Neke primjene derivacija

Matematika 2

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Sadržaj

prvi zadatak

Zadatak 1

Odredite derivaciju arkus sinus funkcije pomoću formule za derivaciju inverzne funkcije.

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drugi zadatak

Zadatak 2

Neka je $f(x) = (-8x + 1)^3$ *i neka je* g *inverzna funkcija od funkcije* f*. Odredite* $g'(1)$ *bez direktnog određivanja pravila pridruživanja od funkcije* g*.*

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Rješenje

- $f(x) = (-8x+1)^3$

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Rješenje

- $f(x) = (-8x+1)^3, \quad g = f^{-1}$

$$(f^{-1})'(y) = \frac{1}{f'(x)}, \; y = f(x)$$

Zadatak 2

Neka je $f(x) = (-8x+1)^3$ i neka je g inverzna funkcija od funkcije f. Odredite $g'(1)$ bez direktnog određivanja pravila pridruživanja od funkcije g.

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$$(-8x+1)^3 = 1$$

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$$(-8x+1)^3 = 1 \;/\; \sqrt[3]{}$$

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Rješenje

$$\begin{aligned} (-8x+1)^3 &= 1 \ / \sqrt[3]{\ } \\ -8x+1 &= 1 \\ -8x &= 0 \end{aligned}$$

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$$(f^{-1})'(y) = \frac{1}{f'(x)}, \ y = f(x)$$

Zadatak 2

Neka je $f(x) = (-8x+1)^3$ *i neka je* g *inverzna funkcija od funkcije* f*. Odredite* $g'(1)$ *bez direktnog određivanja pravila pridruživanja od funkcije* g*.*

Rješenje

$$\begin{aligned}(-8x+1)^3 &= 1 \ \Big/ \sqrt[3]{\ } \\ -8x+1 &= 1 \\ -8x &= 0 \\ x &= 0\end{aligned}$$

- $f(x) = (-8x+1)^3, \quad g = f^{-1}$
- $g'(1) = (f^{-1})'(1) = \frac{1}{\quad}$

$$(f^{-1})'(y) = \frac{1}{f'(x)}, \ y = f(x)$$

Zadatak 2

Neka je $f(x) = (-8x+1)^3$ *i neka je* g *inverzna funkcija od funkcije* f. *Odredite* $g'(1)$ *bez direktnog određivanja pravila pridruživanja od funkcije* g.

Rješenje

$$(-8x+1)^3 = 1 \ / \sqrt[3]{\ }$$
$$-8x+1 = 1$$
$$-8x = 0$$
$$x = 0$$

- $f(x) = (-8x+1)^3, \quad g = f^{-1}, \quad f(0) = 1$
- $g'(1) = (f^{-1})'(1) = \frac{1}{}$

$$(f^{-1})'(y) = \frac{1}{f'(x)}, \; y = f(x)$$

Zadatak 2

Neka je $f(x) = (-8x+1)^3$ *i neka je* g *inverzna funkcija od funkcije* f*. Odredite* $g'(1)$ *bez direktnog određivanja pravila pridruživanja od funkcije* g*.*

Rješenje

- $f(x) = (-8x+1)^3, \quad g = f^{-1}, \quad f(\overset{x}{\downarrow}0) = 1$
- $g'(1) = (f^{-1})'(1) = \frac{1}{}$

$$\begin{aligned} (-8x+1)^3 &= 1 \; / \sqrt[3]{} \\ -8x+1 &= 1 \\ -8x &= 0 \\ x &= 0 \end{aligned}$$

$$(f^{-1})'(y) = \frac{1}{f'(x)}, \ y = f(x)$$

Zadatak 2

Neka je $f(x) = (-8x+1)^3$ *i neka je* g *inverzna funkcija od funkcije* f*. Odredite* $g'(1)$ *bez direktnog određivanja pravila pridruživanja od funkcije* g*.*

Rješenje

- $f(x) = (-8x+1)^3, \quad g = f^{-1}, \quad f(\overset{x}{\downarrow}0) = \overset{y}{\downarrow}1$

- $g'(1) = (f^{-1})'(1) = \frac{1}{}$

$$(-8x+1)^3 = 1 \ / \sqrt[3]{}$$
$$-8x+1 = 1$$
$$-8x = 0$$
$$x = 0$$

$$(f^{-1})'(y) = \frac{1}{f'(x)}, \ y = f(x)$$

Zadatak 2

Neka je $f(x) = (-8x+1)^3$ *i neka je* g *inverzna funkcija od funkcije* f. *Odredite* $g'(1)$ *bez direktnog određivanja pravila pridruživanja od funkcije* g.

Rješenje

- $f(x) = (-8x+1)^3, \quad g = f^{-1}, \quad f(\underset{x}{0}) = \underset{y}{1}$
- $g'(1) = (f^{-1})'(1) = \dfrac{1}{f'(0)}$

$$\begin{aligned} (-8x+1)^3 &= 1 \ / \sqrt[3]{} \\ -8x+1 &= 1 \\ -8x &= 0 \\ x &= 0 \end{aligned}$$

$$(f^{-1})'(y) = \frac{1}{f'(x)}, \ y = f(x)$$

Zadatak 2

Neka je $f(x) = (-8x+1)^3$ i neka je g inverzna funkcija od funkcije f. Odredite $g'(1)$ bez direktnog određivanja pravila pridruživanja od funkcije g.

Rješenje

- $f(x) = (-8x+1)^3, \quad g = f^{-1}, \quad f(\underset{x}{0}) = \underset{y}{1}$
- $g'(1) = (f^{-1})'(1) = \dfrac{1}{f'(0)} = \dfrac{1}{}$

$$\begin{aligned} (-8x+1)^3 &= 1 \ / \sqrt[3]{\ } \\ -8x+1 &= 1 \\ -8x &= 0 \\ x &= 0 \end{aligned}$$

$$(f^{-1})'(y) = \frac{1}{f'(x)}, \; y = f(x)$$

Zadatak 2

Neka je $f(x) = (-8x+1)^3$ i neka je g inverzna funkcija od funkcije f. Odredite $g'(1)$ bez direktnog određivanja pravila pridruživanja od funkcije g.

Rješenje

$$(-8x+1)^3 = 1 \; / \sqrt[3]{\;}$$
$$-8x+1 = 1$$
$$-8x = 0$$
$$x = 0$$

- $f(x) = (-8x+1)^3, \quad g = f^{-1}, \quad f(\underset{x}{0}) = \underset{y}{1}$
- $g'(1) = (f^{-1})'(1) = \dfrac{1}{f'(0)} = \dfrac{1}{\quad}$

$f'(x) =$

$$(f^{-1})'(y) = \frac{1}{f'(x)}, \ y = f(x)$$

Zadatak 2

Neka je $f(x) = (-8x+1)^3$ *i neka je* g *inverzna funkcija od funkcije* f*. Odredite* $g'(1)$ *bez direktnog određivanja pravila pridruživanja od funkcije* g*.*

Rješenje

- $f(x) = (-8x+1)^3, \quad g = f^{-1}, \quad f(\underset{x}{0}) = \underset{y}{1}$

- $g'(1) = (f^{-1})'(1) = \dfrac{1}{f'(0)} = \dfrac{1}{}$

$$(-8x+1)^3 = 1 \ / \sqrt[3]{\ }$$
$$-8x+1 = 1$$
$$-8x = 0$$
$$x = 0$$

$$f'(x) = 3 \cdot (-8x+1)^2$$

$$(f^{-1})'(y) = \frac{1}{f'(x)}, \; y = f(x)$$

Zadatak 2

Neka je $f(x) = (-8x+1)^3$ *i neka je* g *inverzna funkcija od funkcije* f. *Odredite* $g'(1)$ *bez direktnog određivanja pravila pridruživanja od funkcije* g.

Rješenje

$$(-8x+1)^3 = 1 \;/\; \sqrt[3]{}$$
$$-8x+1 = 1$$
$$-8x = 0$$
$$x = 0$$

- $f(x) = (-8x+1)^3, \quad g = f^{-1}, \quad f(\underset{x}{0}) = \underset{y}{1}$
- $g'(1) = (f^{-1})'(1) = \dfrac{1}{f'(0)} = \dfrac{1}{}$

$$f'(x) = 3 \cdot (-8x+1)^2 \cdot (-8x+1)'$$

$$(f^{-1})'(y) = \frac{1}{f'(x)}, \ y = f(x)$$

Zadatak 2

Neka je $f(x) = (-8x+1)^3$ *i neka je* g *inverzna funkcija od funkcije* f*. Odredite* $g'(1)$ *bez direktnog određivanja pravila pridruživanja od funkcije* g*.*

Rješenje

$$(-8x+1)^3 = 1 \ / \sqrt[3]{}$$
$$-8x+1 = 1$$
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$$x = 0$$

- $f(x) = (-8x+1)^3, \quad g = f^{-1}, \quad f(\overset{x}{0}) = \overset{y}{1}$
- $g'(1) = (f^{-1})'(1) = \dfrac{1}{f'(0)} = \dfrac{1}{}$

$$f'(x) = 3 \cdot (-8x+1)^2 \cdot (-8x+1)' = 3 \cdot (-8x+1)^2$$

$$(f^{-1})'(y) = \frac{1}{f'(x)}, \; y = f(x)$$

Zadatak 2

Neka je $f(x) = (-8x+1)^3$ *i neka je* g *inverzna funkcija od funkcije* f*. Odredite* $g'(1)$ *bez direktnog određivanja pravila pridruživanja od funkcije* g*.*

Rješenje

$$(-8x+1)^3 = 1 \;/\; \sqrt[3]{}$$
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- $f(x) = (-8x+1)^3, \quad g = f^{-1}, \quad f(\overset{x}{\downarrow}0) = \overset{y}{\downarrow}1$
- $g'(1) = (f^{-1})'(1) = \dfrac{1}{f'(0)} = \dfrac{1}{}$

$$f'(x) = 3\cdot(-8x+1)^2\cdot(-8x+1)' = 3\cdot(-8x+1)^2\cdot(-8)$$

$$(f^{-1})'(y) = \frac{1}{f'(x)}, \; y = f(x)$$

Zadatak 2

Neka je $f(x) = (-8x+1)^3$ i neka je g inverzna funkcija od funkcije f. Odredite $g'(1)$ bez direktnog određivanja pravila pridruživanja od funkcije g.

Rješenje

$$(-8x+1)^3 = 1 \;\Big/ \sqrt[3]{}$$
$$-8x+1 = 1$$
$$-8x = 0$$
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- $f(x) = (-8x+1)^3, \quad g = f^{-1}, \quad f(\overset{x}{0}) = \overset{y}{1}$
- $g'(1) = (f^{-1})'(1) = \dfrac{1}{f'(0)} = \dfrac{1}{}$

$$f'(x) = 3 \cdot (-8x+1)^2 \cdot (-8x+1)' = 3 \cdot (-8x+1)^2 \cdot (-8)$$

$$f'(x) = -24 \cdot (-8x+1)^2$$

$$(f^{-1})'(y) = \frac{1}{f'(x)}, \; y = f(x)$$

Zadatak 2

Neka je $f(x) = (-8x+1)^3$ *i neka je* g *inverzna funkcija od funkcije* f. *Odredite* $g'(1)$ *bez direktnog određivanja pravila pridruživanja od funkcije* g.

Rješenje

$$(-8x+1)^3 = 1 \;/\; \sqrt[3]{}$$
$$-8x+1 = 1$$
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- $f(x) = (-8x+1)^3, \quad g = f^{-1}, \quad f(\underset{x}{0}) = \underset{y}{1}$
- $g'(1) = (f^{-1})'(1) = \dfrac{1}{f'(0)} = \dfrac{1}{}$

$$f'(x) = 3 \cdot (-8x+1)^2 \cdot (-8x+1)' = 3 \cdot (-8x+1)^2 \cdot (-8)$$

$$\boxed{f'(x) = -24 \cdot (-8x+1)^2}$$

$$(f^{-1})'(y) = \frac{1}{f'(x)}, \; y = f(x)$$

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Neka je $f(x) = (-8x+1)^3$ *i neka je* g *inverzna funkcija od funkcije* f. *Odredite* $g'(1)$ *bez direktnog određivanja pravila pridruživanja od funkcije* g.

