+2) - x^2\right)\mathrm{d}x = \int_{-1}^{2} \left(-x^2 + x + 2\right)\mathrm{d}x =$$

$$= \left(-\frac{x^3}{3} + \frac{x^2}{2} + 2x\right)\Bigg|_{-1}^{2} =$$

$$= \left(-\frac{2^3}{3} + \frac{2^2}{2} + \right.$$

$$P = \int_{-1}^{2} \left((x+2) - x^2\right)\mathrm{d}x = \int_{-1}^{2} \left(-x^2 + x + 2\right)\mathrm{d}x =$$

$$= \left(-\frac{x^3}{3} + \frac{x^2}{2} + 2x\right)\Big|_{-1}^{2} =$$

$$= \Big(-\frac{2^3}{3} + \frac{2^2}{2} + 2 \cdot 2$$

$$P = \int_{-1}^{2} \left((x+2) - x^2\right)\mathrm{d}x = \int_{-1}^{2} \left(-x^2 + x + 2\right)\mathrm{d}x =$$

$$= \left(-\frac{x^3}{3} + \frac{x^2}{2} + 2x\right)\Bigg|_{-1}^{2} =$$

$$= \left(-\frac{2^3}{3} + \frac{2^2}{2} + 2 \cdot 2\right)$$

$$P = \int_{-1}^{2} \left((x+2) - x^2\right)\mathrm{d}x = \int_{-1}^{2} \left(-x^2 + x + 2\right)\mathrm{d}x =$$

$$= \left(-\frac{x^3}{3} + \frac{x^2}{2} + 2x\right)\Bigg|_{-1}^{2} =$$

$$= \left(-\frac{2^3}{3} + \frac{2^2}{2} + 2 \cdot 2\right) - \Bigg($$

$$P = \int\limits_{-1}^{2} \left((x+2) - x^2\right)\mathrm{d}x = \int\limits_{-1}^{2} \left(-x^2 + x + 2\right)\mathrm{d}x =$$

$$= \left(-\frac{x^3}{3} + \frac{x^2}{2} + 2x\right)\Bigg|_{-1}^{2} =$$

$$= \left(-\frac{2^3}{3} + \frac{2^2}{2} + 2\cdot 2\right) - \left(-\frac{(-1)^3}{3}\right.$$

$$P = \int_{-1}^{2} \big((x+2) - x^2\big)\mathrm{d}x = \int_{-1}^{2} \big(-x^2 + x + 2\big)\mathrm{d}x =$$

$$= \left(-\frac{x^3}{3} + \frac{x^2}{2} + 2x\right)\Bigg|_{-1}^{2} =$$

$$= \left(-\frac{2^3}{3} + \frac{2^2}{2} + 2\cdot 2\right) - \left(-\frac{(-1)^3}{3} + \right.$$

$$P = \int_{-1}^{2} \left((x+2) - x^2\right)\mathrm{d}x = \int_{-1}^{2} \left(-x^2 + x + 2\right)\mathrm{d}x =$$

$$= \left(-\frac{x^3}{3} + \frac{x^2}{2} + 2x\right)\Big|_{-1}^{2} =$$

$$= \left(-\frac{2^3}{3} + \frac{2^2}{2} + 2 \cdot 2\right) - \left(-\frac{(-1)^3}{3} + \frac{(-1)^2}{2}\right.$$

$$P = \int_{-1}^{2} \left((x+2) - x^2\right)\mathrm{d}x = \int_{-1}^{2} \left(-x^2 + x + 2\right)\mathrm{d}x =$$

$$= \left(-\frac{x^3}{3} + \frac{x^2}{2} + 2x\right)\Big|_{-1}^{2} =$$

$$= \left(-\frac{2^3}{3} + \frac{2^2}{2} + 2 \cdot 2\right) - \left(-\frac{(-1)^3}{3} + \frac{(-1)^2}{2} +\right.$$

$$P = \int\limits_{-1}^{2} \left((x+2) - x^2\right)\mathrm{d}x = \int\limits_{-1}^{2} \left(-x^2 + x + 2\right)\mathrm{d}x =$$

$$= \left(-\frac{x^3}{3} + \frac{x^2}{2} + 2x\right)\Bigg|_{-1}^{2} =$$

$$= \left(-\frac{2^3}{3} + \frac{2^2}{2} + 2\cdot 2\right) - \left(-\frac{(-1)^3}{3} + \frac{(-1)^2}{2} + 2\cdot(-1)\right)$$

$$P = \int_{-1}^{2} \left((x+2) - x^2\right)\mathrm{d}x = \int_{-1}^{2} \left(-x^2 + x + 2\right)\mathrm{d}x =$$

$$= \left(-\frac{x^3}{3} + \frac{x^2}{2} + 2x\right)\Bigg|_{-1}^{2} =$$

$$= \left(-\frac{2^3}{3} + \frac{2^2}{2} + 2\cdot 2\right) - \left(-\frac{(-1)^3}{3} + \frac{(-1)^2}{2} + 2\cdot(-1)\right)$$

$$P = \int_{-1}^{2} \left((x+2) - x^2\right)\mathrm{d}x = \int_{-1}^{2} \left(-x^2 + x + 2\right)\mathrm{d}x =$$

$$= \left(-\frac{x^3}{3} + \frac{x^2}{2} + 2x\right)\Big|_{-1}^{2} =$$

$$= \left(-\frac{2^3}{3} + \frac{2^2}{2} + 2 \cdot 2\right) - \left(-\frac{(-1)^3}{3} + \frac{(-1)^2}{2} + 2 \cdot (-1)\right) =$$

$$= \Bigg($$

$$P = \int\limits_{-1}^{2} \left((x+2) - x^2\right)\mathrm{d}x = \int\limits_{-1}^{2} \left(-x^2 + x + 2\right)\mathrm{d}x =$$

$$= \left(-\frac{x^3}{3} + \frac{x^2}{2} + 2x\right)\Bigg|_{-1}^{2} =$$

$$= \left(-\frac{2^3}{3} + \frac{2^2}{2} + 2 \cdot 2\right) - \left(-\frac{(-1)^3}{3} + \frac{(-1)^2}{2} + 2 \cdot (-1)\right) =$$

$$= \Bigg(-\frac{8}{3}$$

$$P=\int\limits_{-1}^{2}\left((x+2)-x^2\right)\mathrm{d}x=\int\limits_{-1}^{2}\left(-x^2+x+2\right)\mathrm{d}x=$$

$$=\left(-\frac{x^3}{3}+\frac{x^2}{2}+2x\right)\Big|_{-1}^{2}=$$

$$=\left(-\frac{2^3}{3}+\frac{2^2}{2}+2\cdot 2\right)-\left(-\frac{(-1)^3}{3}+\frac{(-1)^2}{2}+2\cdot(-1)\right)=$$

$$=\left(-\frac{8}{3}+\frac{4}{2}\right.$$

$$P = \int_{-1}^{2} \left((x+2) - x^2\right)\mathrm{d}x = \int_{-1}^{2} \left(-x^2 + x + 2\right)\mathrm{d}x =$$

$$= \left(-\frac{x^3}{3} + \frac{x^2}{2} + 2x\right)\Big|_{-1}^{2} =$$

$$= \left(-\frac{2^3}{3} + \frac{2^2}{2} + 2\cdot 2\right) - \left(-\frac{(-1)^3}{3} + \frac{(-1)^2}{2} + 2\cdot(-1)\right) =$$

$$= \left(-\frac{8}{3} + \frac{4}{2} + 4\right)$$

$$P = \int_{-1}^{2} \left((x+2) - x^2\right)\mathrm{d}x = \int_{-1}^{2} \left(-x^2 + x + 2\right)\mathrm{d}x =$$

$$= \left(-\frac{x^3}{3} + \frac{x^2}{2} + 2x\right)\Bigg|_{-1}^{2} =$$

$$= \left(-\frac{2^3}{3} + \frac{2^2}{2} + 2 \cdot 2\right) - \left(-\frac{(-1)^3}{3} + \frac{(-1)^2}{2} + 2 \cdot (-1)\right) =$$

$$= \left(-\frac{8}{3} + \frac{4}{2} + 4\right) - \Bigg($$

$$P = \int_{-1}^{2} \left((x+2) - x^2\right)\mathrm{d}x = \int_{-1}^{2} \left(-x^2 + x + 2\right)\mathrm{d}x =$$

$$= \left(-\frac{x^3}{3} + \frac{x^2}{2} + 2x\right)\Big|_{-1}^{2} =$$

$$= \left(-\frac{2^3}{3} + \frac{2^2}{2} + 2 \cdot 2\right) - \left(-\frac{(-1)^3}{3} + \frac{(-1)^2}{2} + 2 \cdot (-1)\right) =$$

$$= \left(-\frac{8}{3} + \frac{4}{2} + 4\right) - \left(\frac{1}{3}\right.$$

$$P = \int_{-1}^{2} \left((x+2) - x^2\right)\mathrm{d}x = \int_{-1}^{2} \left(-x^2 + x + 2\right)\mathrm{d}x =$$

$$= \left(-\frac{x^3}{3} + \frac{x^2}{2} + 2x\right)\Big|_{-1}^{2} =$$

$$= \left(-\frac{2^3}{3} + \frac{2^2}{2} + 2\cdot 2\right) - \left(-\frac{(-1)^3}{3} + \frac{(-1)^2}{2} + 2\cdot(-1)\right) =$$

$$= \left(-\frac{8}{3} + \frac{4}{2} + 4\right) - \left(\frac{1}{3} + \frac{1}{2}\right.$$

$$P = \int_{-1}^{2} \left((x+2) - x^2\right)\mathrm{d}x = \int_{-1}^{2} \left(-x^2 + x + 2\right)\mathrm{d}x =$$

$$= \left(-\frac{x^3}{3} + \frac{x^2}{2} + 2x\right)\Big|_{-1}^{2} =$$

$$= \left(-\frac{2^3}{3} + \frac{2^2}{2} + 2 \cdot 2\right) - \left(-\frac{(-1)^3}{3} + \frac{(-1)^2}{2} + 2 \cdot (-1)\right) =$$

$$= \left(-\frac{8}{3} + \frac{4}{2} + 4\right) - \left(\frac{1}{3} + \frac{1}{2} - 2\right)$$

$$P = \int_{-1}^{2} \left((x+2) - x^2\right) \mathrm{d}x = \int_{-1}^{2} \left(-x^2 + x + 2\right) \mathrm{d}x =$$

$$= \left(-\frac{x^3}{3} + \frac{x^2}{2} + 2x\right) \Bigg|_{-1}^{2} =$$

$$= \left(-\frac{2^3}{3} + \frac{2^2}{2} + 2 \cdot 2\right) - \left(-\frac{(-1)^3}{3} + \frac{(-1)^2}{2} + 2 \cdot (-1)\right) =$$

$$= \left(-\frac{8}{3} + \frac{4}{2} + 4\right) - \left(\frac{1}{3} + \frac{1}{2} - 2\right) =$$

$$= -\frac{8}{3} + 6$$

$$P = \int_{-1}^{2} \left((x+2) - x^2\right)\mathrm{d}x = \int_{-1}^{2} \left(-x^2 + x + 2\right)\mathrm{d}x =$$

$$= \left(-\frac{x^3}{3} + \frac{x^2}{2} + 2x\right)\Big|_{-1}^{2} =$$

$$= \left(-\frac{2^3}{3} + \frac{2^2}{2} + 2\cdot 2\right) - \left(-\frac{(-1)^3}{3} + \frac{(-1)^2}{2} + 2\cdot(-1)\right) =$$

$$= \left(-\frac{8}{3} + \frac{4}{2} + 4\right) - \left(\frac{1}{3} + \frac{1}{2} - 2\right) =$$

$$= -\frac{8}{3} + 6 - \frac{1}{3}$$

$$P = \int_{-1}^{2} \left((x+2) - x^2\right)\mathrm{d}x = \int_{-1}^{2} \left(-x^2 + x + 2\right)\mathrm{d}x =$$

$$= \left(-\frac{x^3}{3} + \frac{x^2}{2} + 2x\right)\Bigg|_{-1}^{2} =$$

$$= \left(-\frac{2^3}{3} + \frac{2^2}{2} + 2\cdot 2\right) - \left(-\frac{(-1)^3}{3} + \frac{(-1)^2}{2} + 2\cdot(-1)\right) =$$

$$= \left(-\frac{8}{3} + \frac{4}{2} + 4\right) - \left(\frac{1}{3} + \frac{1}{2} - 2\right) =$$

$$= -\frac{8}{3} + 6 - \frac{1}{3} - \frac{1}{2}$$

$$P = \int_{-1}^{2} \left((x+2) - x^2\right)\mathrm{d}x = \int_{-1}^{2} \left(-x^2 + x + 2\right)\mathrm{d}x =$$

$$= \left(-\frac{x^3}{3} + \frac{x^2}{2} + 2x\right)\Big|_{-1}^{2} =$$

$$= \left(-\frac{2^3}{3} + \frac{2^2}{2} + 2\cdot 2\right) - \left(-\frac{(-1)^3}{3} + \frac{(-1)^2}{2} + 2\cdot(-1)\right) =$$

$$= \left(-\frac{8}{3} + \frac{4}{2} + 4\right) - \left(\frac{1}{3} + \frac{1}{2} - 2\right) =$$

$$= -\frac{8}{3} + 6 - \frac{1}{3} - \frac{1}{2} + 2$$

$$P = \int_{-1}^{2} \left((x+2) - x^2\right)\mathrm{d}x = \int_{-1}^{2} \left(-x^2 + x + 2\right)\mathrm{d}x =$$