Rješenje

- $f(x) = (-8x+1)^3, \quad g = f^{-1}, \quad f(\underset{x}{0}) = \underset{y}{1}$
- $g'(1) = (f^{-1})'(1) = \dfrac{1}{f'(0)} = \dfrac{1}{}$

$$(-8x+1)^3 = 1 \;/\; \sqrt[3]{}$$
$$-8x+1 = 1$$
$$-8x = 0$$
$$x = 0$$

$$f'(x) = 3\cdot(-8x+1)^2\cdot(-8x+1)' = 3\cdot(-8x+1)^2\cdot(-8)$$

$$\boxed{f'(x) = -24\cdot(-8x+1)^2} \qquad f'(0) =$$

$$(f^{-1})'(y) = \frac{1}{f'(x)}, \ y = f(x)$$

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Rješenje

$$(-8x+1)^3 = 1 \ / \sqrt[3]{\ }$$
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- $f(x) = (-8x+1)^3, \quad g = f^{-1}, \quad f(\overset{x}{\downarrow}0) = \overset{y}{\downarrow}1$
- $g'(1) = (f^{-1})'(1) = \dfrac{1}{f'(0)} = \dfrac{1}{\quad}$

$$f'(x) = 3\cdot(-8x+1)^2\cdot(-8x+1)' = 3\cdot(-8x+1)^2\cdot(-8)$$

$$\boxed{f'(x) = -24\cdot(-8x+1)^2} \qquad f'(0) = -24\cdot(-8\cdot 0+1)^2$$

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Rješenje

$$(-8x+1)^3 = 1 \;/\; \sqrt[3]{}$$
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- $f(x) = (-8x+1)^3, \quad g = f^{-1}, \quad f(\underset{x}{0}) = \underset{y}{1}$
- $g'(1) = (f^{-1})'(1) = \dfrac{1}{f'(0)} = \dfrac{1}{}$

$$f'(x) = 3\cdot(-8x+1)^2\cdot(-8x+1)' = 3\cdot(-8x+1)^2\cdot(-8)$$

$$\boxed{f'(x) = -24\cdot(-8x+1)^2} \qquad f'(0) = -24\cdot(-8\cdot 0+1)^2$$

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$$(f^{-1})'(y) = \frac{1}{f'(x)}, \; y = f(x)$$

Zadatak 2

Neka je $f(x) = (-8x+1)^3$ *i neka je* g *inverzna funkcija od funkcije* f. *Odredite* $g'(1)$ *bez direktnog određivanja pravila pridruživanja od funkcije* g.

Rješenje

- $f(x) = (-8x+1)^3, \quad g = f^{-1}, \quad f(\underset{x}{0}) = \underset{y}{1}$
- $g'(1) = (f^{-1})'(1) = \dfrac{1}{f'(0)} = \dfrac{1}{}$

$$(-8x+1)^3 = 1 \;\Big/\; \sqrt[3]{}$$
$$-8x+1 = 1$$
$$-8x = 0$$
$$x = 0$$

$$f'(x) = 3\cdot(-8x+1)^2\cdot(-8x+1)' = 3\cdot(-8x+1)^2\cdot(-8)$$

$$\boxed{f'(x) = -24\cdot(-8x+1)^2} \qquad f'(0) = -24\cdot(-8\cdot 0+1)^2$$

$$\boxed{f'(0) = -24}$$

$$(f^{-1})'(y) = \frac{1}{f'(x)}, \; y = f(x)$$

Zadatak 2

Neka je $f(x) = (-8x+1)^3$ i neka je g inverzna funkcija od funkcije f. Odredite $g'(1)$ bez direktnog određivanja pravila pridruživanja od funkcije g.

Rješenje

$$(-8x+1)^3 = 1 \;/\; \sqrt[3]{\;}$$
$$-8x+1 = 1$$
$$-8x = 0$$
$$x = 0$$

- $f(x) = (-8x+1)^3, \quad g = f^{-1}, \quad f(\underset{x}{0}) = \underset{y}{1}$
- $g'(1) = (f^{-1})'(1) = \dfrac{1}{f'(0)} = \dfrac{1}{-24}$

$$f'(x) = 3\cdot(-8x+1)^2\cdot(-8x+1)' = 3\cdot(-8x+1)^2\cdot(-8)$$

$$\boxed{f'(x) = -24\cdot(-8x+1)^2} \qquad f'(0) = -24\cdot(-8\cdot 0+1)^2$$

$$\boxed{f'(0) = -24}$$

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Neka je $f(x) = (-8x+1)^3$ *i neka je* g *inverzna funkcija od funkcije* f. *Odredite* $g'(1)$ *bez direktnog određivanja pravila pridruživanja od funkcije* g.

Rješenje

- $f(x) = (-8x+1)^3, \quad g = f^{-1}, \quad f(\underset{x}{0}) = \underset{y}{1}$
- $g'(1) = (f^{-1})'(1) = \dfrac{1}{f'(0)} = \dfrac{1}{-24} = -\dfrac{1}{24}$

$$(-8x+1)^3 = 1 \ / \sqrt[3]{\ }$$
$$-8x+1 = 1$$
$$-8x = 0$$
$$x = 0$$

$$f'(x) = 3\cdot(-8x+1)^2\cdot(-8x+1)' = 3\cdot(-8x+1)^2\cdot(-8)$$

$$\boxed{f'(x) = -24\cdot(-8x+1)^2} \qquad f'(0) = -24\cdot(-8\cdot 0+1)^2$$

$$\boxed{f'(0) = -24}$$

$$(f^{-1})'(y) = \frac{1}{f'(x)}, \ y = f(x)$$

Zadatak 2

Neka je $f(x) = (-8x+1)^3$ i neka je g inverzna funkcija od funkcije f. Odredite $g'(1)$ bez direktnog određivanja pravila pridruživanja od funkcije g.

$$g'(1) = -\frac{1}{24}$$

Rješenje

- $f(x) = (-8x+1)^3, \quad g = f^{-1}, \quad f(\underset{x}{0}) = \underset{y}{1}$
- $g'(1) = (f^{-1})'(1) = \dfrac{1}{f'(0)} = \dfrac{1}{-24} = -\dfrac{1}{24}$

$$(-8x+1)^3 = 1 \ / \sqrt[3]{\ }$$

$$-8x+1 = 1$$

$$-8x = 0$$

$$x = 0$$

$$f'(x) = 3\cdot(-8x+1)^2\cdot(-8x+1)' = 3\cdot(-8x+1)^2\cdot(-8)$$

$$\boxed{f'(x) = -24\cdot(-8x+1)^2} \qquad f'(0) = -24\cdot(-8\cdot 0+1)^2$$

$$\boxed{f'(0) = -24}$$

$$(f^{-1})'(y) = \frac{1}{f'(x)}, \; y = f(x)$$

Zadatak 2

Neka je $f(x) = (-8x+1)^3$ *i neka je* g *inverzna funkcija od funkcije* f*. Odredite* $g'(1)$ *bez direktnog određivanja pravila pridruživanja od funkcije* g*.*

$$g'(1) = -\frac{1}{24}$$

Rješenje

- $f(x) = (-8x+1)^3, \quad g = f^{-1}, \quad f(\underset{x}{0}) = \underset{y}{1}$
- $g'(1) = (f^{-1})'(1) = \dfrac{1}{f'(0)} = \dfrac{1}{-24} = -\dfrac{1}{24}$

$$\begin{aligned}(-8x+1)^3 &= 1 \quad / \sqrt[3]{\;} \\ -8x+1 &= 1 \\ -8x &= 0 \\ x &= 0\end{aligned}$$

$$f'(x) = 3\cdot(-8x+1)^2\cdot(-8x+1)' = 3\cdot(-8x+1)^2\cdot(-8)$$

$$f'(x) = -24\cdot(-8x+1)^2 \qquad f'(0) = -24\cdot(-8\cdot 0+1)^2$$

$$f'(0) = -24$$

trećі zadatak

Zadatak 3

Izračunajte sljedeće limese:

a) $\lim\limits_{x\to-\infty} \dfrac{\ln\left(8+e^{x}\right)}{2x}$

b) $\lim\limits_{x\to+\infty} \dfrac{\ln\left(8+e^{x}\right)}{2x}$

Zadatak 3

Izračunajte sljedeće limese:

a) $\lim\limits_{x \to -\infty} \dfrac{\ln(8 + e^x)}{2x}$

b) $\lim\limits_{x \to +\infty} \dfrac{\ln(8 + e^x)}{2x}$

Rješenje

a)

$$\lim_{x \to -\infty} \frac{\ln(8 + e^x)}{2x} =$$

Zadatak 3

Izračunajte sljedeće limese:

a) $\lim\limits_{x \to -\infty} \dfrac{\ln(8 + e^x)}{2x}$

b) $\lim\limits_{x \to +\infty} \dfrac{\ln(8 + e^x)}{2x}$

Rješenje

a)

$$\lim_{x \to -\infty} \frac{\ln(8 + e^x)}{2x} = \underline{\hspace{5cm}}$$

Zadatak 3

Izračunajte sljedeće limese:

a) $\lim\limits_{x \to -\infty} \dfrac{\ln(8 + e^x)}{2x}$

b) $\lim\limits_{x \to +\infty} \dfrac{\ln(8 + e^x)}{2x}$

Rješenje

a)

$$\lim_{x \to -\infty} \frac{\ln(8 + e^x)}{2x} = \frac{\ln(8 + e^{-\infty})}{}$$

Zadatak 3

Izračunajte sljedeće limese:

a) $\lim\limits_{x \to -\infty} \dfrac{\ln\left(8 + e^x\right)}{2x}$

b) $\lim\limits_{x \to +\infty} \dfrac{\ln\left(8 + e^x\right)}{2x}$

Rješenje

a)

$$\lim_{x \to -\infty} \frac{\ln\left(8 + e^x\right)}{2x} = \frac{\ln\left(8 + e^{-\infty}\right)}{2 \cdot (-\infty)}$$

Zadatak 3

Izračunajte sljedeće limese:

a) $\lim\limits_{x\to-\infty} \dfrac{\ln(8+e^x)}{2x}$

b) $\lim\limits_{x\to+\infty} \dfrac{\ln(8+e^x)}{2x}$

Rješenje

a)

$$\lim_{x\to-\infty} \frac{\ln(8+e^x)}{2x} = \frac{\ln(8+e^{-\infty})}{2\cdot(-\infty)} = \underline{\qquad\qquad}$$

Zadatak 3

Izračunajte sljedeće limese:

a) $\lim\limits_{x \to -\infty} \dfrac{\ln(8+e^x)}{2x}$

b) $\lim\limits_{x \to +\infty} \dfrac{\ln(8+e^x)}{2x}$

Rješenje

a)

$$\lim_{x \to -\infty} \frac{\ln(8+e^x)}{2x} = \frac{\ln(8+e^{-\infty})}{2 \cdot (-\infty)} = \frac{\ln(8+0)}{}$$

Zadatak 3

Izračunajte sljedeće limese:

a) $\lim\limits_{x\to-\infty} \dfrac{\ln(8+e^x)}{2x}$

b) $\lim\limits_{x\to+\infty} \dfrac{\ln(8+e^x)}{2x}$

Rješenje

a)

$$\lim_{x\to-\infty} \frac{\ln(8+e^x)}{2x} = \frac{\ln(8+e^{-\infty})}{2\cdot(-\infty)} = \frac{\ln(8+0)}{2\cdot(-\infty)}$$

Zadatak 3

Izračunajte sljedeće limese:

a) $\lim\limits_{x\to-\infty} \dfrac{\ln(8+e^x)}{2x}$

b) $\lim\limits_{x\to+\infty} \dfrac{\ln(8+e^x)}{2x}$

Rješenje

a)

$$\lim_{x\to-\infty} \frac{\ln(8+e^x)}{2x} = \frac{\ln(8+e^{-\infty})}{2\cdot(-\infty)} = \frac{\ln(8+0)}{2\cdot(-\infty)} = \underline{\qquad}$$

Zadatak 3

Izračunajte sljedeće limese:

a) $\lim\limits_{x \to -\infty} \dfrac{\ln(8 + e^x)}{2x}$

b) $\lim\limits_{x \to +\infty} \dfrac{\ln(8 + e^x)}{2x}$

Rješenje

a)

$$\lim_{x \to -\infty} \frac{\ln(8 + e^x)}{2x} = \frac{\ln(8 + e^{-\infty})}{2 \cdot (-\infty)} = \frac{\ln(8 + 0)}{2 \cdot (-\infty)} = \frac{\ln 8}{}$$

Zadatak 3

Izračunajte sljedeće limese:

a) $\lim\limits_{x \to -\infty} \dfrac{\ln(8+e^x)}{2x}$

b) $\lim\limits_{x \to +\infty} \dfrac{\ln(8+e^x)}{2x}$

Rješenje

a)

$$\lim_{x \to -\infty} \frac{\ln(8+e^x)}{2x} = \frac{\ln(8+e^{-\infty})}{2 \cdot (-\infty)} = \frac{\ln(8+0)}{2 \cdot (-\infty)} = \frac{\ln 8}{-\infty}$$

Zadatak 3

Izračunajte sljedeće limese:

a) $\lim\limits_{x \to -\infty} \dfrac{\ln(8 + e^x)}{2x}$

b) $\lim\limits_{x \to +\infty} \dfrac{\ln(8 + e^x)}{2x}$

Rješenje

a)

$$\lim_{x \to -\infty} \frac{\ln(8 + e^x)}{2x} = \frac{\ln(8 + e^{-\infty})}{2 \cdot (-\infty)} = \frac{\ln(8 + 0)}{2 \cdot (-\infty)} = \frac{\ln 8}{-\infty} = 0$$

b)

$$\lim_{x \to +\infty} \frac{\ln(8 + e^x)}{2x} =$$

b)

$$\boxed{\frac{\infty}{\infty}}$$

$$\lim_{x \to +\infty} \frac{\ln(8 + e^x)}{2x} =$$

b)

$$\frac{\infty}{\infty}$$

$$\lim_{x\to+\infty} \frac{\ln\left(8+e^{x}\right)}{2x} = \lim_{x\to+\infty} \underline{\qquad\qquad}$$

b)

$\frac{\infty}{\infty}$

L'Hospitalovo pravilo

$$\lim_{x\to+\infty} \frac{\ln(8+e^x)}{2x} = \lim_{x\to+\infty} \underline{\hspace{4cm}}$$

b)

$\frac{\infty}{\infty}$ L'Hospitalovo pravilo

$$\lim_{x\to+\infty} \frac{\ln(8+e^x)}{2x} = \lim_{x\to+\infty} \frac{\big(\ln(8+e^x)\big)'}{}$$

b)

$\frac{\infty}{\infty}$

L'Hospitalovo pravilo

$$\lim_{x \to +\infty} \frac{\ln(8 + e^x)}{2x} = \lim_{x \to +\infty} \frac{\left(\ln(8 + e^x)\right)'}{(2x)'}$$

b)

$\frac{\infty}{\infty}$

L'Hospitalovo pravilo

$$\lim_{x\to+\infty}\frac{\ln(8+e^x)}{2x} = \lim_{x\to+\infty}\frac{\big(\ln(8+e^x)\big)'}{(2x)'} =$$

$$= \lim_{x\to+\infty}\frac{\qquad}{\qquad}$$

b)

$\frac{\infty}{\infty}$

L'Hospitalovo pravilo

$$\lim_{x\to+\infty} \frac{\ln(8+e^x)}{2x} = \lim_{x\to+\infty} \frac{\big(\ln(8+e^x)\big)'}{(2x)'} =$$

$$= \lim_{x\to+\infty} \frac{\qquad}{2}$$

b)

$\frac{\infty}{\infty}$

L'Hospitalovo pravilo

$$\Big(\ln(\text{nešto})\Big)' = \frac{1}{\text{nešto}} \cdot (\text{nešto})'$$

$$\lim_{x\to+\infty} \frac{\ln(8+e^x)}{2x} = \lim_{x\to+\infty} \frac{\Big(\ln(8+e^x)\Big)'}{(2x)'} =$$

$$(\ln x)' = \frac{1}{x}$$

$$= \lim_{x\to+\infty} \frac{\frac{1}{8+e^x}}{2}$$

b)

$\frac{\infty}{\infty}$

L'Hospitalovo pravilo

$$\big(\ln(\text{nešto})\big)' = \frac{1}{\text{nešto}} \cdot (\text{nešto})'$$

$$(\ln x)' = \frac{1}{x}$$

$$\lim_{x\to+\infty} \frac{\ln(8+e^x)}{2x} = \lim_{x\to+\infty} \frac{\big(\ln(8+e^x)\big)'}{(2x)'} =$$

$$= \lim_{x\to+\infty} \frac{\frac{1}{8+e^x} \cdot (8+e^x)'}{2}$$

b)

$\frac{\infty}{\infty}$

L'Hospitalovo pravilo

$\big(\ln(\text{nešto})\big)' = \frac{1}{\text{nešto}} \cdot (\text{nešto})'$

$$\lim_{x\to+\infty} \frac{\ln(8+e^x)}{2x} = \lim_{x\to+\infty} \frac{\big(\ln(8+e^x)\big)'}{(2x)'} =$$

$(\ln x)' = \frac{1}{x}$

$$= \lim_{x\to+\infty} \frac{\frac{1}{8+e^x}\cdot(8+e^x)'}{2} = \lim_{x\to+\infty} \frac{\quad}{\quad}$$

b)

$\frac{\infty}{\infty}$

L'Hospitalovo pravilo

$\big(\ln(\text{nešto})\big)' = \frac{1}{\text{nešto}} \cdot (\text{nešto})'$

$(\ln x)' = \frac{1}{x}$

$$\lim_{x\to+\infty} \frac{\ln(8+e^x)}{2x} = \lim_{x\to+\infty} \frac{\big(\ln(8+e^x)\big)'}{(2x)'} =$$

$$= \lim_{x\to+\infty} \frac{\frac{1}{8+e^x} \cdot (8+e^x)'}{2} = \lim_{x\to+\infty} \frac{}{2}$$

b)

$\frac{\infty}{\infty}$

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$$\Big(\ln(\text{nešto})\Big)' = \frac{1}{\text{nešto}} \cdot (\text{nešto})'$$

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$$= \lim_{x\to+\infty} \frac{\frac{1}{8+e^x}\cdot(8+e^x)'}{2} = \lim_{x\to+\infty} \frac{\frac{e^x}{8+e^x}}{2}$$

b)

$\frac{\infty}{\infty}$

L'Hospitalovo pravilo

$\left(\ln(\text{nešto})\right)' = \frac{1}{\text{nešto}} \cdot (\text{nešto})'$

$(\ln x)' = \frac{1}{x}$

$$\lim_{x\to+\infty} \frac{\ln(8+e^x)}{2x} = \lim_{x\to+\infty} \frac{\left(\ln(8+e^x)\right)'}{(2x)'} =$$

$$= \lim_{x\to+\infty} \frac{\frac{1}{8+e^x}\cdot(8+e^x)'}{2} = \lim_{x\to+\infty} \frac{\frac{e^x}{8+e^x}}{2} =$$

$$= \lim_{x\to+\infty} \frac{\quad}{\quad}$$

b)

$\frac{\infty}{\infty}$

L'Hospitalovo pravilo

$\left(\ln\left(\text{nešto}\right)\right)' = \frac{1}{\text{nešto}} \cdot (\text{nešto})'$

$$\lim_{x\to+\infty} \frac{\ln\left(8+e^x\right)}{2x} = \lim_{x\to+\infty} \frac{\left(\ln\left(8+e^x\right)\right)'}{(2x)'} =$$

$(\ln x)' = \frac{1}{x}$

$$= \lim_{x\to+\infty} \frac{\frac{1}{8+e^x} \cdot (8+e^x)'}{2} = \lim_{x\to+\infty} \frac{\frac{e^x}{8+e^x}}{2} =$$

$$= \lim_{x\to+\infty} \frac{e^x}{}$$

b)

$\frac{\infty}{\infty}$

L'Hospitalovo pravilo

$$\left(\ln(\text{nešto})\right)' = \frac{1}{\text{nešto}} \cdot (\text{nešto})'$$

$$(\ln x)' = \frac{1}{x}$$

$$\lim_{x\to+\infty} \frac{\ln(8+e^x)}{2x} = \lim_{x\to+\infty} \frac{\left(\ln(8+e^x)\right)'}{(2x)'} =$$

$$= \lim_{x\to+\infty} \frac{\frac{1}{8+e^x}\cdot(8+e^x)'}{2} = \lim_{x\to+\infty} \frac{\frac{e^x}{8+e^x}}{2} =$$

$$= \lim_{x\to+\infty} \frac{e^x}{2\cdot(8+e^x)}$$

b)

$\frac{\infty}{\infty}$

L'Hospitalovo pravilo

$\Big(\ln(\text{nešto})\Big)' = \frac{1}{\text{nešto}} \cdot (\text{nešto})'$

$(\ln x)' = \frac{1}{x}$

$$\lim_{x\to+\infty} \frac{\ln(8+e^x)}{2x} = \lim_{x\to+\infty} \frac{\Big(\ln(8+e^x)\Big)'}{(2x)'} =$$

$$= \lim_{x\to+\infty} \frac{\frac{1}{8+e^x}\cdot(8+e^x)'}{2} = \lim_{x\to+\infty} \frac{\frac{e^x}{8+e^x}}{2} =$$

$$= \lim_{x\to+\infty} \frac{e^x}{2\cdot(8+e^x)} = \frac{1}{2}\lim_{x\to+\infty} \frac{}{}$$

b)

$\frac{\infty}{\infty}$ — L'Hospitalovo pravilo

$\big(\ln(\text{nešto})\big)' = \frac{1}{\text{nešto}} \cdot (\text{nešto})'$

$(\ln x)' = \frac{1}{x}$

$$\lim_{x\to+\infty} \frac{\ln(8+e^x)}{2x} = \lim_{x\to+\infty} \frac{\big(\ln(8+e^x)\big)'}{(2x)'} =$$

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$$= \lim_{x\to+\infty} \frac{e^x}{2\cdot(8+e^x)} = \frac{1}{2}\lim_{x\to+\infty} \frac{e^x}{}$$

b)

$\frac{\infty}{\infty}$

L'Hospitalovo pravilo

$$\left(\ln(\text{nešto})\right)' = \frac{1}{\text{nešto}} \cdot (\text{nešto})'$$

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$$= \lim_{x\to+\infty} \frac{e^x}{2\cdot(8+e^x)} = \frac{1}{2}\lim_{x\to+\infty} \frac{e^x}{8+e^x}$$

b)

$\frac{\infty}{\infty}$

L'Hospitalovo pravilo

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$\frac{\infty}{\infty}$

$$= \lim_{x\to+\infty} \frac{e^x}{2\cdot(8+e^x)} = \frac{1}{2}\lim_{x\to+\infty} \frac{e^x}{8+e^x}$$

b)

$\frac{\infty}{\infty}$ — L'Hospitalovo pravilo

$\big(\ln(\text{nešto})\big)' = \frac{1}{\text{nešto}} \cdot (\text{nešto})'$

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$$= \lim_{x\to+\infty} \frac{e^x}{2\cdot(8+e^x)} = \frac{1}{2}\lim_{x\to+\infty} \frac{e^x}{8+e^x} = \frac{1}{2}\lim_{x\to+\infty} \frac{}{}$$

b)

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L'Hospitalovo pravilo

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b)

$\frac{\infty}{\infty}$

L'Hospitalovo pravilo

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$\frac{\infty}{\infty}$

L'Hospitalovo pravilo

$$= \lim_{x\to+\infty} \frac{e^x}{2\cdot(8+e^x)} = \frac{1}{2}\lim_{x\to+\infty} \frac{e^x}{8+e^x} = \frac{1}{2}\lim_{x\to+\infty} \frac{(e^x)'}{}$$

b)

$\frac{\infty}{\infty}$ | L'Hospitalovo pravilo

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$\frac{\infty}{\infty}$ | L'Hospitalovo pravilo

$$= \lim_{x\to+\infty} \frac{e^x}{2\cdot(8+e^x)} = \frac{1}{2}\lim_{x\to+\infty} \frac{e^x}{8+e^x} = \frac{1}{2}\lim_{x\to+\infty} \frac{(e^x)'}{(8+e^x)'}$$

b)

$\frac{\infty}{\infty}$

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$\frac{\infty}{\infty}$

L'Hospitalovo pravilo

$$= \lim_{x\to+\infty} \frac{e^x}{2\cdot(8+e^x)} = \frac{1}{2}\lim_{x\to+\infty} \frac{e^x}{8+e^x} = \frac{1}{2}\lim_{x\to+\infty} \frac{(e^x)'}{(8+e^x)'} =$$

$$= \frac{1}{2}\lim_{x\to+\infty} \frac{\quad}{\quad}$$

b)

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$\frac{\infty}{\infty}$ L'Hospitalovo pravilo

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$$= \frac{1}{2}\lim_{x\to+\infty} \frac{e^x}{}$$

b)

$\frac{\infty}{\infty}$ L'Hospitalovo pravilo

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$\frac{\infty}{\infty}$ L'Hospitalovo pravilo

$$= \lim_{x\to+\infty} \frac{e^x}{2\cdot(8+e^x)} = \frac{1}{2}\lim_{x\to+\infty} \frac{e^x}{8+e^x} = \frac{1}{2}\lim_{x\to+\infty} \frac{(e^x)'}{(8+e^x)'} =$$

$$= \frac{1}{2}\lim_{x\to+\infty} \frac{e^x}{e^x}$$

b)