$$= \left(-\frac{x^3}{3} + \frac{x^2}{2} + 2x\right)\Big|_{-1}^{2} =$$

$$= \left(-\frac{2^3}{3} + \frac{2^2}{2} + 2\cdot 2\right) - \left(-\frac{(-1)^3}{3} + \frac{(-1)^2}{2} + 2\cdot(-1)\right) =$$

$$= \left(-\frac{8}{3} + \frac{4}{2} + 4\right) - \left(\frac{1}{3} + \frac{1}{2} - 2\right) =$$

$$= -\frac{8}{3} + 6 - \frac{1}{3} - \frac{1}{2} + 2 = \frac{9}{2}$$

četvrti zadatak

Zadatak 4

Izračunajte površinu lika kojeg omeđuju krivulje

$$y = \frac{1}{x}, \quad y = 2^{x-1}, \quad y = 4.$$

Zadatak 4

Izračunajte površinu lika kojeg omeđuju krivulje

$$y = \frac{1}{x}, \quad y = 2^{x-1}, \quad y = 4.$$

Rješenje

- Presjek krivulja

  $y = \dfrac{1}{x}$ i $y = 4$

Zadatak 4

Izračunajte površinu lika kojeg omeđuju krivulje

$$y = \frac{1}{x}, \quad y = 2^{x-1}, \quad y = 4.$$

Rješenje

- Presjek krivulja

  $y = \frac{1}{x}$ i $y = 4$

  $$\frac{1}{x} = 4$$

Zadatak 4

Izračunajte površinu lika kojeg omeđuju krivulje

$$y = \frac{1}{x}, \quad y = 2^{x-1}, \quad y = 4.$$

Rješenje

- Presjek krivulja

  $y = \dfrac{1}{x}$ i $y = 4$

  $$\frac{1}{x} = 4 \Big/ \cdot x$$

  $$4x = 1$$

Zadatak 4

Izračunajte površinu lika kojeg omeđuju krivulje

$$y = \frac{1}{x}, \quad y = 2^{x-1}, \quad y = 4.$$

Rješenje

- Presjek krivulja

  $y = \frac{1}{x}$ i $y = 4$

$$\frac{1}{x} = 4 \Big/ \cdot x$$

$$4x = 1$$

$$x = \frac{1}{4}$$

Zadatak 4

Izračunajte površinu lika kojeg omeđuju krivulje

$$y = \frac{1}{x}, \quad y = 2^{x-1}, \quad y = 4.$$

Rješenje

- Presjek krivulja

$y = \frac{1}{x}$ i $y = 4$

$$\frac{1}{x} = 4 \Big/ \cdot x$$

$$4x = 1$$

$$x = \frac{1}{4}$$

$$\boxed{\left(\frac{1}{4}, 4\right)}$$

Zadatak 4

Izračunajte površinu lika kojeg omeđuju krivulje

$$y = \frac{1}{x}, \quad y = 2^{x-1}, \quad y = 4.$$

Rješenje

- Presjek krivulja

$y = \frac{1}{x}$ i $y = 4$

$$\frac{1}{x} = 4 \;\Big/ \cdot x$$

$$4x = 1$$

$$x = \frac{1}{4}$$

$$\boxed{\left(\frac{1}{4}, 4\right)}$$

- Presjek krivulja

$y = 2^{x-1}$ i $y = 4$

Zadatak 4

Izračunajte površinu lika kojeg omeđuju krivulje

$$y = \frac{1}{x}, \quad y = 2^{x-1}, \quad y = 4.$$

Rješenje

- Presjek krivulja

$y = \dfrac{1}{x}$ i $y = 4$

$$\frac{1}{x} = 4 \Big/ \cdot x$$

$$4x = 1$$

$$x = \frac{1}{4}$$

$$\boxed{\left(\frac{1}{4}, 4\right)}$$

- Presjek krivulja

$y = 2^{x-1}$ i $y = 4$

$$2^{x-1} = 4$$

Zadatak 4

Izračunajte površinu lika kojeg omeđuju krivulje

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Rješenje

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$$x = \frac{1}{4}$$

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- Presjek krivulja

$y = 2^{x-1}$ i $y = 4$

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Zadatak 4

Izračunajte površinu lika kojeg omeđuju krivulje

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Rješenje

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$$x - 1 = \log_2 4$$

$$x = 2 + 1$$

Zadatak 4

Izračunajte površinu lika kojeg omeđuju krivulje

$$y = \frac{1}{x}, \quad y = 2^{x-1}, \quad y = 4.$$

Rješenje

- Presjek krivulja

$$y = \frac{1}{x} \text{ i } y = 4$$

$$\frac{1}{x} = 4 \;\Big/ \cdot x$$

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$$x = \frac{1}{4}$$

$$\boxed{\left(\frac{1}{4}, 4\right)}$$

- Presjek krivulja

$$y = 2^{x-1} \text{ i } y = 4$$

$$2^{x-1} = 4$$

$$x - 1 = \log_2 4$$

$$x = 2 + 1$$

$$x = 3$$

Zadatak 4

Izračunajte površinu lika kojeg omeđuju krivulje

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Rješenje

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Zadatak 4

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- Presjek krivulja

$$y = 2^{x-1} \text{ i } y = \frac{1}{x}$$

Zadatak 4

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Rješenje

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- Presjek krivulja

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Zadatak 4

Izračunajte površinu lika kojeg omeđuju krivulje

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Rješenje

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- Presjek krivulja

$$y = 2^{x-1} \text{ i } y = \frac{1}{x}$$

$$2^{x-1} = \frac{1}{x}$$

pogađamo rješenje

Zadatak 4

Izračunajte površinu lika kojeg omeđuju krivulje

$$y = \frac{1}{x}, \quad y = 2^{x-1}, \quad y = 4.$$

Rješenje

- Presjek krivulja

$$y = \frac{1}{x} \text{ i } y = 4$$

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$$y = 2^{x-1} \text{ i } y = 4$$

$$2^{x-1} = 4$$

$$x - 1 = \log_2 4$$

$$x = 2 + 1$$

$$x = 3$$

$$\boxed{(3, 4)}$$

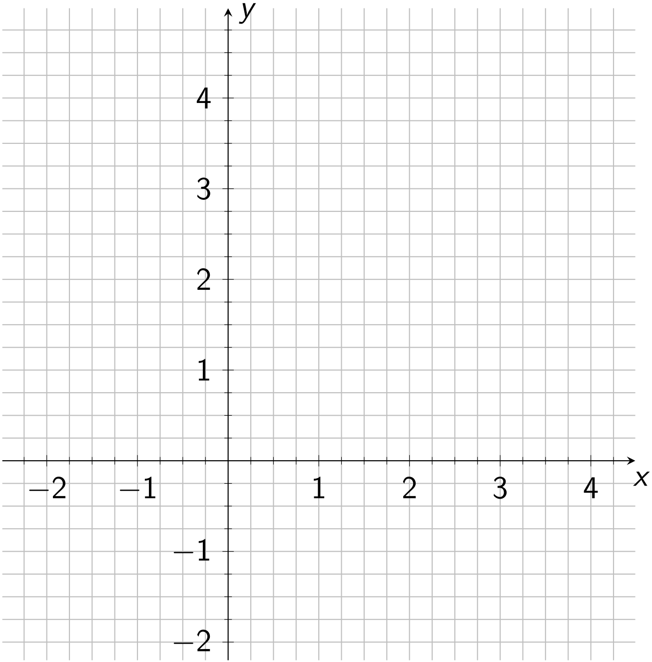
- Presjek krivulja

$$y = 2^{x-1} \text{ i } y = \frac{1}{x}$$

$$2^{x-1} = \frac{1}{x}$$

pogađamo rješenje

$$\boxed{(1, 1)}$$


y
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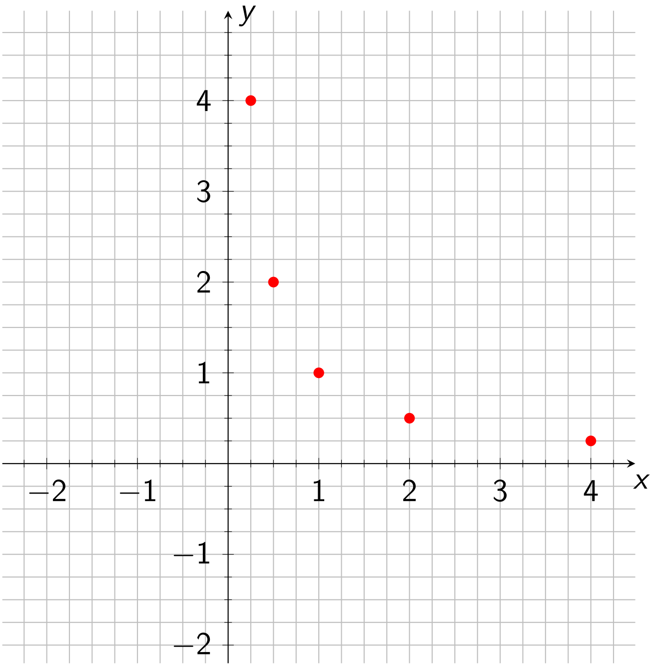
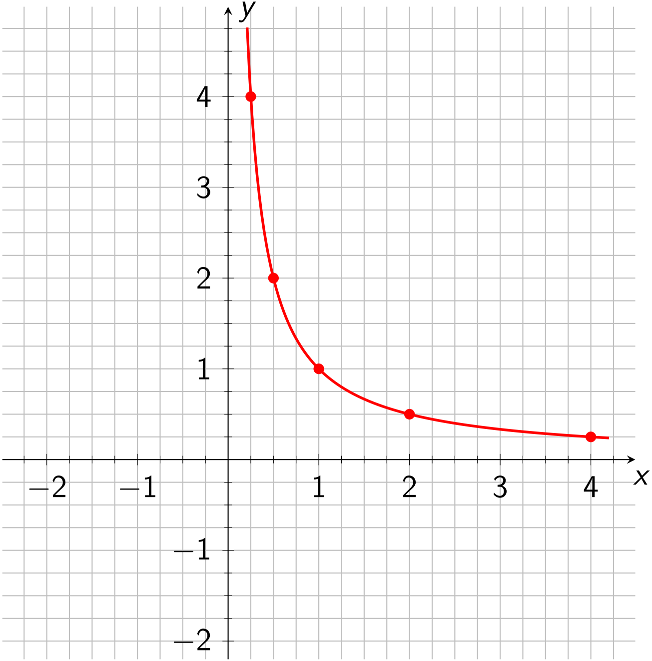
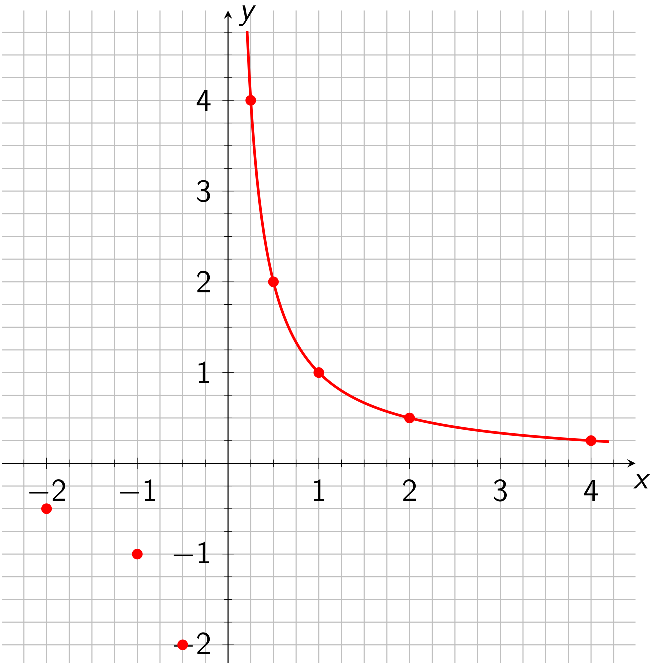