$\frac{\infty}{\infty}$ L'Hospitalovo pravilo

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$\frac{\infty}{\infty}$ L'Hospitalovo pravilo

$$= \lim_{x\to+\infty} \frac{e^x}{2\cdot(8+e^x)} = \frac{1}{2}\lim_{x\to+\infty} \frac{e^x}{8+e^x} = \frac{1}{2}\lim_{x\to+\infty} \frac{(e^x)'}{(8+e^x)'} =$$

$$= \frac{1}{2}\lim_{x\to+\infty} \frac{e^x}{e^x} = \frac{1}{2}\lim_{x\to+\infty} 1$$

b)

$\frac{\infty}{\infty}$ — L'Hospitalovo pravilo

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$\frac{\infty}{\infty}$ — L'Hospitalovo pravilo

$$= \lim_{x\to+\infty} \frac{e^x}{2\cdot(8+e^x)} = \frac{1}{2}\lim_{x\to+\infty} \frac{e^x}{8+e^x} = \frac{1}{2}\lim_{x\to+\infty} \frac{(e^x)'}{(8+e^x)'} =$$

$$= \frac{1}{2}\lim_{x\to+\infty} \frac{e^x}{e^x} = \frac{1}{2}\lim_{x\to+\infty} 1 = \frac{1}{2}\cdot 1$$

b)

$\frac{\infty}{\infty}$ L'Hospitalovo pravilo

$\left(\ln(\text{nešto})\right)' = \frac{1}{\text{nešto}} \cdot (\text{nešto})'$

$(\ln x)' = \frac{1}{x}$

$$\lim_{x\to+\infty} \frac{\ln(8+e^x)}{2x} = \lim_{x\to+\infty} \frac{\left(\ln(8+e^x)\right)'}{(2x)'} =$$

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$\frac{\infty}{\infty}$ L'Hospitalovo pravilo

$$= \lim_{x\to+\infty} \frac{e^x}{2\cdot(8+e^x)} = \frac{1}{2}\lim_{x\to+\infty} \frac{e^x}{8+e^x} = \frac{1}{2}\lim_{x\to+\infty} \frac{(e^x)'}{(8+e^x)'} =$$

$$= \frac{1}{2}\lim_{x\to+\infty} \frac{e^x}{e^x} = \frac{1}{2}\lim_{x\to+\infty} 1 = \frac{1}{2}\cdot 1 = \frac{1}{2}$$

četvrti zadatak

Napomena

- Ako je $\lim\limits_{x \to c} \big(f(x) \cdot g(x)\big)$ oblika $0 \cdot \infty$, tada na njega ne možemo direktno primijeniti L'Hospitalovo pravilo.

- Ako želimo na neodređeni oblik $0 \cdot \infty$ primijeniti L'Hospitalovo pravilo, moramo ga prije primjene tog pravila svesti na neki od neodređenih oblika $\frac{0}{0}$ ili $\frac{\infty}{\infty}$.

$$\underbrace{\lim_{x \to c} \big(f(x) \cdot g(x)\big)}_{0 \cdot \infty} = \underbrace{\lim_{x \to c} \frac{f(x)}{\frac{1}{g(x)}}}_{\frac{0}{0}} = \underbrace{\lim_{x \to c} \frac{g(x)}{\frac{1}{f(x)}}}_{\frac{\infty}{\infty}}$$

Zadatak 4

Pomoću L'Hospitalovog pravila izračunajte limes

$$\lim_{x\to\pm\infty}\left(3x^2e^{-x^2}\right).$$

Zadatak 4

Pomoću L'Hospitalovog pravila izračunajte limes

$$\lim_{x\to\pm\infty}\left(3x^2 e^{-x^2}\right).$$

Rješenje

- $\lim_{x\to\pm\infty}\left(3x^2\right) =$

Zadatak 4

Pomoću L'Hospitalovog pravila izračunajte limes

$$\lim_{x \to \pm\infty} \left(3x^2 e^{-x^2}\right).$$

Rješenje

- $\lim_{x \to \pm\infty} \left(3x^2\right) = 3 \cdot (\pm\infty)^2$

Zadatak 4

Pomoću L'Hospitalovog pravila izračunajte limes

$$\lim_{x\to\pm\infty}\left(3x^2e^{-x^2}\right).$$

Rješenje

- $\lim_{x\to\pm\infty}\left(3x^2\right)=3\cdot(\pm\infty)^2=3\cdot(+\infty)$

Zadatak 4

Pomoću L'Hospitalovog pravila izračunajte limes

$$\lim_{x\to\pm\infty}\left(3x^2e^{-x^2}\right).$$

Rješenje

- $\lim_{x\to\pm\infty}\left(3x^2\right) = 3\cdot(\pm\infty)^2 = 3\cdot(+\infty) = +\infty$

Zadatak 4

Pomoću L'Hospitalovog pravila izračunajte limes

$$\lim_{x\to\pm\infty}\left(3x^2e^{-x^2}\right).$$

Rješenje

- $\lim_{x\to\pm\infty}\left(3x^2\right) = 3\cdot(\pm\infty)^2 = 3\cdot(+\infty) = +\infty$
- $\lim_{x\to\pm\infty} e^{-x^2} =$

Zadatak 4

Pomoću L'Hospitalovog pravila izračunajte limes

$$\lim_{x\to\pm\infty}\left(3x^2e^{-x^2}\right).$$

Rješenje

- $\lim\limits_{x\to\pm\infty}\left(3x^2\right) = 3\cdot(\pm\infty)^2 = 3\cdot(+\infty) = +\infty$
- $\lim\limits_{x\to\pm\infty} e^{-x^2} = e^{-(\pm\infty)^2}$

Zadatak 4

Pomoću L'Hospitalovog pravila izračunajte limes

$$\lim_{x \to \pm\infty} \left(3x^2 e^{-x^2} \right).$$

Rješenje

- $\lim\limits_{x \to \pm\infty} \left(3x^2 \right) = 3 \cdot (\pm\infty)^2 = 3 \cdot (+\infty) = +\infty$
- $\lim\limits_{x \to \pm\infty} e^{-x^2} = e^{-(\pm\infty)^2} = e^{-\infty}$

Zadatak 4

Pomoću L'Hospitalovog pravila izračunajte limes

$$\lim_{x \to \pm\infty} \left(3x^2 e^{-x^2}\right).$$

Rješenje

- $\lim\limits_{x \to \pm\infty} \left(3x^2\right) = 3 \cdot (\pm\infty)^2 = 3 \cdot (+\infty) = +\infty$
- $\lim\limits_{x \to \pm\infty} e^{-x^2} = e^{-(\pm\infty)^2} = e^{-\infty} = 0$

Zadatak 4

Pomoću L'Hospitalovog pravila izračunajte limes

$$\lim_{x\to\pm\infty}\left(3x^2e^{-x^2}\right).$$

Rješenje

- $\lim_{x\to\pm\infty}\left(3x^2\right) = 3\cdot(\pm\infty)^2 = 3\cdot(+\infty) = +\infty$
- $\lim_{x\to\pm\infty} e^{-x^2} = e^{-(\pm\infty)^2} = e^{-\infty} = 0$
- Radi se o neodređenom obliku $0\cdot\infty$ pa ćemo ga prije primjene L'Hospitalovog pravila svesti na neodređeni oblik $\frac{\infty}{\infty}$.

$$\lim_{x \to \pm\infty} \left(3x^2 e^{-x^2} \right) =$$

$0 \cdot \infty$

$$\lim_{x \to \pm\infty} \left(3x^2 e^{-x^2} \right) =$$

$0 \cdot \infty$

$$\lim_{x \to \pm\infty} \left(3x^2 e^{-x^2}\right) = \lim_{x \to \pm\infty} \frac{}{}$$

$0 \cdot \infty$

$$\lim_{x \to \pm\infty} \left(3x^2 e^{-x^2}\right) = \lim_{x \to \pm\infty} \frac{3x^2}{}$$

$$e^{-x^2} = \frac{1}{e^{x^2}}$$

$0 \cdot \infty$

$$\lim_{x \to \pm\infty} \left(3x^2 e^{-x^2} \right) = \lim_{x \to \pm\infty} \frac{3x^2}{}$$

$$e^{-x^2} = \frac{1}{e^{x^2}}$$

$0 \cdot \infty$

$$\lim_{x \to \pm\infty} \left(3x^2 e^{-x^2}\right) = \lim_{x \to \pm\infty} \frac{3x^2}{e^{x^2}}$$

$$e^{-x^2} = \frac{1}{e^{x^2}}$$

$0 \cdot \infty$

$\frac{\infty}{\infty}$

$$\lim_{x \to \pm\infty} \left(3x^2 e^{-x^2}\right) = \lim_{x \to \pm\infty} \frac{3x^2}{e^{x^2}}$$

$$e^{(\pm\infty)^2} = e^{+\infty} = +\infty$$

$$e^{-x^2} = \frac{1}{e^{x^2}}$$

$0 \cdot \infty$

$\frac{\infty}{\infty}$

$$\lim_{x\to\pm\infty}\left(3x^2 e^{-x^2}\right) = \lim_{x\to\pm\infty}\frac{3x^2}{e^{x^2}} = \lim_{x\to\pm\infty}\frac{}{}$$

$$e^{(\pm\infty)^2} = e^{+\infty} = +\infty$$

$$e^{-x^2} = \frac{1}{e^{x^2}}$$

$0 \cdot \infty$

$\frac{\infty}{\infty}$

L'Hospitalovo pravilo

$$\lim_{x \to \pm\infty} \left(3x^2 e^{-x^2}\right) = \lim_{x \to \pm\infty} \frac{3x^2}{e^{x^2}} = \lim_{x \to \pm\infty} \frac{}{}$$

$$e^{(\pm\infty)^2} = e^{+\infty} = +\infty$$

$$e^{-x^2} = \frac{1}{e^{x^2}}$$

$0 \cdot \infty$ $\frac{\infty}{\infty}$ L'Hospitalovo pravilo

$$\lim_{x\to\pm\infty} \left(3x^2 e^{-x^2}\right) = \lim_{x\to\pm\infty} \frac{3x^2}{e^{x^2}} = \lim_{x\to\pm\infty} \frac{(3x^2)'}{}$$

$$e^{(\pm\infty)^2} = e^{+\infty} = +\infty$$

$$e^{-x^2} = \frac{1}{e^{x^2}}$$

$0 \cdot \infty$; $\frac{\infty}{\infty}$; L'Hospitalovo pravilo

$$\lim_{x\to\pm\infty}\left(3x^2e^{-x^2}\right) = \lim_{x\to\pm\infty}\frac{3x^2}{e^{x^2}} = \lim_{x\to\pm\infty}\frac{(3x^2)'}{(e^{x^2})'}$$

$$e^{(\pm\infty)^2} = e^{+\infty} = +\infty$$

$$e^{-x^2} = \frac{1}{e^{x^2}}$$

$0 \cdot \infty$ $\frac{\infty}{\infty}$ L'Hospitalovo pravilo

$$\lim_{x\to\pm\infty}\left(3x^2 e^{-x^2}\right) = \lim_{x\to\pm\infty}\frac{3x^2}{e^{x^2}} = \lim_{x\to\pm\infty}\frac{(3x^2)'}{(e^{x^2})'} =$$

$$= \lim_{x\to\pm\infty}\frac{}{}$$

$$e^{(\pm\infty)^2} = e^{+\infty} = +\infty$$

$$e^{-x^2} = \frac{1}{e^{x^2}}$$

$0 \cdot \infty$

$\frac{\infty}{\infty}$

L'Hospitalovo pravilo

$$\lim_{x \to \pm\infty} \left(3x^2 e^{-x^2}\right) = \lim_{x \to \pm\infty} \frac{3x^2}{e^{x^2}} = \lim_{x \to \pm\infty} \frac{(3x^2)'}{(e^{x^2})'} =$$

$$= \lim_{x \to \pm\infty} \frac{6x}{}$$

$$e^{(\pm\infty)^2} = e^{+\infty} = +\infty$$

$$\left(e^{\text{nešto}}\right)' = e^{\text{nešto}} \cdot (\text{nešto})'$$

$$e^{-x^2} = \frac{1}{e^{x^2}}$$

$$(e^x)' = e^x$$

$0 \cdot \infty$ $\qquad \frac{\infty}{\infty}$ $\qquad$ L'Hospitalovo pravilo

$$\lim_{x\to\pm\infty}\left(3x^2e^{-x^2}\right) = \lim_{x\to\pm\infty}\frac{3x^2}{e^{x^2}} = \lim_{x\to\pm\infty}\frac{(3x^2)'}{(e^{x^2})'} =$$

$$= \lim_{x\to\pm\infty}\frac{6x}{e^{x^2}}$$

$$e^{(\pm\infty)^2} = e^{+\infty} = +\infty$$

$$\left(e^{\text{nešto}}\right)' = e^{\text{nešto}} \cdot (\text{nešto})'$$

$$e^{-x^2} = \frac{1}{e^{x^2}}$$

$$(e^x)' = e^x$$

$0 \cdot \infty$ $\qquad \frac{\infty}{\infty}$ $\qquad$ L'Hospitalovo pravilo

$$\lim_{x \to \pm\infty} \left(3x^2 e^{-x^2}\right) = \lim_{x \to \pm\infty} \frac{3x^2}{e^{x^2}} = \lim_{x \to \pm\infty} \frac{(3x^2)'}{(e^{x^2})'} =$$

$$= \lim_{x \to \pm\infty} \frac{6x}{e^{x^2} \cdot (x^2)'}$$

$$e^{(\pm\infty)^2} = e^{+\infty} = +\infty$$

$$\left(e^{\text{nešto}}\right)' = e^{\text{nešto}} \cdot (\text{nešto})'$$

$$e^{-x^2} = \frac{1}{e^{x^2}}$$

$$(e^x)' = e^x$$

$0 \cdot \infty$ $\qquad \frac{\infty}{\infty}$ $\qquad$ L'Hospitalovo pravilo

$$\lim_{x\to\pm\infty} \left(3x^2 e^{-x^2}\right) = \lim_{x\to\pm\infty} \frac{3x^2}{e^{x^2}} = \lim_{x\to\pm\infty} \frac{(3x^2)'}{(e^{x^2})'} =$$

$$= \lim_{x\to\pm\infty} \frac{6x}{e^{x^2} \cdot (x^2)'} = \lim_{x\to\pm\infty} \frac{}{}$$

$$e^{(\pm\infty)^2} = e^{+\infty} = +\infty$$

$$\left(e^{\text{nešto}}\right)' = e^{\text{nešto}} \cdot (\text{nešto})'$$

$$e^{-x^2} = \frac{1}{e^{x^2}}$$

$$(e^x)' = e^x$$

$0 \cdot \infty$ $\qquad \frac{\infty}{\infty}$ $\qquad$ L'Hospitalovo pravilo

$$\lim_{x\to\pm\infty}\left(3x^2e^{-x^2}\right) = \lim_{x\to\pm\infty}\frac{3x^2}{e^{x^2}} = \lim_{x\to\pm\infty}\frac{(3x^2)'}{(e^{x^2})'} =$$

$$= \lim_{x\to\pm\infty}\frac{6x}{e^{x^2}\cdot(x^2)'} = \lim_{x\to\pm\infty}\frac{6x}{\phantom{e^{x^2}}}$$

$$e^{(\pm\infty)^2} = e^{+\infty} = +\infty$$

$$\left(e^{\text{nešto}}\right)' = e^{\text{nešto}} \cdot (\text{nešto})'$$

$$(e^x)' = e^x$$

$$e^{-x^2} = \frac{1}{e^{x^2}}$$

$0 \cdot \infty$

$\frac{\infty}{\infty}$

L'Hospitalovo pravilo

$$\lim_{x\to\pm\infty}\left(3x^2e^{-x^2}\right) = \lim_{x\to\pm\infty}\frac{3x^2}{e^{x^2}} = \lim_{x\to\pm\infty}\frac{(3x^2)'}{(e^{x^2})'} =$$

$$= \lim_{x\to\pm\infty}\frac{6x}{e^{x^2}\cdot(x^2)'} = \lim_{x\to\pm\infty}\frac{6x}{2xe^{x^2}}$$

$$e^{(\pm\infty)^2} = e^{+\infty} = +\infty$$

$$\left(e^{\text{nešto}}\right)' = e^{\text{nešto}} \cdot (\text{nešto})'$$

$$e^{-x^2} = \frac{1}{e^{x^2}}$$

$$(e^x)' = e^x$$

$0 \cdot \infty$

$\dfrac{\infty}{\infty}$

L'Hospitalovo pravilo

$$\lim_{x\to\pm\infty}\left(3x^2e^{-x^2}\right) = \lim_{x\to\pm\infty}\frac{3x^2}{e^{x^2}} = \lim_{x\to\pm\infty}\frac{(3x^2)'}{(e^{x^2})'} =$$

$$= \lim_{x\to\pm\infty}\frac{6x}{e^{x^2}\cdot(x^2)'} = \lim_{x\to\pm\infty}\frac{6x}{2xe^{x^2}} = \lim_{x\to\pm\infty}\frac{\quad}{\quad}$$

$$e^{(\pm\infty)^2} = e^{+\infty} = +\infty$$

$$\left(e^{\text{nešto}}\right)' = e^{\text{nešto}} \cdot (\text{nešto})'$$

$$e^{-x^2} = \frac{1}{e^{x^2}}$$

$$(e^x)' = e^x$$

$0 \cdot \infty$

$\frac{\infty}{\infty}$

L'Hospitalovo pravilo

$$\lim_{x\to\pm\infty}\left(3x^2e^{-x^2}\right) = \lim_{x\to\pm\infty}\frac{3x^2}{e^{x^2}} = \lim_{x\to\pm\infty}\frac{(3x^2)'}{(e^{x^2})'} =$$

$$= \lim_{x\to\pm\infty}\frac{6x}{e^{x^2}\cdot(x^2)'} = \lim_{x\to\pm\infty}\frac{6x}{2xe^{x^2}} = \lim_{x\to\pm\infty}\frac{3}{}$$

$$e^{(\pm\infty)^2} = e^{+\infty} = +\infty$$

$$\left(e^{\text{nešto}}\right)' = e^{\text{nešto}} \cdot (\text{nešto})'$$

$$e^{-x^2} = \frac{1}{e^{x^2}}$$

$$(e^x)' = e^x$$

$0 \cdot \infty$

$\frac{\infty}{\infty}$

L'Hospitalovo pravilo

$$\lim_{x\to\pm\infty}\left(3x^2e^{-x^2}\right) = \lim_{x\to\pm\infty}\frac{3x^2}{e^{x^2}} = \lim_{x\to\pm\infty}\frac{(3x^2)'}{(e^{x^2})'} =$$

$$= \lim_{x\to\pm\infty}\frac{6x}{e^{x^2}\cdot(x^2)'} = \lim_{x\to\pm\infty}\frac{6x}{2xe^{x^2}} = \lim_{x\to\pm\infty}\frac{3}{e^{x^2}}$$

$$e^{(\pm\infty)^2} = e^{+\infty} = +\infty$$

$$\left(e^{\text{nešto}}\right)' = e^{\text{nešto}} \cdot (\text{nešto})'$$

$$e^{-x^2} = \frac{1}{e^{x^2}}$$

$$(e^x)' = e^x$$

$0 \cdot \infty$ $\qquad \frac{\infty}{\infty}$ $\qquad$ L'Hospitalovo pravilo

$$\lim_{x\to\pm\infty}\left(3x^2 e^{-x^2}\right) = \lim_{x\to\pm\infty}\frac{3x^2}{e^{x^2}} = \lim_{x\to\pm\infty}\frac{(3x^2)'}{(e^{x^2})'} =$$

$$= \lim_{x\to\pm\infty}\frac{6x}{e^{x^2}\cdot(x^2)'} = \lim_{x\to\pm\infty}\frac{6x}{2xe^{x^2}} = \lim_{x\to\pm\infty}\frac{3}{e^{x^2}} = \text{———}$$

$$e^{(\pm\infty)^2} = e^{+\infty} = +\infty$$

$$\left(e^{\text{nešto}}\right)' = e^{\text{nešto}} \cdot (\text{nešto})'$$

$$e^{-x^2} = \frac{1}{e^{x^2}}$$

$$(e^x)' = e^x$$

$0 \cdot \infty$ $\quad \frac{\infty}{\infty}$ $\quad$ L'Hospitalovo pravilo

$$\lim_{x\to\pm\infty}\left(3x^2e^{-x^2}\right) = \lim_{x\to\pm\infty}\frac{3x^2}{e^{x^2}} = \lim_{x\to\pm\infty}\frac{(3x^2)'}{(e^{x^2})'} =$$

$$= \lim_{x\to\pm\infty}\frac{6x}{e^{x^2}\cdot(x^2)'} = \lim_{x\to\pm\infty}\frac{6x}{2xe^{x^2}} = \lim_{x\to\pm\infty}\frac{3}{e^{x^2}} = \frac{3}{}$$

$$e^{(\pm\infty)^2} = e^{+\infty} = +\infty$$

$$\left(e^{\text{nešto}}\right)' = e^{\text{nešto}} \cdot (\text{nešto})'$$

$$(e^x)' = e^x$$

$$e^{-x^2} = \frac{1}{e^{x^2}}$$

$0 \cdot \infty$

$\dfrac{\infty}{\infty}$

L'Hospitalovo pravilo

$$\lim_{x\to\pm\infty}\left(3x^2 e^{-x^2}\right) = \lim_{x\to\pm\infty}\frac{3x^2}{e^{x^2}} = \lim_{x\to\pm\infty}\frac{(3x^2)'}{(e^{x^2})'} =$$

$$= \lim_{x\to\pm\infty}\frac{6x}{e^{x^2}\cdot(x^2)'} = \lim_{x\to\pm\infty}\frac{6x}{2xe^{x^2}} = \lim_{x\to\pm\infty}\frac{3}{e^{x^2}} = \frac{3}{+\infty}$$

$$e^{(\pm\infty)^2} = e^{+\infty} = +\infty$$

$\left(e^{\text{nešto}}\right)' = e^{\text{nešto}} \cdot (\text{nešto})'$

$(e^x)' = e^x$

$$e^{-x^2} = \frac{1}{e^{x^2}}$$

$0 \cdot \infty$ — $\frac{\infty}{\infty}$ — L'Hospitalovo pravilo

$$\lim_{x\to\pm\infty}\left(3x^2e^{-x^2}\right) = \lim_{x\to\pm\infty}\frac{3x^2}{e^{x^2}} = \lim_{x\to\pm\infty}\frac{(3x^2)'}{(e^{x^2})'} =$$

$$= \lim_{x\to\pm\infty}\frac{6x}{e^{x^2}\cdot(x^2)'} = \lim_{x\to\pm\infty}\frac{6x}{2xe^{x^2}} = \lim_{x\to\pm\infty}\frac{3}{e^{x^2}} = \frac{3}{+\infty} = 0$$

$$e^{(\pm\infty)^2} = e^{+\infty} = +\infty$$

peti zadatak

Napomena

- Ako je $\lim_{x\to c} f(x)^{g(x)}$ nekog od oblika $1^\infty, \infty^0$ ili 0^0, tada na njega ne možemo direktno primijeniti L'Hospitalovo pravilo.