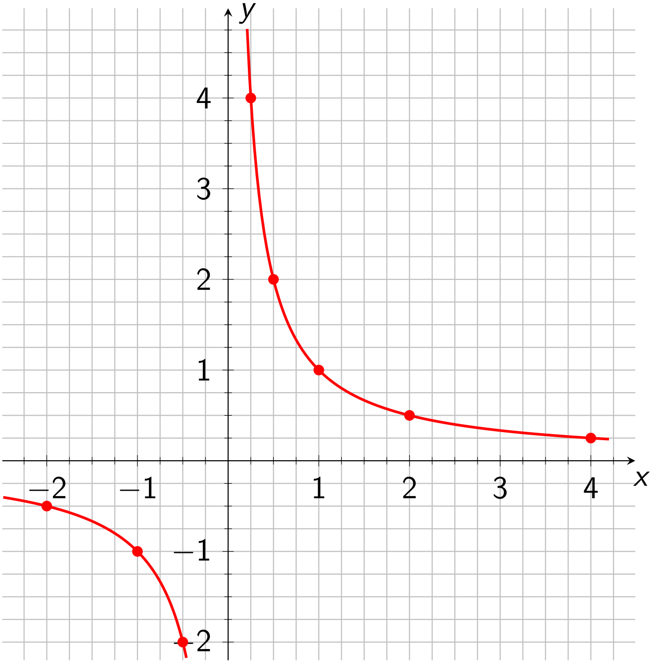
$$y = \frac{1}{x}$$



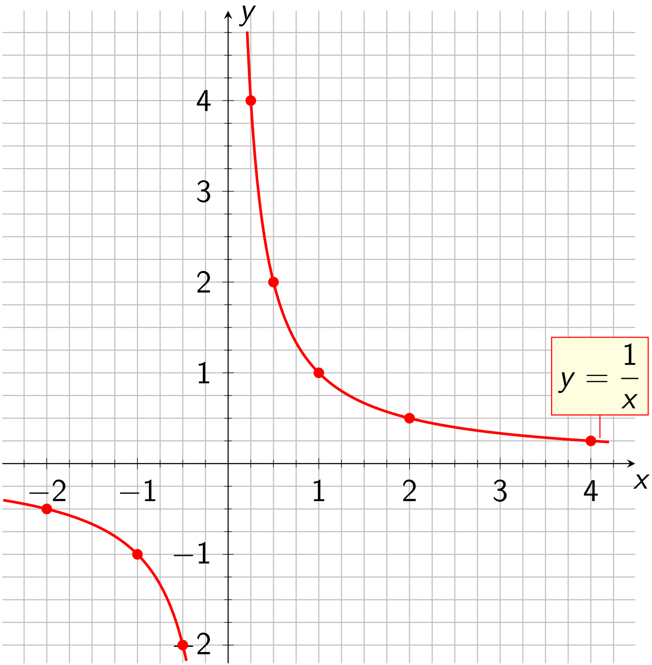
$$y = \frac{1}{x}$$



$$y = \frac{1}{x}$$



$$y = \frac{1}{x}$$


y
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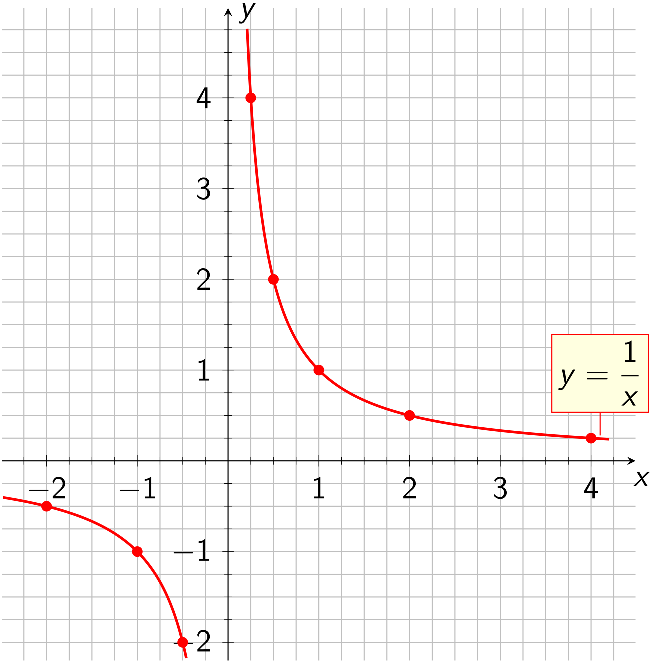
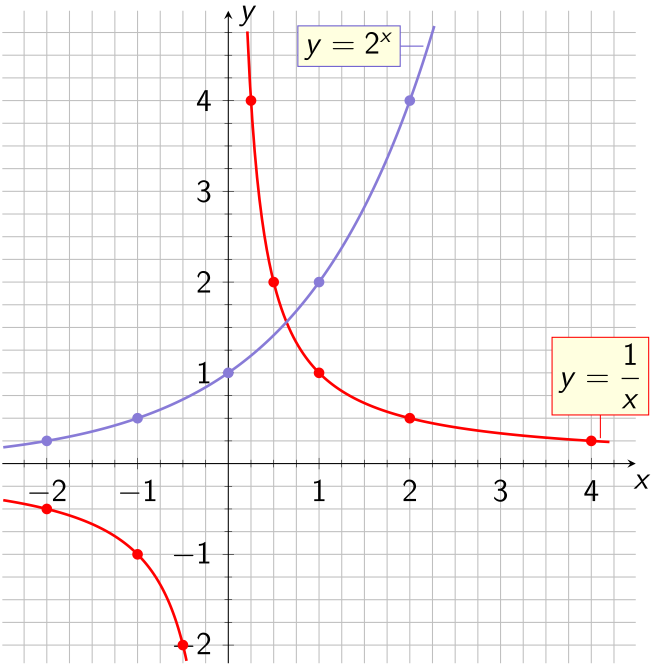
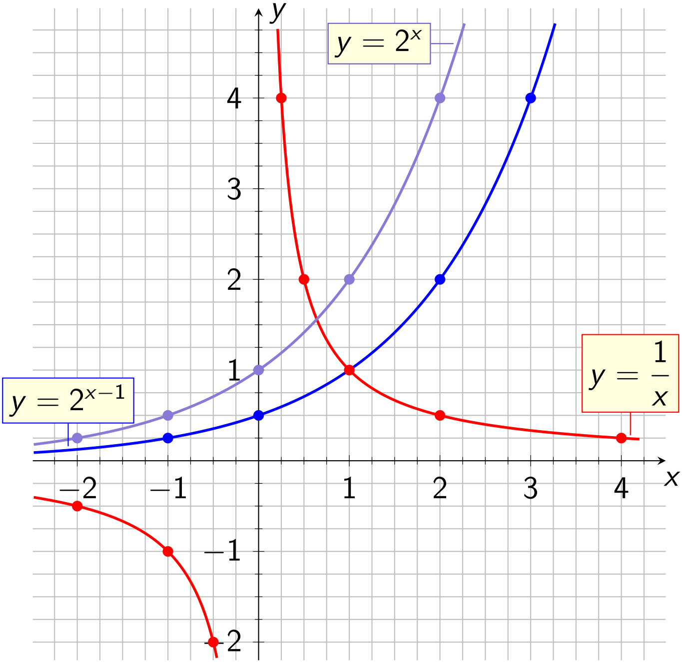
4
$y = \frac{1}{x}$



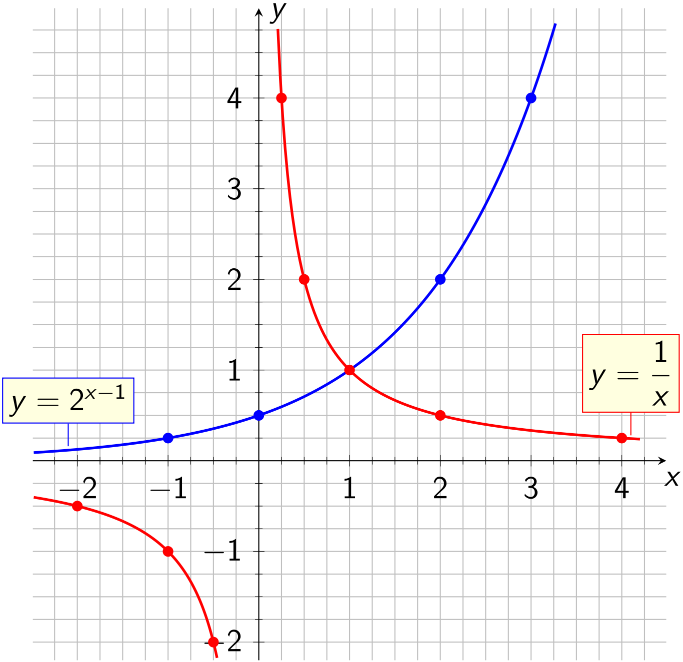
$y = 2^{x-1}$



$$y = 2^{x-1}$$



$y = 2^{x-1}$



y
x
4
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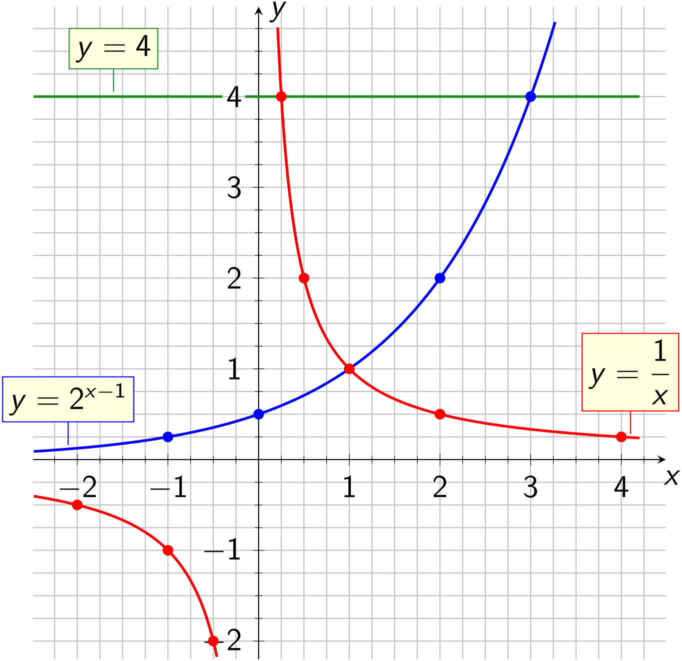
4
$y = 2^{x-1}$
$y = \frac{1}{x}$



$y = 4$
$y = 2^{x-1}$
$y = \frac{1}{x}$
y
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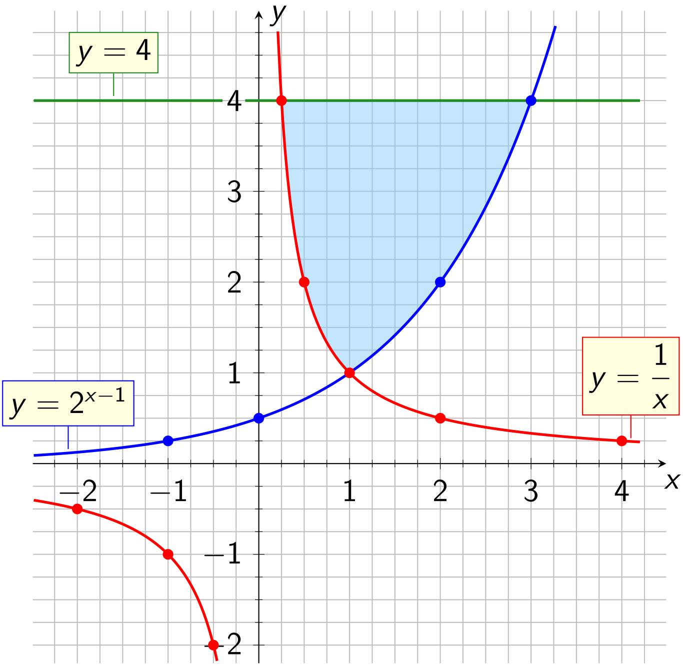

y
$y = 4$
4
3
2
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$y = 2^{x-1}$
$y = \dfrac{1}{x}$
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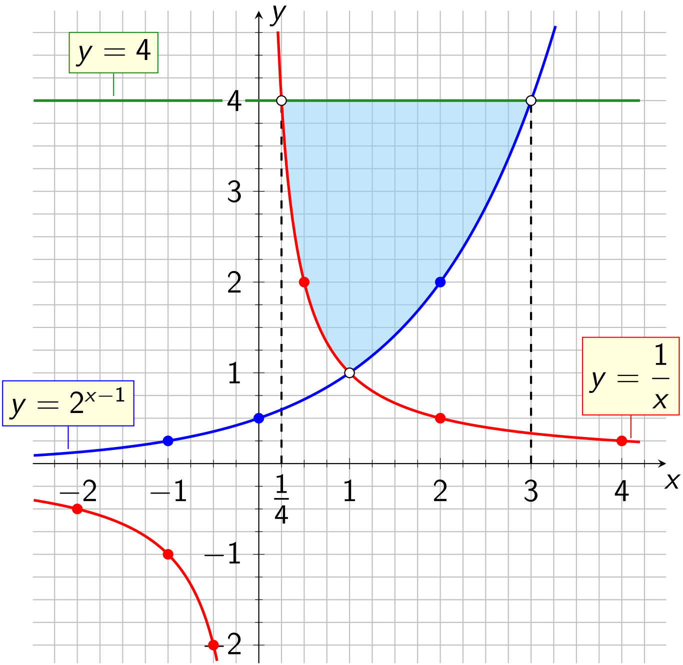
x