- Najprije treba logaritmirati izraz $f(x)^{g(x)}$ i izračunati limes logaritmiranog izraza koji je oblika $0 \cdot \infty$. Ranije je već objašnjeno kako se primijenjuje L'Hospitalovo pravilo na neodređeni izraz oblika $0 \cdot \infty$.

$$\log_a x^k = k \cdot \log_a x$$

$1^\infty, \infty^0, 0^0$

$$\lim_{x\to c} f(x)^{g(x)}$$

$0 \cdot \infty$

$$\lim_{x\to c} \left(\ln f(x)^{g(x)} \right) = \lim_{x\to c} \left(g(x) \cdot \ln f(x) \right)$$

Napomena

- Pretpostavimo da je limes logaritmiranog izraza jednak A, tj.

$$\lim_{x \to c} \left(\ln f(x)^{g(x)} \right) = A.$$

- Kako limes i neprekidna funkcija komutiraju, dalje dobivamo

$$\ln \left(\lim_{x \to c} f(x)^{g(x)} \right) = A,$$

odnosno

$$\lim_{x \to c} f(x)^{g(x)} = e^{A}.$$

Zadatak 5

Pomoću L'Hospitalovog pravila izračunajte limes

$$\lim_{x \to \frac{3}{2}\pi} (1 + \sin x)^{2 \operatorname{ctg} x}.$$

Zadatak 5

Pomoću L'Hospitalovog pravila izračunajte limes

$$\lim_{x\to\frac{3}{2}\pi}(1+\sin x)^{2\operatorname{ctg} x}.$$

Rješenje

- $\lim\limits_{x\to\frac{3}{2}\pi}\left(1+\sin x\right) =$

Zadatak 5

Pomoću L'Hospitalovog pravila izračunajte limes

$$\lim_{x \to \frac{3}{2}\pi} (1 + \sin x)^{2 \operatorname{ctg} x}.$$

Rješenje

- $\lim\limits_{x \to \frac{3}{2}\pi} \big(1 + \sin x\big) = 1 + \sin \dfrac{3}{2}\pi$

Zadatak 5

Pomoću L'Hospitalovog pravila izračunajte limes

$$\lim_{x \to \frac{3}{2}\pi} (1 + \sin x)^{2 \operatorname{ctg} x}.$$

Rješenje

- $\lim\limits_{x \to \frac{3}{2}\pi} \left(1 + \sin x\right) = 1 + \sin \dfrac{3}{2}\pi = 1 + (-1)$

Zadatak 5

Pomoću L'Hospitalovog pravila izračunajte limes

$$\lim_{x\to\frac{3}{2}\pi}(1+\sin x)^{2\operatorname{ctg} x}.$$

Rješenje

- $\lim\limits_{x\to\frac{3}{2}\pi}\big(1+\sin x\big) = 1+\sin\dfrac{3}{2}\pi = 1+(-1) = 0$

Zadatak 5

Pomoću L'Hospitalovog pravila izračunajte limes

$$\lim_{x\to\frac{3}{2}\pi}(1+\sin x)^{2\operatorname{ctg}x}.$$

Rješenje

- $\lim_{x\to\frac{3}{2}\pi}\big(1+\sin x\big) = 1+\sin\frac{3}{2}\pi = 1+(-1) = 0$
- $\lim_{x\to\frac{3}{2}\pi}(2\operatorname{ctg}x) =$

Zadatak 5

Pomoću L'Hospitalovog pravila izračunajte limes

$$\lim_{x \to \frac{3}{2}\pi} (1 + \sin x)^{2 \operatorname{ctg} x}.$$

Rješenje

- $\displaystyle \lim_{x \to \frac{3}{2}\pi} \big(1 + \sin x\big) = 1 + \sin \frac{3}{2}\pi = 1 + (-1) = 0$
- $\displaystyle \lim_{x \to \frac{3}{2}\pi} (2 \operatorname{ctg} x) = 2 \operatorname{ctg} \frac{3}{2}\pi$

Zadatak 5

Pomoću L'Hospitalovog pravila izračunajte limes

$$\lim_{x \to \frac{3}{2}\pi} (1 + \sin x)^{2 \operatorname{ctg} x}.$$

Rješenje

- $\lim\limits_{x \to \frac{3}{2}\pi} \big(1 + \sin x\big) = 1 + \sin \frac{3}{2}\pi = 1 + (-1) = 0$
- $\lim\limits_{x \to \frac{3}{2}\pi} (2 \operatorname{ctg} x) = 2 \operatorname{ctg} \frac{3}{2}\pi = 2 \cdot 0$

Zadatak 5

Pomoću L'Hospitalovog pravila izračunajte limes

$$\lim_{x \to \frac{3}{2}\pi} (1 + \sin x)^{2 \operatorname{ctg} x}.$$

Rješenje

- $\lim\limits_{x \to \frac{3}{2}\pi} \big(1 + \sin x\big) = 1 + \sin \dfrac{3}{2}\pi = 1 + (-1) = 0$
- $\lim\limits_{x \to \frac{3}{2}\pi} (2 \operatorname{ctg} x) = 2 \operatorname{ctg} \dfrac{3}{2}\pi = 2 \cdot 0 = 0$

Zadatak 5

Pomoću L'Hospitalovog pravila izračunajte limes

$$\lim_{x\to\frac{3}{2}\pi}(1+\sin x)^{2\operatorname{ctg}x}.$$

Rješenje

- $\lim\limits_{x\to\frac{3}{2}\pi}\big(1+\sin x\big)=1+\sin\frac{3}{2}\pi=1+(-1)=0$
- $\lim\limits_{x\to\frac{3}{2}\pi}(2\operatorname{ctg}x)=2\operatorname{ctg}\frac{3}{2}\pi=2\cdot 0=0$
- Radi se o neodređenom obliku 0^0 pa ćemo najprije izračunati limes logaritmiranog izraza $\ln(1+\sin x)^{2\operatorname{ctg}x}$.

$$\lim_{x \to \frac{3}{2}\pi} \left(\ln (1 + \sin x)^{2 \operatorname{ctg} x} \right) =$$

$$\lim_{x\to\frac{3}{2}\pi}\left(\ln(1+\sin x)^{2\operatorname{ctg} x}\right) = \lim_{x\to\frac{3}{2}\pi}\left(\qquad\qquad\qquad\qquad \right)$$

$$\lim_{x\to\frac{3}{2}\pi}\left(\ln\left(1+\sin x\right)^{2\operatorname{ctg}x}\right)=\lim_{x\to\frac{3}{2}\pi}\left(2\operatorname{ctg}x\qquad\qquad\qquad\right)$$

$$\lim_{x\to\frac{3}{2}\pi}\left(\ln\left(1+\sin x\right)^{2\operatorname{ctg}x}\right)=\lim_{x\to\frac{3}{2}\pi}\left(2\operatorname{ctg}x\cdot\ln\left(1+\sin x\right)\right)$$

$0 \cdot \infty$

$$\lim_{x \to \frac{3}{2}\pi} \left(\ln\left(1+\sin x\right)^{2\operatorname{ctg} x} \right) = \lim_{x \to \frac{3}{2}\pi} \left(2\operatorname{ctg} x \cdot \ln\left(1+\sin x\right) \right)$$

$0 \cdot \infty$

$$\lim_{x \to \frac{3}{2}\pi} \left(\ln \left(1 + \sin x\right)^{2 \operatorname{ctg} x} \right) = \lim_{x \to \frac{3}{2}\pi} \left(2 \operatorname{ctg} x \cdot \ln \left(1 + \sin x\right)\right) =$$

$$= \lim_{x \to \frac{3}{2}\pi} \frac{}{}$$

$0 \cdot \infty$

$$\lim_{x \to \frac{3}{2}\pi} \left(\ln (1 + \sin x)^{2 \operatorname{ctg} x} \right) = \lim_{x \to \frac{3}{2}\pi} \Big(2 \operatorname{ctg} x \cdot \ln (1 + \sin x) \Big) =$$

$$= \lim_{x \to \frac{3}{2}\pi} \frac{2 \ln (1 + \sin x)}{}$$

$\operatorname{ctg} x = \dfrac{1}{\operatorname{tg} x}$

$0 \cdot \infty$

$$\lim_{x \to \frac{3}{2}\pi} \left(\ln (1 + \sin x)^{2 \operatorname{ctg} x} \right) = \lim_{x \to \frac{3}{2}\pi} \Big(2 \operatorname{ctg} x \cdot \ln (1 + \sin x) \Big) =$$

$$= \lim_{x \to \frac{3}{2}\pi} \frac{2 \ln (1 + \sin x)}{}$$

$$\operatorname{ctg} x = \frac{1}{\operatorname{tg} x}$$

$0 \cdot \infty$

$$\lim_{x \to \frac{3}{2}\pi} \left(\ln (1 + \sin x)^{2 \operatorname{ctg} x} \right) = \lim_{x \to \frac{3}{2}\pi} \Big(2 \operatorname{ctg} x \cdot \ln (1 + \sin x) \Big) =$$

$$= \lim_{x \to \frac{3}{2}\pi} \frac{2 \ln (1 + \sin x)}{\operatorname{tg} x}$$

$$\operatorname{ctg} x = \frac{1}{\operatorname{tg} x}$$

$$0 \cdot \infty$$

$$\lim_{x \to \frac{3}{2}\pi} \left(\ln \left(1 + \sin x\right)^{2 \operatorname{ctg} x} \right) = \lim_{x \to \frac{3}{2}\pi} \Big(2 \operatorname{ctg} x \cdot \ln \left(1 + \sin x\right) \Big) =$$

$$\frac{\infty}{\infty}$$

$$= \lim_{x \to \frac{3}{2}\pi} \frac{2 \ln \left(1 + \sin x\right)}{\operatorname{tg} x}$$

$$\operatorname{ctg} x = \frac{1}{\operatorname{tg} x}$$

$0 \cdot \infty$

$$\lim_{x \to \frac{3}{2}\pi} \left(\ln (1 + \sin x)^{2 \operatorname{ctg} x} \right) = \lim_{x \to \frac{3}{2}\pi} \Big(2 \operatorname{ctg} x \cdot \ln (1 + \sin x) \Big) =$$

$\dfrac{\infty}{\infty}$

$$= \lim_{x \to \frac{3}{2}\pi} \frac{2 \ln (1 + \sin x)}{\operatorname{tg} x} = \lim_{x \to \frac{3}{2}\pi} \frac{}{}$$

$$\operatorname{ctg} x = \frac{1}{\operatorname{tg} x}$$

$0 \cdot \infty$

$$\lim_{x \to \frac{3}{2}\pi} \left(\ln (1 + \sin x)^{2 \operatorname{ctg} x} \right) = \lim_{x \to \frac{3}{2}\pi} \Big(2 \operatorname{ctg} x \cdot \ln (1 + \sin x) \Big) =$$

$\frac{\infty}{\infty}$

L'Hospitalovo pravilo

$$= \lim_{x \to \frac{3}{2}\pi} \frac{2 \ln (1 + \sin x)}{\operatorname{tg} x} = \lim_{x \to \frac{3}{2}\pi} \frac{}{}$$

$$\operatorname{ctg} x = \frac{1}{\operatorname{tg} x}$$

$0 \cdot \infty$

$$\lim_{x \to \frac{3}{2}\pi} \left(\ln{(1 + \sin x)}^{2 \operatorname{ctg} x} \right) = \lim_{x \to \frac{3}{2}\pi} \Big(2 \operatorname{ctg} x \cdot \ln{(1 + \sin x)} \Big) =$$

$\frac{\infty}{\infty}$

L'Hospitalovo pravilo

$$= \lim_{x \to \frac{3}{2}\pi} \frac{2 \ln{(1 + \sin x)}}{\operatorname{tg} x} = \lim_{x \to \frac{3}{2}\pi} \frac{\big(2 \ln{(1 + \sin x)}\big)'}{}$$

$$\operatorname{ctg} x = \frac{1}{\operatorname{tg} x}$$

$0 \cdot \infty$

$$\lim_{x\to\frac{3}{2}\pi} \left(\ln (1+\sin x)^{2\operatorname{ctg} x} \right) = \lim_{x\to\frac{3}{2}\pi} \Big(2\operatorname{ctg} x \cdot \ln (1+\sin x) \Big) =$$

$\frac{\infty}{\infty}$

L'Hospitalovo pravilo

$$= \lim_{x\to\frac{3}{2}\pi} \frac{2\ln (1+\sin x)}{\operatorname{tg} x} = \lim_{x\to\frac{3}{2}\pi} \frac{\big(2\ln (1+\sin x)\big)'}{(\operatorname{tg} x)'}$$

$$\operatorname{ctg} x = \frac{1}{\operatorname{tg} x}$$

$0 \cdot \infty$

$$\lim_{x \to \frac{3}{2}\pi} \left(\ln (1 + \sin x)^{2 \operatorname{ctg} x} \right) = \lim_{x \to \frac{3}{2}\pi} \Big(2 \operatorname{ctg} x \cdot \ln (1 + \sin x) \Big) =$$

$\frac{\infty}{\infty}$

L'Hospitalovo pravilo

$$= \lim_{x \to \frac{3}{2}\pi} \frac{2 \ln (1 + \sin x)}{\operatorname{tg} x} = \lim_{x \to \frac{3}{2}\pi} \frac{\big(2 \ln (1 + \sin x)\big)'}{(\operatorname{tg} x)'} =$$

$$= \lim_{x \to \frac{3}{2}\pi} \frac{}{}$$

$$\operatorname{ctg} x = \frac{1}{\operatorname{tg} x}$$

$$(\operatorname{tg} x)' = \frac{1}{\cos^2 x}$$

$0 \cdot \infty$

$$\lim_{x\to\frac{3}{2}\pi}\left(\ln{(1+\sin x)}^{2\operatorname{ctg} x}\right) = \lim_{x\to\frac{3}{2}\pi}\Big(2\operatorname{ctg} x \cdot \ln{(1+\sin x)}\Big) =$$