$y = 4$
$y = 2^{x-1}$
$y = \frac{1}{x}$
x
y
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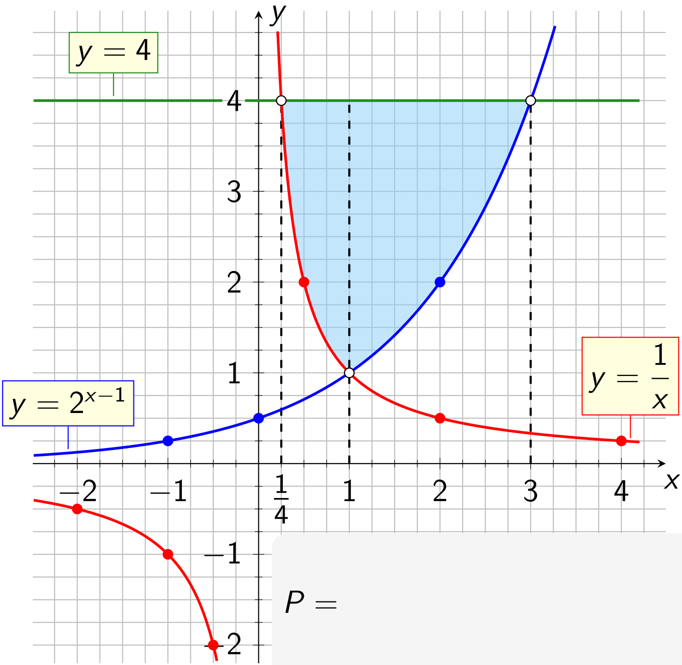
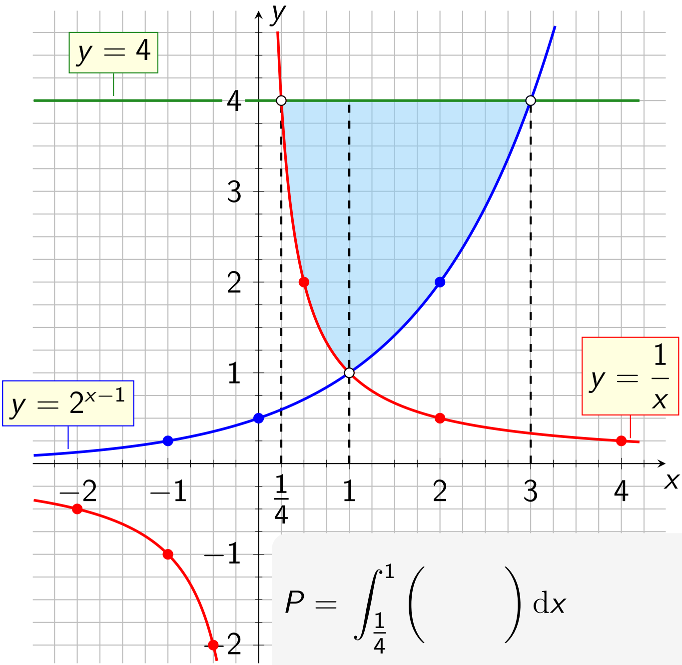
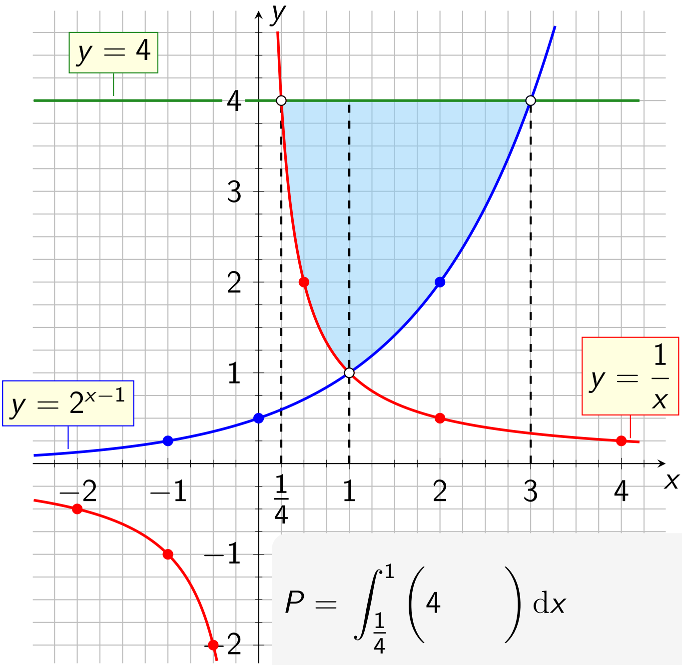


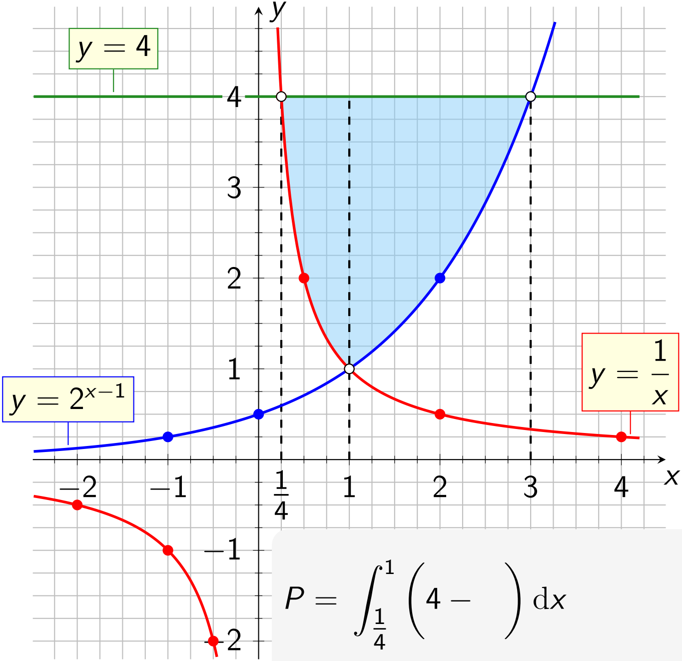
$P =$



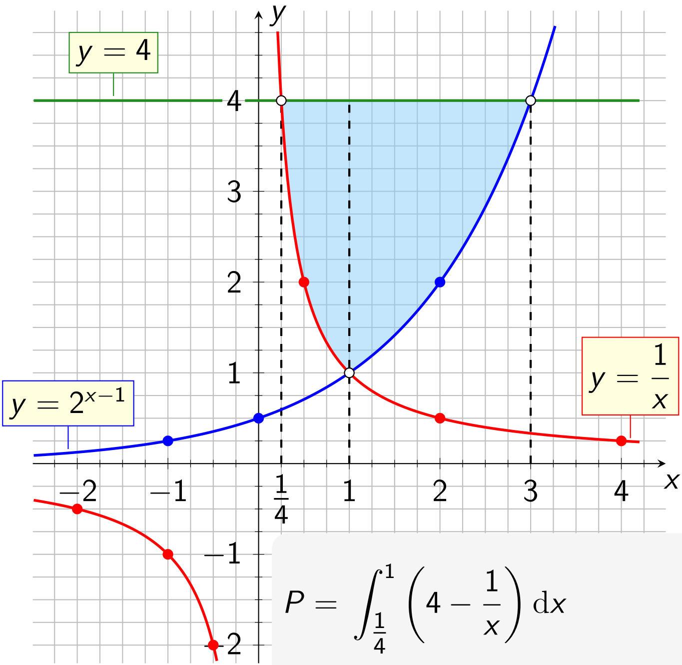
$$P = \int_{\frac{1}{4}}^{1} \Big(\qquad \Big)\, \mathrm{d}x$$



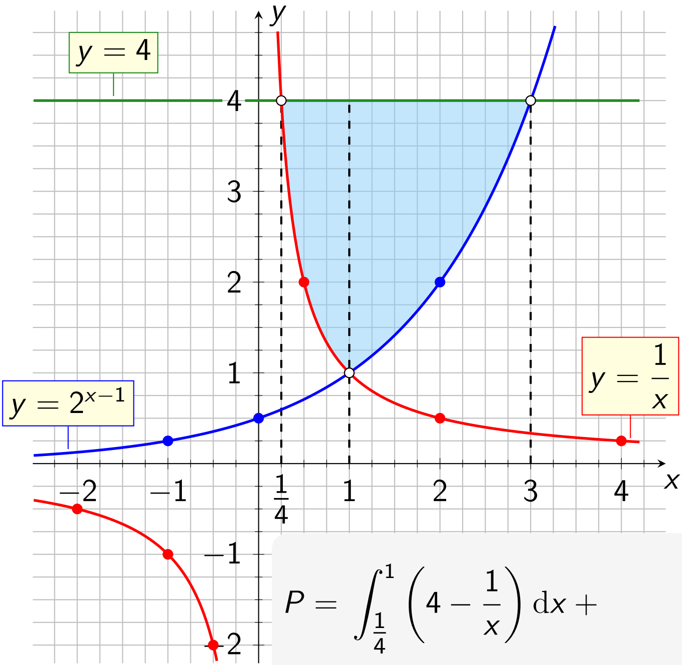
$$P = \int_{\frac{1}{4}}^{1} \left(4 \qquad \right) \mathrm{d}x$$



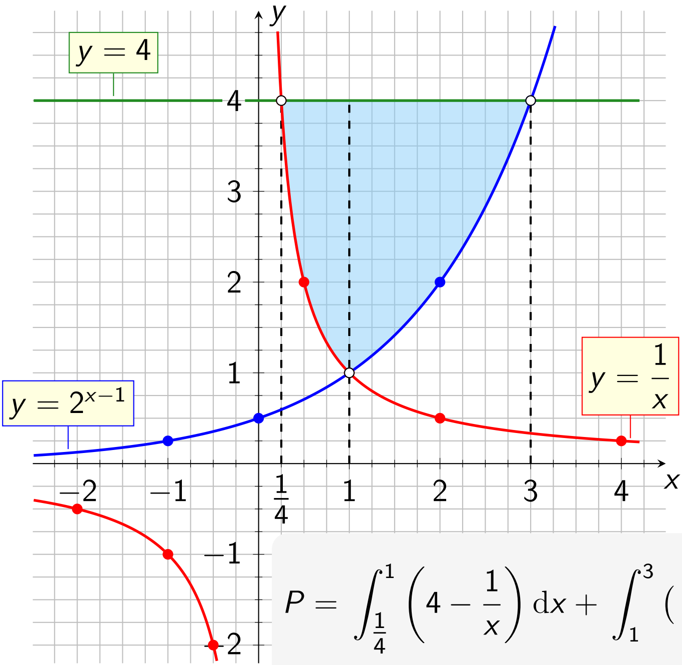
$$P = \int_{\frac{1}{4}}^{1} \left(4 - \right) \mathrm{d}x$$



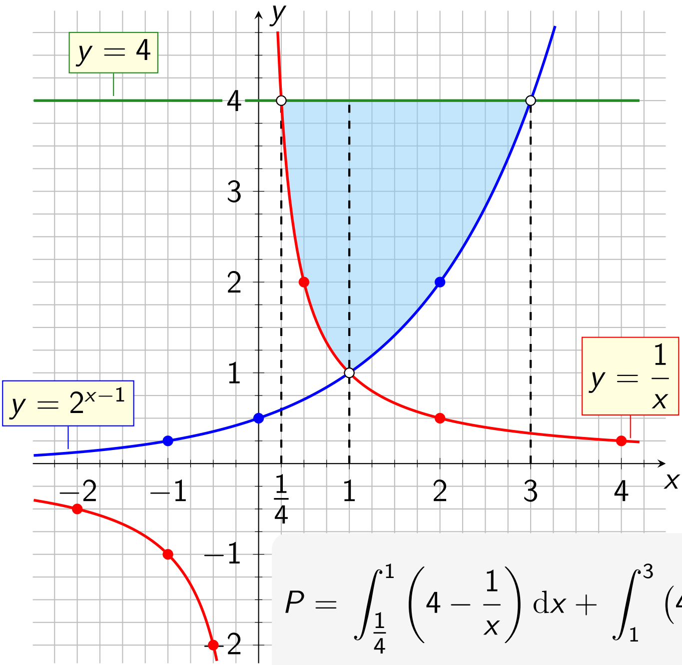
$$P = \int_{\frac{1}{4}}^{1} \left(4 - \frac{1}{x}\right) \mathrm{d}x$$



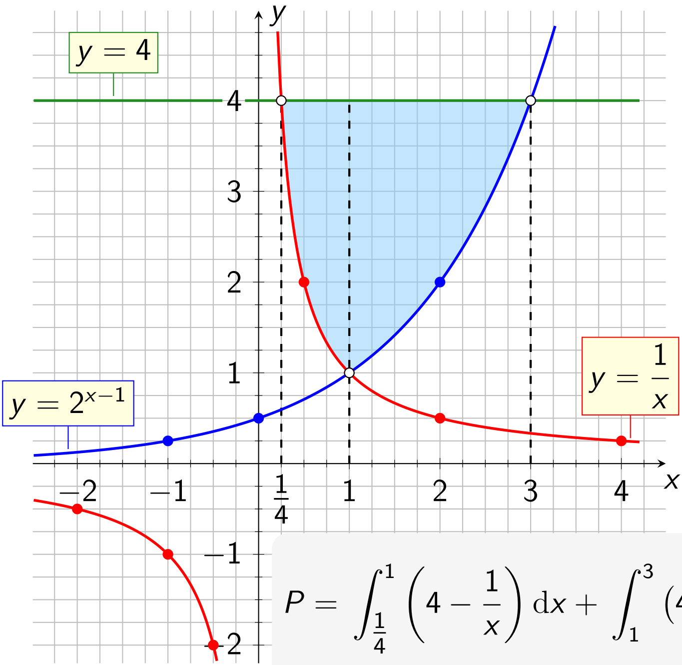
$$P = \int_{\frac{1}{4}}^{1} \left(4 - \frac{1}{x}\right) \mathrm{d}x +$$



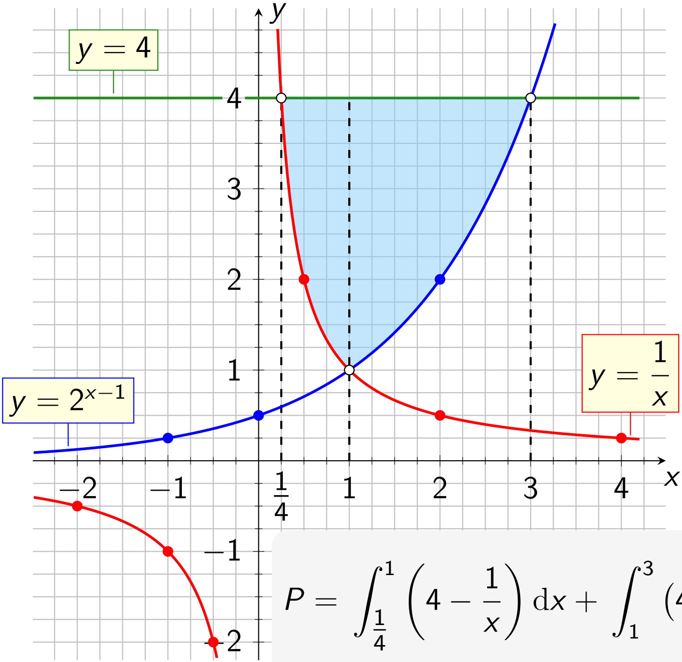
$$P = \int_{\frac{1}{4}}^{1} \left(4 - \frac{1}{x}\right) \mathrm{d}x + \int_{1}^{3} \left(\qquad \right) \mathrm{d}x$$



$$P = \int_{\frac{1}{4}}^{1} \left(4 - \frac{1}{x}\right) \mathrm{d}x + \int_{1}^{3} \left(4 \qquad\qquad \right) \mathrm{d}x$$



$$P = \int_{\frac{1}{4}}^{1} \left(4 - \frac{1}{x}\right) \mathrm{d}x + \int_{1}^{3} \left(4 - \qquad\right) \mathrm{d}x$$



$$P = \int_{\frac{1}{4}}^{1} \left(4 - \frac{1}{x}\right) \mathrm{d}x + \int_{1}^{3} \left(4 - 2^{x-1}\right) \mathrm{d}x$$

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$$P=\int_{\frac{1}{4}}^{1}\left(4-\frac{1}{x}\right)\mathrm{d}x+\int_{1}^{3}\left(4-2^{x-1}\right)\mathrm{d}x=$$

$$=($$

$$P = \int_{\frac{1}{4}}^{1} \left(4 - \frac{1}{x}\right) \mathrm{d}x + \int_{1}^{3} \left(4 - 2^{x-1}\right) \mathrm{d}x =$$

$$= (4x$$

$$P = \int_{\frac{1}{4}}^{1} \left(4 - \frac{1}{x}\right) \mathrm{d}x + \int_{1}^{3} \left(4 - 2^{x-1}\right) \mathrm{d}x =$$

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$$= (4x - \ln|x|$$

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$$= \left(4x - \ln|x|\right) \Big|_{\frac{1}{4}}^{1}$$

$$P = \int_{\frac{1}{4}}^{1} \left(4 - \frac{1}{x}\right) \mathrm{d}x + \int_{1}^{3} \left(4 - 2^{x-1}\right) \mathrm{d}x =$$

$$= (4x - \ln|x|)\Big|_{\frac{1}{4}}^{1} + \Bigg($$

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$$= (4x - \ln|x|)\Big|_{\frac{1}{4}}^{1} + \Bigg(4x -$$

$$\boxed{\int a^x \, \mathrm{d}x = \frac{a^x}{\ln a} + C}$$

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$$\int 2^{x-1} \, \mathrm{d}x =$$

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$$\boxed{\int a^x \, \mathrm{d}x = \frac{a^x}{\ln a} + C}$$

$$= (4x - \ln|x|)\Big|_{\frac{1}{4}}^{1} + \Big(4x -$$

$$\int 2^{x-1} \, \mathrm{d}x = \Big[\; x - 1 = t$$

$$P = \int_{\frac{1}{4}}^{1} \left(4 - \frac{1}{x}\right) \mathrm{d}x + \int_{1}^{3} \left(4 - 2^{x-1}\right) \mathrm{d}x =$$

$$\boxed{\int a^x \, \mathrm{d}x = \frac{a^x}{\ln a} + C}$$

$$= (4x - \ln|x|)\Big|_{\frac{1}{4}}^{1} + \Big(4x -$$

$$\int 2^{x-1} \, \mathrm{d}x = \Big[\; x - 1 = t \; / \, '$$

$$P = \int_{\frac{1}{4}}^{1} \left(4 - \frac{1}{x}\right) \mathrm{d}x + \int_{1}^{3} \left(4 - 2^{x-1}\right) \mathrm{d}x =$$

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$$= (4x - \ln|x|)\Big|_{\frac{1}{4}}^{1} + \Big(4x -$$

$$\int 2^{x-1} \, \mathrm{d}x = \left[\begin{array}{c} x - 1 = t \,/\, ' \\ \mathrm{d}x = \mathrm{d}t \end{array} \right] =$$

$$\boxed{\int a^x \,\mathrm{d}x = \frac{a^x}{\ln a} + C}$$

$$P = \int_{\frac{1}{4}}^{1} \left(4 - \frac{1}{x}\right) \mathrm{d}x + \int_{1}^{3} \left(4 - 2^{x-1}\right) \mathrm{d}x =$$

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$$\int 2^{x-1} \,\mathrm{d}x = \left[\begin{array}{c} x - 1 = t \,/\,' \\ \mathrm{d}x = \mathrm{d}t \end{array}\right] = \int$$

$$P = \int_{\frac{1}{4}}^{1} \left(4 - \frac{1}{x}\right) \mathrm{d}x + \int_{1}^{3} \left(4 - 2^{x-1}\right) \mathrm{d}x =$$

$$\boxed{\int a^x \, \mathrm{d}x = \frac{a^x}{\ln a} + C}$$

$$= \left(4x - \ln|x|\right)\Big|_{\frac{1}{4}}^{1} + \Big(4x -$$

$$\int 2^{x-1} \, \mathrm{d}x = \left[\begin{array}{c} x - 1 = t \,/\, ' \\ \mathrm{d}x = \mathrm{d}t \end{array} \right] = \int 2^t$$

$$\boxed{\int a^x \, \mathrm{d}x = \frac{a^x}{\ln a} + C}$$

$$P = \int_{\frac{1}{4}}^{1} \left(4 - \frac{1}{x}\right) \mathrm{d}x + \int_{1}^{3} \left(4 - 2^{x-1}\right) \mathrm{d}x =$$

$$= \left(4x - \ \ln|x|\right)\Big|_{\frac{1}{4}}^{1} + \Big(4x -$$

$$\int 2^{x-1} \, \mathrm{d}x = \left[\begin{array}{c} x - 1 = t \, /\, ' \\ \mathrm{d}x = \mathrm{d}t \end{array}\right] = \int 2^t \, \mathrm{d}t$$

$$\boxed{\int a^x \, \mathrm{d}x = \frac{a^x}{\ln a} + C}$$

$$P = \int_{\frac{1}{4}}^{1} \left(4 - \frac{1}{x}\right) \mathrm{d}x + \int_{1}^{3} \left(4 - 2^{x-1}\right) \mathrm{d}x =$$

$$= (4x - \ln|x|)\Big|_{\frac{1}{4}}^{1} + \Big(4x -$$

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$$= \Bigg($$

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$$= \Big((4 - \ln 1)$$

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$$= \left((4 - \ln 1) - \left(1 - \ln \frac{1}{4}\right)\right.$$

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$P\approx 5.28562$

peti zadatak

Zadatak 5

Pomoću određenog integrala dokažite da je površina kruga polumjera r jednaka $r^2\pi$.

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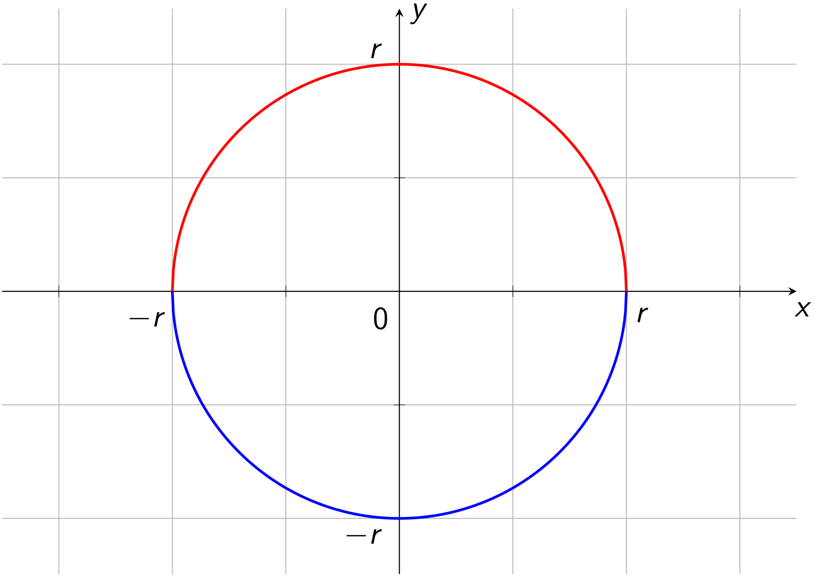
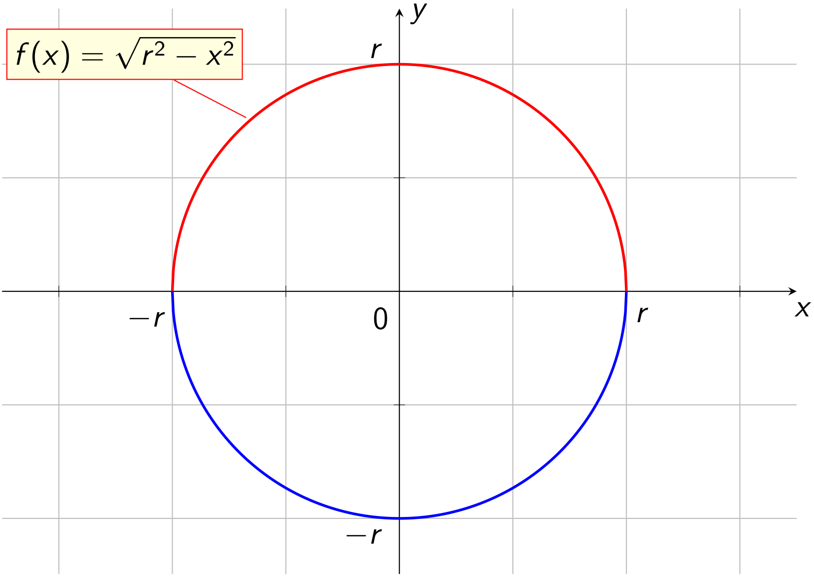
$+ \rightsquigarrow$ gornja polukružnica

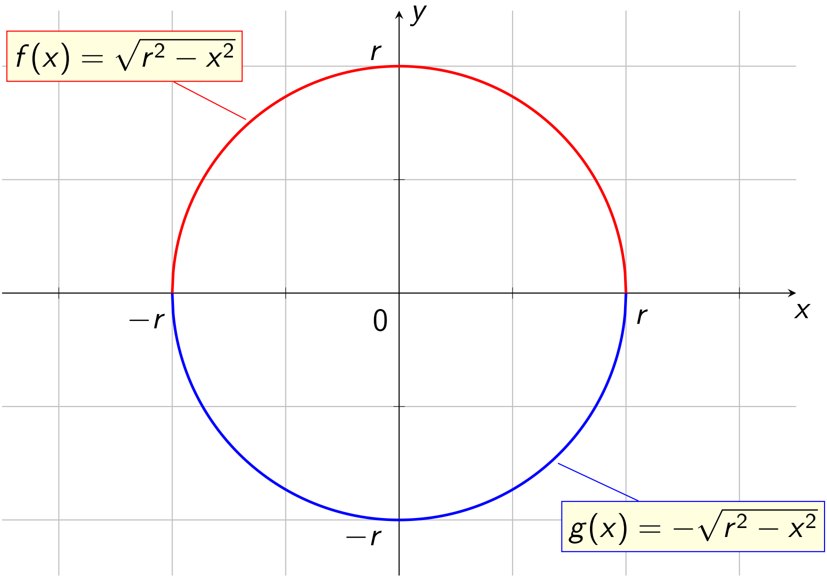
$- \rightsquigarrow$ donja polukružnica



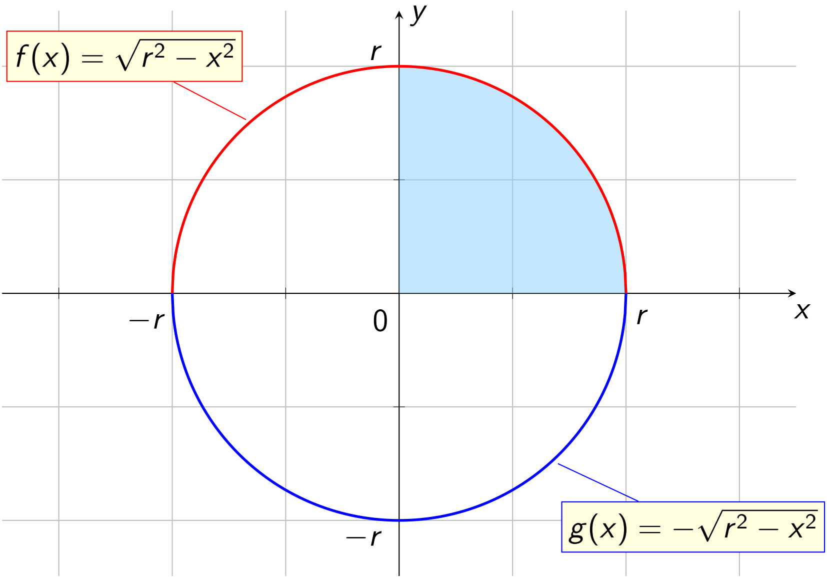
y
r
x
−r
0
r
−r



$f(x) = \sqrt{r^2 - x^2}$
y
r
$-r$
0
r
x
$-r$



$f(x) = \sqrt{r^2 - x^2}$
y
r
x
$-r$
0
r
$-r$
$g(x) = -\sqrt{r^2 - x^2}$



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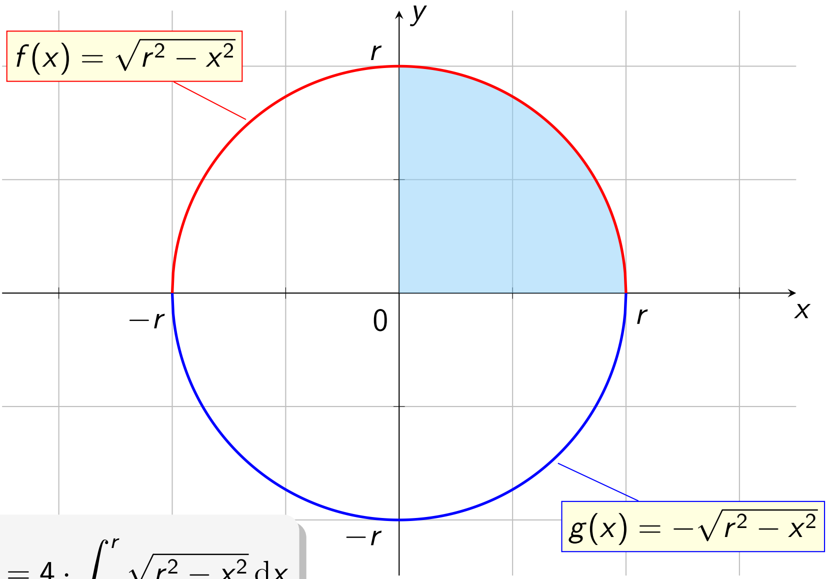

$$P = 4 \cdot \int_0^r \sqrt{r^2 - x^2}\,\mathrm{d}x$$

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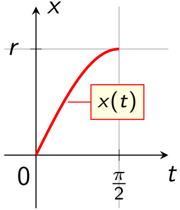