$\frac{\infty}{\infty}$ L'Hospitalovo pravilo

$$= \lim_{x\to\frac{3}{2}\pi}\frac{2\ln{(1+\sin x)}}{\operatorname{tg} x} = \lim_{x\to\frac{3}{2}\pi}\frac{\big(2\ln{(1+\sin x)}\big)'}{(\operatorname{tg} x)'} =$$

$$= \lim_{x\to\frac{3}{2}\pi}\frac{}{\frac{1}{\cos^2 x}}$$

$$\operatorname{ctg} x = \frac{1}{\operatorname{tg} x}$$

$$(\operatorname{tg} x)' = \frac{1}{\cos^2 x}$$

$0 \cdot \infty$

$$\lim_{x\to\frac{3}{2}\pi} \left(\ln (1+\sin x)^{2\operatorname{ctg} x} \right) = \lim_{x\to\frac{3}{2}\pi} \Big(2\operatorname{ctg} x \cdot \ln (1+\sin x) \Big) =$$

$\frac{\infty}{\infty}$

L'Hospitalovo pravilo

$$= \lim_{x\to\frac{3}{2}\pi} \frac{2\ln(1+\sin x)}{\operatorname{tg} x} = \lim_{x\to\frac{3}{2}\pi} \frac{\big(2\ln(1+\sin x)\big)'}{(\operatorname{tg} x)'} =$$

$$= \lim_{x\to\frac{3}{2}\pi} \frac{\qquad}{\frac{1}{\cos^2 x}}$$

$$\big(\ln(\text{nešto})\big)' = \frac{1}{\text{nešto}} \cdot (\text{nešto})'$$

$$(\ln x)' = \frac{1}{x}$$

$$\operatorname{ctg} x = \frac{1}{\operatorname{tg} x}$$

$$(\operatorname{tg} x)' = \frac{1}{\cos^2 x}$$

$0 \cdot \infty$

$$\lim_{x\to\frac{3}{2}\pi} \left(\ln\left(1+\sin x\right)^{2\operatorname{ctg} x} \right) = \lim_{x\to\frac{3}{2}\pi} \Big(2 \operatorname{ctg} x \cdot \ln\left(1+\sin x\right)\Big) =$$

$\frac{\infty}{\infty}$

L'Hospitalovo pravilo

$$= \lim_{x\to\frac{3}{2}\pi} \frac{2\ln\left(1+\sin x\right)}{\operatorname{tg} x} = \lim_{x\to\frac{3}{2}\pi} \frac{\big(2\ln\left(1+\sin x\right)\big)'}{(\operatorname{tg} x)'} =$$

$$= \lim_{x\to\frac{3}{2}\pi} \frac{\frac{2}{1+\sin x}}{\frac{1}{\cos^2 x}}$$

$$\big(\ln(\text{nešto})\big)' = \frac{1}{\text{nešto}} \cdot (\text{nešto})'$$

$$(\ln x)' = \frac{1}{x}$$

$$\operatorname{ctg} x = \frac{1}{\operatorname{tg} x}$$

$$(\operatorname{tg} x)' = \frac{1}{\cos^2 x}$$

$0 \cdot \infty$

$$\lim_{x \to \frac{3}{2}\pi} \left(\ln (1 + \sin x)^{2 \operatorname{ctg} x} \right) = \lim_{x \to \frac{3}{2}\pi} \Big(2 \operatorname{ctg} x \cdot \ln (1 + \sin x) \Big) =$$

$\frac{\infty}{\infty}$

L'Hospitalovo pravilo

$$= \lim_{x \to \frac{3}{2}\pi} \frac{2 \ln (1 + \sin x)}{\operatorname{tg} x} = \lim_{x \to \frac{3}{2}\pi} \frac{\big(2 \ln (1 + \sin x)\big)'}{(\operatorname{tg} x)'} =$$

$$= \lim_{x \to \frac{3}{2}\pi} \frac{\frac{2}{1 + \sin x} \cdot (1 + \sin x)'}{\frac{1}{\cos^2 x}}$$

$$\big(\ln (\text{nešto}) \big)' = \frac{1}{\text{nešto}} \cdot (\text{nešto})'$$

$$(\ln x)' = \frac{1}{x}$$

$$\operatorname{ctg} x = \frac{1}{\operatorname{tg} x}$$

$$(\operatorname{tg} x)' = \frac{1}{\cos^2 x}$$

$0 \cdot \infty$

$$\lim_{x\to\frac{3}{2}\pi} \left(\ln (1+\sin x)^{2\operatorname{ctg} x} \right) = \lim_{x\to\frac{3}{2}\pi} \Big(2\operatorname{ctg} x \cdot \ln(1+\sin x) \Big) =$$

$\frac{\infty}{\infty}$

L'Hospitalovo pravilo

$$= \lim_{x\to\frac{3}{2}\pi} \frac{2\ln(1+\sin x)}{\operatorname{tg} x} = \lim_{x\to\frac{3}{2}\pi} \frac{\big(2\ln(1+\sin x)\big)'}{(\operatorname{tg} x)'} =$$

$$= \lim_{x\to\frac{3}{2}\pi} \frac{\frac{2}{1+\sin x} \cdot (1+\sin x)'}{\frac{1}{\cos^2 x}} = \lim_{x\to\frac{3}{2}\pi} \text{\_\_\_\_\_\_\_\_}$$

$$\big(\ln(\text{nešto})\big)' = \frac{1}{\text{nešto}} \cdot (\text{nešto})'$$

$$(\ln x)' = \frac{1}{x}$$

$\operatorname{ctg} x = \dfrac{1}{\operatorname{tg} x}$

$(\operatorname{tg} x)' = \dfrac{1}{\cos^2 x}$

$0 \cdot \infty$

$$\lim_{x \to \frac{3}{2}\pi} \left(\ln (1 + \sin x)^{2 \operatorname{ctg} x} \right) = \lim_{x \to \frac{3}{2}\pi} \Big(2 \operatorname{ctg} x \cdot \ln (1 + \sin x) \Big) =$$

$\dfrac{\infty}{\infty}$

L'Hospitalovo pravilo

$$= \lim_{x \to \frac{3}{2}\pi} \frac{2 \ln (1 + \sin x)}{\operatorname{tg} x} = \lim_{x \to \frac{3}{2}\pi} \frac{\Big(2 \ln (1 + \sin x) \Big)'}{(\operatorname{tg} x)'} =$$

$$= \lim_{x \to \frac{3}{2}\pi} \frac{\dfrac{2}{1 + \sin x} \cdot (1 + \sin x)'}{\dfrac{1}{\cos^2 x}} = \lim_{x \to \frac{3}{2}\pi} \frac{}{\dfrac{1}{\cos^2 x}}$$

$\Big(\ln (\text{nešto}) \Big)' = \dfrac{1}{\text{nešto}} \cdot (\text{nešto})'$

$(\ln x)' = \dfrac{1}{x}$

$\operatorname{ctg} x = \dfrac{1}{\operatorname{tg} x}$

$(\operatorname{tg} x)' = \dfrac{1}{\cos^2 x}$

$0 \cdot \infty$

$$\lim_{x \to \frac{3}{2}\pi} \left(\ln (1 + \sin x)^{2 \operatorname{ctg} x} \right) = \lim_{x \to \frac{3}{2}\pi} \Big(2 \operatorname{ctg} x \cdot \ln (1 + \sin x) \Big) =$$

$\dfrac{\infty}{\infty}$

L'Hospitalovo pravilo

$$= \lim_{x \to \frac{3}{2}\pi} \frac{2 \ln (1 + \sin x)}{\operatorname{tg} x} = \lim_{x \to \frac{3}{2}\pi} \frac{\big(2 \ln (1 + \sin x)\big)'}{(\operatorname{tg} x)'} =$$

$$= \lim_{x \to \frac{3}{2}\pi} \frac{\dfrac{2}{1 + \sin x} \cdot (1 + \sin x)'}{\dfrac{1}{\cos^2 x}} = \lim_{x \to \frac{3}{2}\pi} \frac{\dfrac{2 \cos x}{1 + \sin x}}{\dfrac{1}{\cos^2 x}}$$

$\big(\ln (\text{nešto}) \big)' = \dfrac{1}{\text{nešto}} \cdot (\text{nešto})'$

$(\ln x)' = \dfrac{1}{x}$

$(\sin x)' = \cos x$

$$\text{ctg}\, x = \frac{1}{\text{tg}\, x}$$

$$(\text{tg}\, x)' = \frac{1}{\cos^2 x}$$

$0 \cdot \infty$

$$\lim_{x \to \frac{3}{2}\pi} \left(\ln (1 + \sin x)^{2\, \text{ctg}\, x} \right) = \lim_{x \to \frac{3}{2}\pi} \Big(2\, \text{ctg}\, x \cdot \ln (1 + \sin x) \Big) =$$

$\frac{\infty}{\infty}$

L'Hospitalovo pravilo

$$= \lim_{x \to \frac{3}{2}\pi} \frac{2 \ln (1 + \sin x)}{\text{tg}\, x} = \lim_{x \to \frac{3}{2}\pi} \frac{\big(2 \ln (1 + \sin x)\big)'}{(\text{tg}\, x)'} =$$

$$= \lim_{x \to \frac{3}{2}\pi} \frac{\frac{2}{1 + \sin x} \cdot (1 + \sin x)'}{\frac{1}{\cos^2 x}} = \lim_{x \to \frac{3}{2}\pi} \frac{\frac{2 \cos x}{1 + \sin x}}{\frac{1}{\cos^2 x}} = \lim_{x \to \frac{3}{2}\pi} \frac{\quad}{\quad}$$

$$\big(\ln (\text{nešto}) \big)' = \frac{1}{\text{nešto}} \cdot (\text{nešto})'$$

$$(\ln x)' = \frac{1}{x}$$

$$(\sin x)' = \cos x$$

$\operatorname{ctg} x = \dfrac{1}{\operatorname{tg} x}$

$(\operatorname{tg} x)' = \dfrac{1}{\cos^2 x}$

$0 \cdot \infty$

$$\lim_{x\to\frac{3}{2}\pi} \left(\ln (1+\sin x)^{2\operatorname{ctg} x} \right) = \lim_{x\to\frac{3}{2}\pi} \Big(2\operatorname{ctg} x \cdot \ln (1+\sin x) \Big) =$$

$\dfrac{\infty}{\infty}$

L'Hospitalovo pravilo

$$= \lim_{x\to\frac{3}{2}\pi} \frac{2\ln (1+\sin x)}{\operatorname{tg} x} = \lim_{x\to\frac{3}{2}\pi} \frac{\big(2\ln (1+\sin x)\big)'}{(\operatorname{tg} x)'} =$$

$$= \lim_{x\to\frac{3}{2}\pi} \frac{\dfrac{2}{1+\sin x} \cdot (1+\sin x)'}{\dfrac{1}{\cos^2 x}} = \lim_{x\to\frac{3}{2}\pi} \frac{\dfrac{2\cos x}{1+\sin x}}{\dfrac{1}{\cos^2 x}} = \lim_{x\to\frac{3}{2}\pi} \frac{2\cos^3 x}{}$$

$\big(\ln(\text{nešto})\big)' = \dfrac{1}{\text{nešto}} \cdot (\text{nešto})'$

$(\ln x)' = \dfrac{1}{x}$

$(\sin x)' = \cos x$

$\operatorname{ctg} x = \dfrac{1}{\operatorname{tg} x}$

$(\operatorname{tg} x)' = \dfrac{1}{\cos^2 x}$

$0 \cdot \infty$

$$\lim_{x \to \frac{3}{2}\pi} \left(\ln (1 + \sin x)^{2 \operatorname{ctg} x} \right) = \lim_{x \to \frac{3}{2}\pi} \Big(2 \operatorname{ctg} x \cdot \ln (1 + \sin x) \Big) =$$

$\dfrac{\infty}{\infty}$

L'Hospitalovo pravilo

$$= \lim_{x \to \frac{3}{2}\pi} \frac{2 \ln (1 + \sin x)}{\operatorname{tg} x} = \lim_{x \to \frac{3}{2}\pi} \frac{\big(2 \ln (1 + \sin x)\big)'}{(\operatorname{tg} x)'} =$$

$$= \lim_{x \to \frac{3}{2}\pi} \frac{\frac{2}{1 + \sin x} \cdot (1 + \sin x)'}{\frac{1}{\cos^2 x}} = \lim_{x \to \frac{3}{2}\pi} \frac{\frac{2 \cos x}{1 + \sin x}}{\frac{1}{\cos^2 x}} = \lim_{x \to \frac{3}{2}\pi} \frac{2 \cos^3 x}{1 + \sin x}$$