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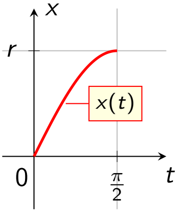

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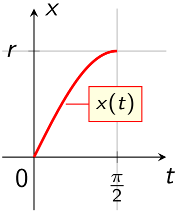

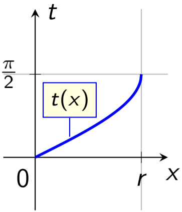

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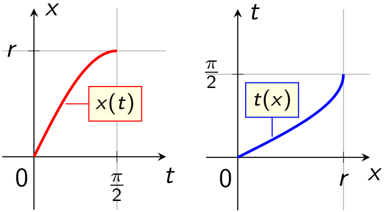

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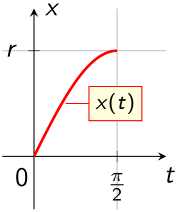

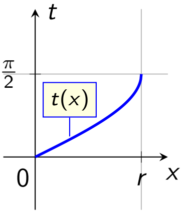

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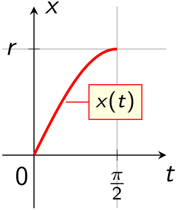

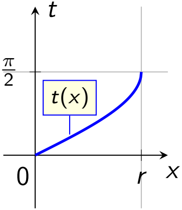

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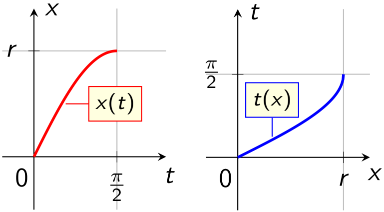

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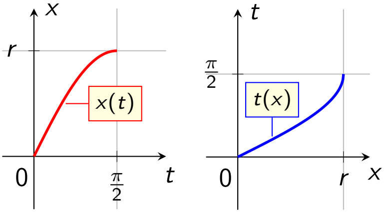

$$= 4 \cdot \int_0^{\frac{\pi}{2}}$$

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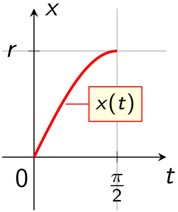

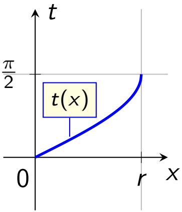

$$= 4 \cdot \int_0^{\frac{\pi}{2}} \sqrt{r^2 - r^2 \sin^2 t}$$

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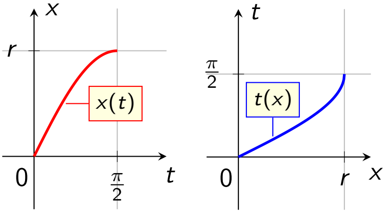

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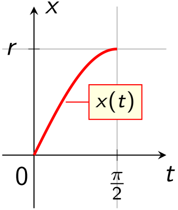

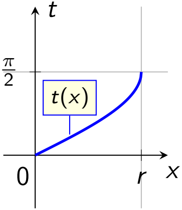

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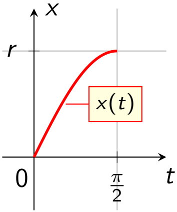

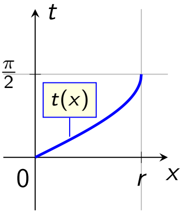

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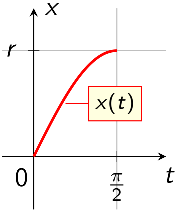

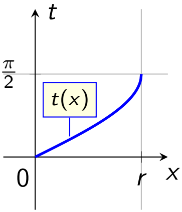

$$= 4 \cdot \int_0^{\frac{\pi}{2}} \sqrt{r^2 - r^2 \sin^2 t} \cdot r \cos t\,\mathrm{d}t =$$

$$= 4 \cdot \int_0^{\frac{\pi}{2}} \sqrt{r^2 \left(1 - \sin^2 t\right)}$$

$$P = 4 \cdot \int_0^r \sqrt{r^2 - x^2}\,\mathrm{d}x =$$

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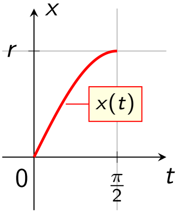

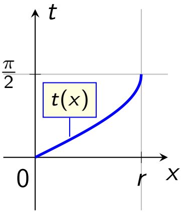

$$= 4 \cdot \int_0^{\frac{\pi}{2}} \sqrt{r^2 - r^2 \sin^2 t} \cdot r \cos t\,\mathrm{d}t =$$

$$= 4 \cdot \int_0^{\frac{\pi}{2}} \sqrt{r^2\left(1 - \sin^2 t\right)} \cdot r \cos t\,\mathrm{d}t$$

$$x : \left[0, \tfrac{\pi}{2}\right] \to [0, r],\ x(t) = r \sin t$$

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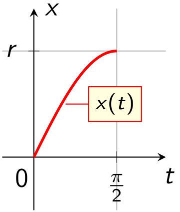

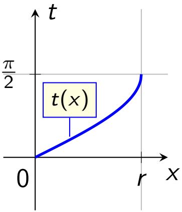

$$= 4 \cdot \int_0^{\frac{\pi}{2}} \sqrt{r^2 - r^2 \sin^2 t} \cdot r \cos t \,\mathrm{d}t =$$

$$= 4 \cdot \int_0^{\frac{\pi}{2}} \sqrt{r^2\left(1 - \sin^2 t\right)} \cdot r \cos t \,\mathrm{d}t = 4 \cdot \int_0^{\frac{\pi}{2}}$$

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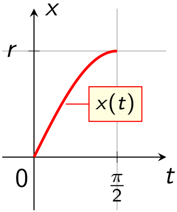

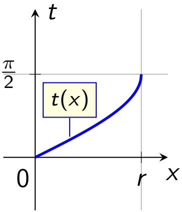

$$= 4 \cdot \int_0^{\frac{\pi}{2}} \sqrt{r^2 - r^2 \sin^2 t} \cdot r \cos t \, \mathrm{d}t =$$

$$= 4 \cdot \int_0^{\frac{\pi}{2}} \sqrt{r^2 \left(1 - \sin^2 t\right)} \cdot r \cos t \, \mathrm{d}t = 4 \cdot \int_0^{\frac{\pi}{2}} \sqrt{r^2 \cos^2 t}$$

$$P = 4 \cdot \int_0^r \sqrt{r^2 - x^2}\,\mathrm{d}x =$$

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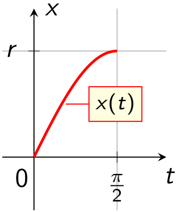

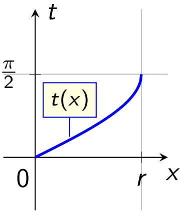

$$= 4 \cdot \int_0^{\frac{\pi}{2}} \sqrt{r^2 - r^2 \sin^2 t} \cdot r \cos t\,\mathrm{d}t =$$

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$$x : \left[0, \tfrac{\pi}{2}\right] \to [0, r],\ \ x(t) = r \sin t$$

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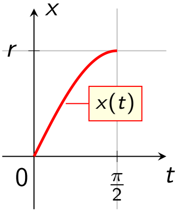

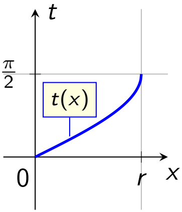

$$= 4 \cdot \int_0^{\frac{\pi}{2}} \sqrt{r^2 - r^2 \sin^2 t} \cdot r \cos t \,\mathrm{d}t =$$

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$$= 4 \cdot \int_0^{\frac{\pi}{2}}$$

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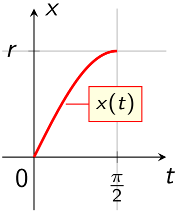

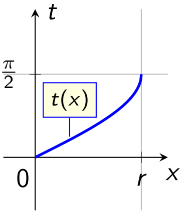

$$= 4 \cdot \int_0^{\frac{\pi}{2}} \sqrt{r^2 - r^2 \sin^2 t} \cdot r \cos t \,\mathrm{d}t =$$

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$$= 4 \cdot \int_0^{\frac{\pi}{2}} \sqrt{r^2} \cdot \sqrt{\cos^2 t}$$

$$\boxed{\begin{array}{c} x : \left[0, \frac{\pi}{2}\right] \to [0, r],\ x(t) = r \sin t \\ t : [0, r] \to \left[0, \frac{\pi}{2}\right],\ t(x) = \arcsin \frac{x}{r} \end{array}}$$

$$P = 4 \cdot \int_0^r \sqrt{r^2 - x^2}\, \mathrm{d}x =$$

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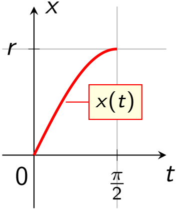

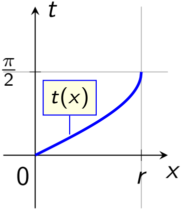

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$$= 4 \cdot \int_0^{\frac{\pi}{2}} \sqrt{r^2 \left(1 - \sin^2 t\right)} \cdot r \cos t \, \mathrm{d}t = 4 \cdot \int_0^{\frac{\pi}{2}} \sqrt{r^2 \cos^2 t} \cdot r \cos t \, \mathrm{d}t =$$

$$= 4 \cdot \int_0^{\frac{\pi}{2}} \sqrt{r^2} \cdot \sqrt{\cos^2 t} \cdot r \cos t \, \mathrm{d}t$$

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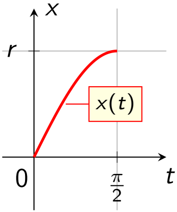
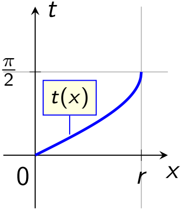
$$P = 4 \cdot \int_0^r \sqrt{r^2 - x^2}\,\mathrm{d}x =$$

$$= \left[\begin{array}{ll} x = r \sin t \,/\, ' & x = 0 \rightsquigarrow t = 0 \\ \mathrm{d}x = r \cos t \,\mathrm{d}t & x = r \rightsquigarrow t = \dfrac{\pi}{2} \end{array}\right] =$$





$$= 4 \cdot \int_0^{\frac{\pi}{2}} \sqrt{r^2 - r^2 \sin^2 t} \cdot r \cos t \,\mathrm{d}t =$$

$$= 4 \cdot \int_0^{\frac{\pi}{2}} \sqrt{r^2\left(1 - \sin^2 t\right)} \cdot r \cos t \,\mathrm{d}t = 4 \cdot \int_0^{\frac{\pi}{2}} \sqrt{r^2 \cos^2 t} \cdot r \cos t \,\mathrm{d}t =$$

$$\sqrt{a^2} = |a|$$

$$= 4 \cdot \int_0^{\frac{\pi}{2}} \sqrt{r^2} \cdot \sqrt{\cos^2 t} \cdot r \cos t \,\mathrm{d}t$$

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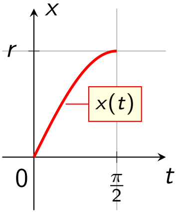

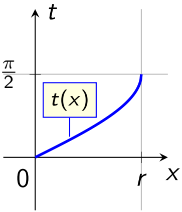

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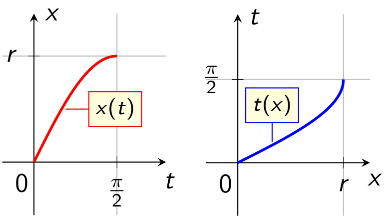

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jer je $r > 0$

$$x : \left[0, \tfrac{\pi}{2}\right] \to [0, r],\ \ x(t) = r \sin t$$

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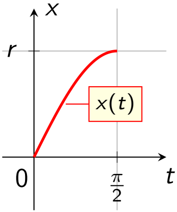

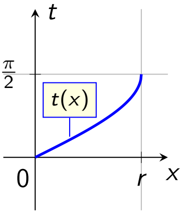

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jer je $r > 0$

$$P = 4 \cdot \int_0^r \sqrt{r^2 - x^2}\,\mathrm{d}x =$$

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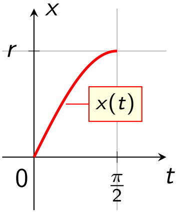

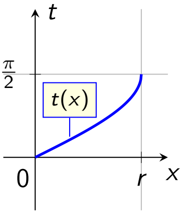

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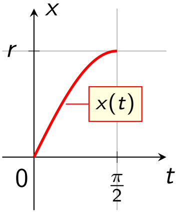

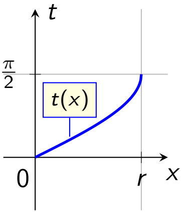

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$\sqrt{a^2} = |a|$

$$= 4 \cdot \int_0^{\frac{\pi}{2}} \underbrace{\sqrt{r^2}}_{=r} \cdot \sqrt{\cos^2 t} \cdot r \cos t \,\mathrm{d}t = 4 \cdot \int_0^{\frac{\pi}{2}} r \cdot |\cos t|$$

jer je $r > 0$

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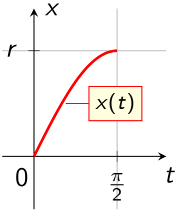

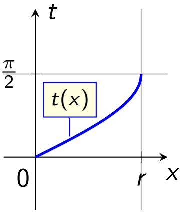

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$$= 4\cdot\int_0^{\frac{\pi}{2}} \underbrace{\sqrt{r^2}}_{=r}\cdot\sqrt{\cos^2 t}\cdot r\cos t\,\mathrm{d}t = 4\cdot\int_0^{\frac{\pi}{2}} r\cdot\left|\cos t\right|\cdot r\cos t\,\mathrm{d}t$$

jer je $r > 0$

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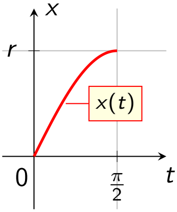

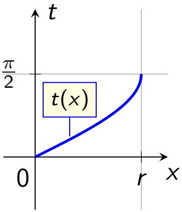

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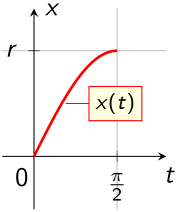

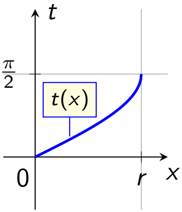

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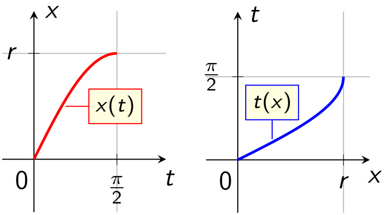

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jer je $r > 0$

jer je $\cos t \geqslant 0$

za $t \in \left[0, \frac{\pi}{2}\right]$

$$x : \left[0, \tfrac{\pi}{2}\right] \to [0, r],\ \ x(t) = r \sin t$$

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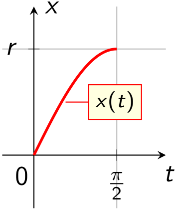

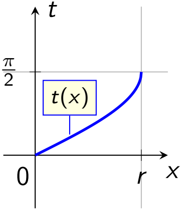

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jer je $r > 0$

jer je $\cos t \geqslant 0$ za $t \in \left[0, \frac{\pi}{2}\right]$

$$= 4 \cdot \int_0^{\frac{\pi}{2}}$$

$$x : \left[0, \tfrac{\pi}{2}\right] \to [0, r], \;\; x(t) = r \sin t$$

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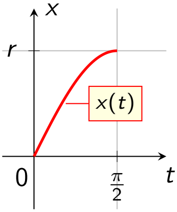

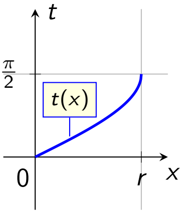

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jer je $r > 0$

jer je $\cos t \geqslant 0$ za $t \in \left[0, \frac{\pi}{2}\right]$

$$= 4 \cdot \int_0^{\frac{\pi}{2}} r^2$$

$$x : \left[0, \tfrac{\pi}{2}\right] \to [0, r], \;\; x(t) = r \sin t$$

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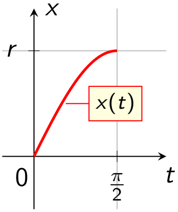

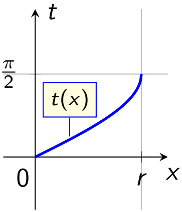

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$$= 4 \cdot \int_0^{\frac{\pi}{2}} \underbrace{\sqrt{r^2}}_{=r} \cdot \sqrt{\cos^2 t} \cdot r \cos t\,\mathrm{d}t = 4 \cdot \int_0^{\frac{\pi}{2}} r \cdot \underbrace{\left|\cos t\right|}_{=\cos t} \cdot r \cos t\,\mathrm{d}t =$$

jer je $r > 0$

jer je $\cos t \geqslant 0$ za $t \in \left[0, \tfrac{\pi}{2}\right]$

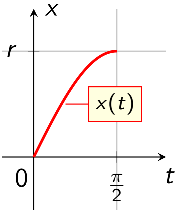
$$= 4 \cdot \int_0^{\frac{\pi}{2}} r^2 \cdot \cos t$$

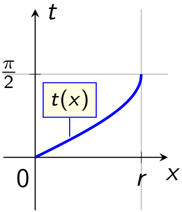
$$x : \left[0, \tfrac{\pi}{2}\right] \to [0, r], \;\; x(t) = r \sin t$$

$$t : [0, r] \to \left[0, \tfrac{\pi}{2}\right], \;\; t(x) = \arcsin \tfrac{x}{r}$$

$$P = 4 \cdot \int_0^r \sqrt{r^2 - x^2}\,\mathrm{d}x =$$

$$= \left[\begin{array}{ll} x = r \sin t \;/\;' & x = 0 \rightsquigarrow t = 0 \\ \mathrm{d}x = r \cos t\,\mathrm{d}t & x = r \rightsquigarrow t = \dfrac{\pi}{2} \end{array}\right] =$$





$$= 4 \cdot \int_0^{\frac{\pi}{2}} \sqrt{r^2 - r^2 \sin^2 t} \cdot r \cos t\,\mathrm{d}t =$$

$$= 4 \cdot \int_0^{\frac{\pi}{2}} \sqrt{r^2\left(1 - \sin^2 t\right)} \cdot r \cos t\,\mathrm{d}t = 4 \cdot \int_0^{\frac{\pi}{2}} \sqrt{r^2 \cos^2 t} \cdot r \cos t\,\mathrm{d}t =$$

$$\sqrt{a^2} = |a|$$

$$= 4 \cdot \int_0^{\frac{\pi}{2}} \underbrace{\sqrt{r^2}}_{=r} \cdot \sqrt{\cos^2 t} \cdot r \cos t\,\mathrm{d}t = 4 \cdot \int_0^{\frac{\pi}{2}} r \cdot \underbrace{|\cos t|}_{=\cos t} \cdot r \cos t\,\mathrm{d}t =$$

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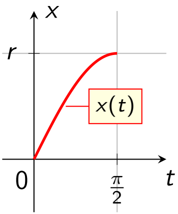

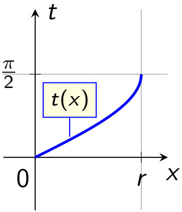

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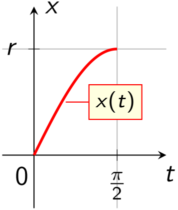

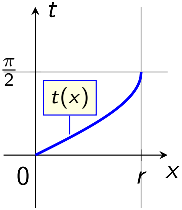

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$$= 4 \cdot \int_0^{\frac{\pi}{2}} r^2 \cdot \cos t \cdot \cos t \,\mathrm{d}t = 4r^2 \cdot$$

$$P = 4 \cdot \int_0^r \sqrt{r^2 - x^2}\,\mathrm{d}x =$$

$$x : \left[0, \tfrac{\pi}{2}\right] \to [0, r], \;\; x(t) = r \sin t$$

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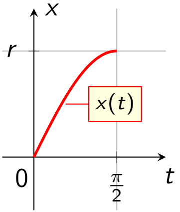

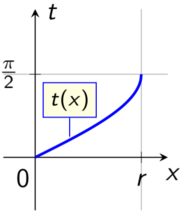

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$$\int \cos^2 t \, \mathrm{d}t =$$

$$\int \cos^2 t \,\mathrm{d}t = \int \frac{\cos 2t + 1}{2} \,\mathrm{d}t$$

$$\int \cos^2 t \,\mathrm{d}t = \int \frac{\cos 2t + 1}{2} \,\mathrm{d}t = \int \left(\frac{1}{2} \cos 2t + \frac{1}{2}\right) \mathrm{d}t$$

$$\int \cos^2 t \,\mathrm{d}t = \int \frac{\cos 2t + 1}{2}\,\mathrm{d}t = \int \left(\frac{1}{2}\cos 2t + \frac{1}{2}\right)\mathrm{d}t =$$

$$=$$

$$\int \cos^2 t \,\mathrm{d}t = \int \frac{\cos 2t + 1}{2} \,\mathrm{d}t = \int \left(\frac{1}{2} \cos 2t + \frac{1}{2} \right) \mathrm{d}t =$$

$$= \frac{1}{2} \int \cos 2t \,\mathrm{d}t$$

$$\int \cos^2 t \,\mathrm{d}t = \int \frac{\cos 2t + 1}{2} \,\mathrm{d}t = \int \left(\frac{1}{2}\cos 2t + \frac{1}{2}\right) \mathrm{d}t =$$

$$= \frac{1}{2}\int \cos 2t \,\mathrm{d}t +$$

$$\int \cos^2 t \, \mathrm{d}t = \int \frac{\cos 2t + 1}{2} \, \mathrm{d}t = \int \left(\frac{1}{2} \cos 2t + \frac{1}{2} \right) \mathrm{d}t =$$

$$= \frac{1}{2} \int \cos 2t \, \mathrm{d}t + \frac{1}{2} \int \mathrm{d}t$$

$$\int \cos^2 t \, \mathrm{d}t = \int \frac{\cos 2t + 1}{2} \, \mathrm{d}t = \int \left(\frac{1}{2} \cos 2t + \frac{1}{2} \right) \mathrm{d}t =$$

$$= \frac{1}{2} \int \cos 2t \, \mathrm{d}t + \frac{1}{2} \int \mathrm{d}t = \frac{1}{2} \cdot$$

$$\int \cos^2 t \, \mathrm{d}t = \int \frac{\cos 2t + 1}{2} \, \mathrm{d}t = \int \left(\frac{1}{2} \cos 2t + \frac{1}{2} \right) \mathrm{d}t =$$

$$= \frac{1}{2} \int \cos 2t \, \mathrm{d}t + \frac{1}{2} \int \mathrm{d}t = \frac{1}{2} \cdot \frac{1}{2} \sin 2t$$

$$\int \cos^2 t \,\mathrm{d}t = \int \frac{\cos 2t + 1}{2} \,\mathrm{d}t = \int \left(\frac{1}{2} \cos 2t + \frac{1}{2}\right) \mathrm{d}t =$$

$$= \frac{1}{2} \int \cos 2t \,\mathrm{d}t + \frac{1}{2} \int \mathrm{d}t = \frac{1}{2} \cdot \frac{1}{2} \sin 2t + \frac{1}{2} t$$

$$\int \cos^2 t \, \mathrm{d}t = \int \frac{\cos 2t + 1}{2} \, \mathrm{d}t = \int \left(\frac{1}{2} \cos 2t + \frac{1}{2} \right) \mathrm{d}t =$$

$$= \frac{1}{2} \int \cos 2t \, \mathrm{d}t + \frac{1}{2} \int \mathrm{d}t = \frac{1}{2} \cdot \frac{1}{2} \sin 2t + \frac{1}{2} t + C$$

$$\int \cos^2 t \, \mathrm{d}t = \int \frac{\cos 2t + 1}{2} \, \mathrm{d}t = \int \left(\frac{1}{2} \cos 2t + \frac{1}{2} \right) \mathrm{d}t =$$

$$= \frac{1}{2} \int \cos 2t \, \mathrm{d}t + \frac{1}{2} \int \mathrm{d}t = \frac{1}{2} \cdot \frac{1}{2} \sin 2t + \frac{1}{2} t + C =$$

$$= \frac{1}{4} \sin 2t + \frac{1}{2} t + C$$

$$\int \cos^2 t \, \mathrm{d}t = \int \frac{\cos 2t + 1}{2} \, \mathrm{d}t = \int \left(\frac{1}{2} \cos 2t + \frac{1}{2} \right) \mathrm{d}t =$$

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$$P = 4r^2 \cdot \int_0^{\frac{\pi}{2}} \cos^2 t \, \mathrm{d}t$$

$$\int \cos^2 t \, \mathrm{d}t = \int \frac{\cos 2t + 1}{2} \, \mathrm{d}t = \int \left(\frac{1}{2} \cos 2t + \frac{1}{2} \right) \mathrm{d}t =$$

$$= \frac{1}{2} \int \cos 2t \, \mathrm{d}t + \frac{1}{2} \int \mathrm{d}t = \frac{1}{2} \cdot \frac{1}{2} \sin 2t + \frac{1}{2} t + C =$$

$$= \frac{1}{4} \sin 2t + \frac{1}{2} t + C, \quad C \in \mathbb{R}$$

$$P = 4r^2 \cdot \int_0^{\frac{\pi}{2}} \cos^2 t \, \mathrm{d}t = 4r^2$$

$$\int \cos^2 t \,\mathrm{d}t = \int \frac{\cos 2t + 1}{2} \,\mathrm{d}t = \int \left(\frac{1}{2}\cos 2t + \frac{1}{2}\right) \mathrm{d}t =$$

$$= \frac{1}{2}\int \cos 2t \,\mathrm{d}t + \frac{1}{2}\int \mathrm{d}t = \frac{1}{2} \cdot \frac{1}{2}\sin 2t + \frac{1}{2}t + C =$$

$$= \frac{1}{4}\sin 2t + \frac{1}{2}t + C, \quad C \in \mathbb{R}$$

$$P = 4r^2 \cdot \int_0^{\frac{\pi}{2}} \cos^2 t \,\mathrm{d}t = 4r^2 \left(\frac{1}{4}\sin 2t + \frac{1}{2}t\right)$$

$$\int \cos^2 t \, \mathrm{d}t = \int \frac{\cos 2t + 1}{2} \, \mathrm{d}t = \int \left(\frac{1}{2} \cos 2t + \frac{1}{2} \right) \mathrm{d}t =$$

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$$P = 4r^2 \cdot \int_0^{\frac{\pi}{2}} \cos^2 t \, \mathrm{d}t = 4r^2 \left(\frac{1}{4} \sin 2t + \frac{1}{2} t \right) \Big|_0^{\frac{\pi}{2}}$$

$$\int \cos^2 t \, \mathrm{d}t = \int \frac{\cos 2t + 1}{2} \, \mathrm{d}t = \int \left(\frac{1}{2} \cos 2t + \frac{1}{2} \right) \mathrm{d}t =$$

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$$=$$

$$\int \cos^2 t \,\mathrm{d}t = \int \frac{\cos 2t + 1}{2}\,\mathrm{d}t = \int \left(\frac{1}{2}\cos 2t + \frac{1}{2}\right)\mathrm{d}t =$$

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$$= 4r^2 \left(\frac{1}{4}\sin\left(2\cdot\frac{\pi}{2}\right) + \frac{1}{2}\cdot\frac{\pi}{2}\right)$$

$$\int \cos^2 t \, \mathrm{d}t = \int \frac{\cos 2t + 1}{2} \, \mathrm{d}t = \int \left(\frac{1}{2} \cos 2t + \frac{1}{2} \right) \mathrm{d}t =$$

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$$= 4r^2 \left(\frac{1}{4} \sin \left(2 \cdot \frac{\pi}{2} \right) + \frac{1}{2} \cdot \frac{\pi}{2} \right) -$$

$$\int \cos^2 t \, \mathrm{d}t = \int \frac{\cos 2t + 1}{2} \, \mathrm{d}t = \int \left(\frac{1}{2} \cos 2t + \frac{1}{2} \right) \mathrm{d}t =$$

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$$P = 4r^2 \cdot \int_0^{\frac{\pi}{2}} \cos^2 t \, \mathrm{d}t = 4r^2 \left(\frac{1}{4} \sin 2t + \frac{1}{2} t \right) \Bigg|_0^{\frac{\pi}{2}} =$$

$$= 4r^2 \left(\frac{1}{4} \sin \left(2 \cdot \frac{\pi}{2} \right) + \frac{1}{2} \cdot \frac{\pi}{2} \right) - 4r^2 \left(\frac{1}{4} \sin (2 \cdot 0) + \frac{1}{2} \cdot 0 \right)$$

$$\int \cos^2 t \, \mathrm{d}t = \int \frac{\cos 2t + 1}{2} \, \mathrm{d}t = \int \left(\frac{1}{2} \cos 2t + \frac{1}{2} \right) \mathrm{d}t =$$

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$$= 4r^2 \left(\frac{1}{4} \sin \left(2 \cdot \frac{\pi}{2} \right) + \frac{1}{2} \cdot \frac{\pi}{2} \right) - 4r^2 \left(\frac{1}{4} \sin \left(2 \cdot 0 \right) + \frac{1}{2} \cdot 0 \right) =$$

$$=$$

$$\int \cos^2 t \,\mathrm{d}t = \int \frac{\cos 2t + 1}{2} \,\mathrm{d}t = \int \left(\frac{1}{2}\cos 2t + \frac{1}{2}\right) \mathrm{d}t =$$

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$$= 4r^2 \left(\frac{1}{4}\sin\left(2\cdot\frac{\pi}{2}\right) + \frac{1}{2}\cdot\frac{\pi}{2}\right) - 4r^2 \left(\frac{1}{4}\sin\left(2\cdot 0\right) + \frac{1}{2}\cdot 0\right) =$$

$$= 4r^2 \left(\frac{1}{4}\sin\pi + \frac{\pi}{4}\right)$$

$$\int \cos^2 t \,\mathrm{d}t = \int \frac{\cos 2t + 1}{2} \,\mathrm{d}t = \int \left(\frac{1}{2}\cos 2t + \frac{1}{2}\right) \mathrm{d}t =$$

$$= \frac{1}{2}\int \cos 2t \,\mathrm{d}t + \frac{1}{2}\int \mathrm{d}t = \frac{1}{2}\cdot\frac{1}{2}\sin 2t + \frac{1}{2}t + C =$$

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$$P = 4r^2 \cdot \int_0^{\frac{\pi}{2}} \cos^2 t \,\mathrm{d}t = 4r^2 \left(\frac{1}{4}\sin 2t + \frac{1}{2}t\right)\Bigg|_0^{\frac{\pi}{2}} =$$

$$= 4r^2 \left(\frac{1}{4}\sin\left(2\cdot\frac{\pi}{2}\right) + \frac{1}{2}\cdot\frac{\pi}{2}\right) - 4r^2\left(\frac{1}{4}\sin\left(2\cdot 0\right) + \frac{1}{2}\cdot 0\right) =$$

$$= 4r^2\left(\frac{1}{4}\sin\pi + \frac{\pi}{4}\right) -$$

$$\int \cos^2 t \,\mathrm{d}t = \int \frac{\cos 2t + 1}{2}\,\mathrm{d}t = \int \left(\frac{1}{2}\cos 2t + \frac{1}{2}\right)\mathrm{d}t =$$

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$$P = 4r^2 \cdot \int_0^{\frac{\pi}{2}} \cos^2 t \,\mathrm{d}t = 4r^2 \left(\frac{1}{4}\sin 2t + \frac{1}{2}t\right)\Bigg|_0^{\frac{\pi}{2}} =$$

$$= 4r^2 \left(\frac{1}{4}\sin\left(2\cdot\frac{\pi}{2}\right) + \frac{1}{2}\cdot\frac{\pi}{2}\right) - 4r^2\left(\frac{1}{4}\sin(2\cdot 0) + \frac{1}{2}\cdot 0\right) =$$

$$= 4r^2\left(\frac{1}{4}\sin\pi + \frac{\pi}{4}\right) - 4r^2\left(\frac{1}{4}\sin 0 + 0\right)$$

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decimale broja π

Dobivanje decimala broja π pomoću integralne sume

- Pokazali smo da je

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- Integral $\int_0^1 \sqrt{1 - x^2}\,\mathrm{d}x$ možemo aproksimirati pomoću integralne sume i na taj način dobiti određeni broj decimala broja π.

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- Možemo uzeti ekvidistantnu razdiobu segmenta $[0, 1]$.

$$\Delta x_i = \frac{1}{n}, \quad x_i = \frac{i}{n}, \quad i = 1, 2, \ldots, n-1, n$$

- U tom slučaju je

$$I_n = \sum_{i=1}^{n} f(\xi_i)\Delta x_i = \sum_{i=1}^{n} f\left(\frac{i}{n}\right) \cdot \frac{1}{n} = \frac{1}{n}\sum_{i=1}^{n} f\left(\frac{i}{n}\right).$$

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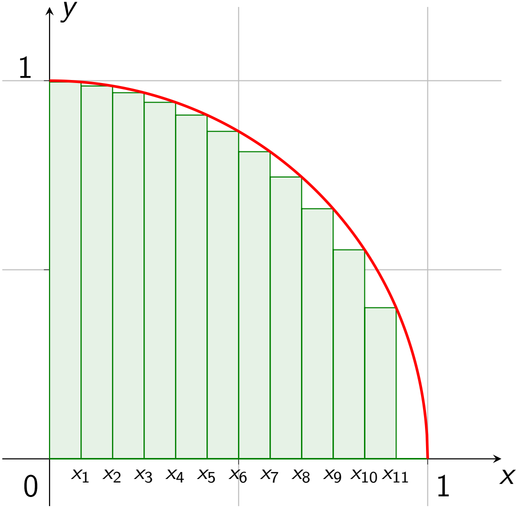
- Stoga za dovoljno veliki $n \in \mathbb{N}$ vrijedi

$$\int_0^1 \sqrt{1-x^2}\,\mathrm{d}x \approx \frac{1}{n}\sum_{i=1}^{n} \sqrt{1-\left(\frac{i}{n}\right)^2}.$$



$$4 \cdot \int_0^1 \sqrt{1 - x^2}\,\mathrm{d}x = \pi$$

$$\frac{4}{n} \sum_{i=1}^{n} \sqrt{1 - \left(\frac{i}{n}\right)^2} \approx \pi$$

konvergencija je spora

za dovoljno veliki $n \in \mathbb{N}$

$$\int_0^1 \sqrt{1 - x^2}\,\mathrm{d}x \approx \frac{1}{n} \sum_{i=1}^{n} \sqrt{1 - \left(\frac{i}{n}\right)^2}$$

C++ kôd za integralnu sumu

```cpp
#include <iostream>
#include <vector>
#include <algorithm>
#include <numeric>
#include <cmath>
#include <iomanip>

// generator za podintegralnu funkciju, u ovom slucaju f(x)=sqrt(1-x^2)
class gen {
private:
  double x, dx;
public:
  gen(double x0, double pomak) : x(x0), dx(pomak) {}
  double operator()() {
    x += dx;
    return sqrt(1.0 - std::min(1.0, x * x));
  }
};

// racunanje vrijednosti integralne sume funkcije f(x)=sqrt(1-x^2) na segmentu [0,1]
double integrate(gen g, int n) {
  std::vector<double> fx(n);
  std::generate(fx.begin(), fx.end(), g);
  return (std::accumulate(fx.begin(), fx.end(), 0.0) / n);
}
```

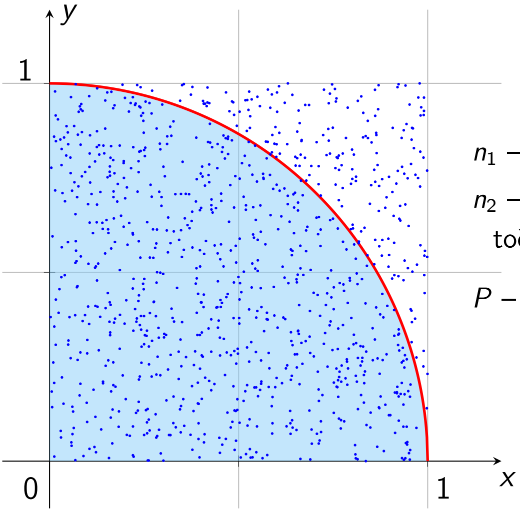
C++ kôd za integralnu sumu

```
int main(void) {
  int n;
  std::cout << "\nNa koliko dijelova podijeliti segment [0,1]: ";
  std::cin >> n;

  gen g(0, 1.0/n);
  std::cout <<std::endl;
  std::cout << "----------------------------------------------------" << std::endl;
  std::cout << "Dobivanje decimala broja PI preko integralne sume" << std::endl;
  std::cout << "----------------------------------------------------" << std::endl;
  std::cout << std::setprecision(17) << 4 * integrate(g, n) << std::endl;
  std::cout << std::endl;

  return 0;
}
```

Monte Carlo integriranje



$$P = \frac{\int_0^1 \sqrt{1-x^2}\,\mathrm{d}x}{\int_0^1 \mathrm{d}x} \approx \frac{n_1}{n_2}$$

n_1 – broj točaka unutar četvrtine kruga

n_2 – ukupni broj slučajno odabranih točaka unutar kvadrata $[0,1] \times [0,1]$

P – vjerojatnost da slučajno odabrana točka iz $[0,1] \times [0,1]$ leži unutar četvrtine kruga

$$\int_0^1 \sqrt{1-x^2}\,\mathrm{d}x = \frac{\pi}{4}$$

$$\boxed{\pi \approx 4 \cdot \frac{n_1}{n_2}}$$

za veliki broj ponavljanja slučajnog pokusa

C++ kôd za Monte Carlo integriranje

```cpp
#include <iostream>
#include <random>
#include <vector>
#include <tuple>
#include <ctime>
#include <cmath>
#include <iomanip>

typedef std::tuple<double, double> point;

std::ostream& operator<<(std::ostream& out, const point& pt) {
  out << "( " << std::get<0>(pt) << ", " <<  std::get<1>(pt) << ") ";
  return out;
}

std::default_random_engine e(time(nullptr));

point random_point() {
  std::uniform_real_distribution<double> u(0,1);
  point temp;
  std::get<0>(temp) = u(e);
  std::get<1>(temp) = u(e);
  return temp;
}
```

C++ kôd za Monte Carlo integriranje

```
double mc_integral(double f(double), std::vector<point>::iterator first,
                   std::vector<point>::iterator last) {
  int total = 0;
  int below = 0;
  for (; first != last; ++first) {
    ++total;
    if (f(std::get<0>(*first)) > std::get<1>(*first))
      ++below;
  }
  return static_cast<double>(below) / total;
}

int main(void) {
  int data_size;
  std::cout << "Koliko slucajnih tocaka zelite generirati? ";
  std::cin >> data_size;
  std::vector<point> data(data_size);

  for (auto& element : data)
    element = random_point();

  std::cout << "PI (Monte Carlo) = " << std :: setprecision (17) <<
     4.0 * mc_integral([](double x){return sqrt(1 - x * x);}, data.begin(), data.end());
  std::cout << std::endl;

  return 0;
}
```