$\big(\ln (\text{nešto}) \big)' = \dfrac{1}{\text{nešto}} \cdot (\text{nešto})'$ $\quad (\ln x)' = \dfrac{1}{x}$ $\quad (\sin x)' = \cos x$

$$\operatorname{ctg} x = \frac{1}{\operatorname{tg} x}$$

$$(\operatorname{tg} x)' = \frac{1}{\cos^2 x}$$

$0 \cdot \infty$

$$\lim_{x \to \frac{3}{2}\pi} \left(\ln (1 + \sin x)^{2 \operatorname{ctg} x} \right) = \lim_{x \to \frac{3}{2}\pi} \Big(2 \operatorname{ctg} x \cdot \ln (1 + \sin x) \Big) =$$

$\frac{\infty}{\infty}$

L'Hospitalovo pravilo

$$= \lim_{x \to \frac{3}{2}\pi} \frac{2 \ln (1 + \sin x)}{\operatorname{tg} x} = \lim_{x \to \frac{3}{2}\pi} \frac{\big(2 \ln (1 + \sin x)\big)'}{(\operatorname{tg} x)'} =$$

$\frac{0}{0}$

$$= \lim_{x \to \frac{3}{2}\pi} \frac{\frac{2}{1 + \sin x} \cdot (1 + \sin x)'}{\frac{1}{\cos^2 x}} = \lim_{x \to \frac{3}{2}\pi} \frac{\frac{2 \cos x}{1 + \sin x}}{\frac{1}{\cos^2 x}} = \lim_{x \to \frac{3}{2}\pi} \frac{2 \cos^3 x}{1 + \sin x}$$

$$\big(\ln (\text{nešto}) \big)' = \frac{1}{\text{nešto}} \cdot (\text{nešto})'$$

$$(\ln x)' = \frac{1}{x}$$

$$(\sin x)' = \cos x$$

$$\operatorname{ctg} x = \frac{1}{\operatorname{tg} x}$$

$$(\operatorname{tg} x)' = \frac{1}{\cos^2 x}$$

$0 \cdot \infty$

$$\lim_{x\to\frac{3}{2}\pi} \left(\ln (1+\sin x)^{2\operatorname{ctg} x} \right) = \lim_{x\to\frac{3}{2}\pi} \Big(2\operatorname{ctg} x \cdot \ln (1+\sin x) \Big) =$$

$\frac{\infty}{\infty}$

L'Hospitalovo pravilo

$$= \lim_{x\to\frac{3}{2}\pi} \frac{2\ln(1+\sin x)}{\operatorname{tg} x} = \lim_{x\to\frac{3}{2}\pi} \frac{\big(2\ln(1+\sin x)\big)'}{(\operatorname{tg} x)'} =$$

$\frac{0}{0}$

$$= \lim_{x\to\frac{3}{2}\pi} \frac{\frac{2}{1+\sin x}\cdot(1+\sin x)'}{\frac{1}{\cos^2 x}} = \lim_{x\to\frac{3}{2}\pi} \frac{\frac{2\cos x}{1+\sin x}}{\frac{1}{\cos^2 x}} = \lim_{x\to\frac{3}{2}\pi} \frac{2\cos^3 x}{1+\sin x} =$$

$$\big(\ln(\text{nešto})\big)' = \frac{1}{\text{nešto}} \cdot (\text{nešto})'$$

$$(\ln x)' = \frac{1}{x}$$

$$(\sin x)' = \cos x$$

$\frac{0}{0}$

$$= \lim_{x \to \frac{3}{2}\pi} \frac{2\cos^3 x}{1 + \sin x} =$$

$\frac{0}{0}$

$$= \lim_{x \to \frac{3}{2}\pi} \frac{2\cos^3 x}{1 + \sin x} = \lim_{x \to \frac{3}{2}\pi} \frac{}{}$$

$\frac{0}{0}$

L'Hospitalovo pravilo

$$= \lim_{x \to \frac{3}{2}\pi} \frac{2\cos^3 x}{1 + \sin x} = \lim_{x \to \frac{3}{2}\pi} \frac{\quad}{\quad}$$

$\frac{0}{0}$

L'Hospitalovo pravilo

$$= \lim_{x \to \frac{3}{2}\pi} \frac{2\cos^3 x}{1 + \sin x} = \lim_{x \to \frac{3}{2}\pi} \frac{(2\cos^3 x)'}{}$$

$\frac{0}{0}$

L'Hospitalovo pravilo

$$= \lim_{x \to \frac{3}{2}\pi} \frac{2\cos^3 x}{1 + \sin x} = \lim_{x \to \frac{3}{2}\pi} \frac{(2\cos^3 x)'}{(1 + \sin x)'}$$

$\frac{0}{0}$

L'Hospitalovo pravilo

$$= \lim_{x \to \frac{3}{2}\pi} \frac{2\cos^3 x}{1 + \sin x} = \lim_{x \to \frac{3}{2}\pi} \frac{(2\cos^3 x)'}{(1 + \sin x)'} = \lim_{x \to \frac{3}{2}\pi} \frac{}{}$$

$\frac{0}{0}$

L'Hospitalovo pravilo

$$= \lim_{x \to \frac{3}{2}\pi} \frac{2\cos^3 x}{1 + \sin x} \stackrel{}{=} \lim_{x \to \frac{3}{2}\pi} \frac{(2\cos^3 x)'}{(1 + \sin x)'} = \lim_{x \to \frac{3}{2}\pi} \frac{}{\cos x}$$

$\cos^3 x = (\cos x)^3$

$\frac{0}{0}$

L'Hospitalovo pravilo

$$= \lim_{x \to \frac{3}{2}\pi} \frac{2\cos^3 x}{1 + \sin x} = \lim_{x \to \frac{3}{2}\pi} \frac{(2\cos^3 x)'}{(1 + \sin x)'} = \lim_{x \to \frac{3}{2}\pi} \frac{\qquad}{\cos x}$$

$\cos^3 x = (\cos x)^3$

$\frac{0}{0}$

L'Hospitalovo pravilo

$$= \lim_{x \to \frac{3}{2}\pi} \frac{2\cos^3 x}{1 + \sin x} = \lim_{x \to \frac{3}{2}\pi} \frac{(2\cos^3 x)'}{(1 + \sin x)'} = \lim_{x \to \frac{3}{2}\pi} \frac{\qquad}{\cos x}$$

$\big((\text{nešto})^n\big)' = n(\text{nešto})^{n-1} \cdot (\text{nešto})'$

$(x^n)' = nx^{n-1}$

$\cos^3 x = (\cos x)^3$

$\frac{0}{0}$

L'Hospitalovo pravilo

$$= \lim_{x \to \frac{3}{2}\pi} \frac{2\cos^3 x}{1 + \sin x} = \lim_{x \to \frac{3}{2}\pi} \frac{(2\cos^3 x)'}{(1 + \sin x)'} = \lim_{x \to \frac{3}{2}\pi} \frac{2 \cdot 3\cos^2 x}{\cos x}$$

$\left((\text{nešto})^n\right)' = n(\text{nešto})^{n-1} \cdot (\text{nešto})'$

$(x^n)' = nx^{n-1}$

$\cos^3 x = (\cos x)^3$

$\frac{0}{0}$

L'Hospitalovo pravilo

$$= \lim_{x \to \frac{3}{2}\pi} \frac{2\cos^3 x}{1 + \sin x} = \lim_{x \to \frac{3}{2}\pi} \frac{(2\cos^3 x)'}{(1 + \sin x)'} = \lim_{x \to \frac{3}{2}\pi} \frac{2 \cdot 3\cos^2 x \cdot (\cos x)'}{\cos x}$$

$\big((\text{nešto})^n\big)' = n(\text{nešto})^{n-1} \cdot (\text{nešto})'$

$(x^n)' = nx^{n-1}$

$\cos^3 x = (\cos x)^3$

$\frac{0}{0}$ L'Hospitalovo pravilo

$$= \lim_{x \to \frac{3}{2}\pi} \frac{2\cos^3 x}{1 + \sin x} = \lim_{x \to \frac{3}{2}\pi} \frac{(2\cos^3 x)'}{(1 + \sin x)'} = \lim_{x \to \frac{3}{2}\pi} \frac{2 \cdot 3\cos^2 x \cdot (\cos x)'}{\cos x} =$$

$$= \lim_{x \to \frac{3}{2}\pi} \frac{}{}$$

$\left((\text{nešto})^n\right)' = n(\text{nešto})^{n-1} \cdot (\text{nešto})'$ $(x^n)' = nx^{n-1}$

$\cos^3 x = (\cos x)^3$

$\frac{0}{0}$ L'Hospitalovo pravilo

$$= \lim_{x\to\frac{3}{2}\pi} \frac{2\cos^3 x}{1+\sin x} = \lim_{x\to\frac{3}{2}\pi} \frac{(2\cos^3 x)'}{(1+\sin x)'} = \lim_{x\to\frac{3}{2}\pi} \frac{2\cdot 3\cos^2 x\cdot(\cos x)'}{\cos x} =$$

$$= \lim_{x\to\frac{3}{2}\pi} \frac{}{\cos x}$$

$\left((\text{nešto})^n\right)' = n(\text{nešto})^{n-1}\cdot(\text{nešto})'$ $(x^n)' = nx^{n-1}$

$\cos^3 x = (\cos x)^3$

$\frac{0}{0}$

L'Hospitalovo pravilo

$$= \lim_{x\to\frac{3}{2}\pi} \frac{2\cos^3 x}{1+\sin x} = \lim_{x\to\frac{3}{2}\pi} \frac{(2\cos^3 x)'}{(1+\sin x)'} = \lim_{x\to\frac{3}{2}\pi} \frac{2\cdot 3\cos^2 x\cdot(\cos x)'}{\cos x} =$$

$$= \lim_{x\to\frac{3}{2}\pi} \frac{6\cos^2 x}{\cos x}$$

$\left((\text{nešto})^n\right)' = n(\text{nešto})^{n-1}\cdot(\text{nešto})'$

$(x^n)' = nx^{n-1}$

$\cos^3 x = (\cos x)^3$

$\frac{0}{0}$ L'Hospitalovo pravilo

$$= \lim_{x\to\frac{3}{2}\pi} \frac{2\cos^3 x}{1+\sin x} = \lim_{x\to\frac{3}{2}\pi} \frac{(2\cos^3 x)'}{(1+\sin x)'} = \lim_{x\to\frac{3}{2}\pi} \frac{2\cdot 3\cos^2 x\cdot(\cos x)'}{\cos x} =$$

$$= \lim_{x\to\frac{3}{2}\pi} \frac{6\cos^2 x\cdot(-\sin x)}{\cos x}$$

$\big((\text{nešto})^n\big)' = n(\text{nešto})^{n-1}\cdot(\text{nešto})'$ $(x^n)' = nx^{n-1}$

$\cos^3 x = (\cos x)^3$

$\frac{0}{0}$

L'Hospitalovo pravilo

$$= \lim_{x\to\frac{3}{2}\pi} \frac{2\cos^3 x}{1+\sin x} = \lim_{x\to\frac{3}{2}\pi} \frac{(2\cos^3 x)'}{(1+\sin x)'} = \lim_{x\to\frac{3}{2}\pi} \frac{2\cdot 3\cos^2 x\cdot(\cos x)'}{\cos x} =$$

$$= \lim_{x\to\frac{3}{2}\pi} \frac{6\cos^2 x\cdot(-\sin x)}{\cos x} = \lim_{x\to\frac{3}{2}\pi} \Big(\qquad\qquad \Big)$$

$\big((\text{nešto})^n\big)' = n(\text{nešto})^{n-1}\cdot(\text{nešto})'$

$(x^n)' = nx^{n-1}$

$\cos^3 x = (\cos x)^3$

$\frac{0}{0}$ L'Hospitalovo pravilo

$$= \lim_{x \to \frac{3}{2}\pi} \frac{2\cos^3 x}{1 + \sin x} = \lim_{x \to \frac{3}{2}\pi} \frac{(2\cos^3 x)'}{(1 + \sin x)'} = \lim_{x \to \frac{3}{2}\pi} \frac{2 \cdot 3\cos^2 x \cdot (\cos x)'}{\cos x} =$$

$$= \lim_{x \to \frac{3}{2}\pi} \frac{6\cos^2 x \cdot (-\sin x)}{\cos x} = \lim_{x \to \frac{3}{2}\pi} \left(-6\sin x \qquad \right)$$

$\left((\text{nešto})^n\right)' = n(\text{nešto})^{n-1} \cdot (\text{nešto})'$ $\quad (x^n)' = nx^{n-1}$

$\cos^3 x = (\cos x)^3$

$\frac{0}{0}$ L'Hospitalovo pravilo

$$= \lim_{x\to\frac{3}{2}\pi} \frac{2\cos^3 x}{1+\sin x} = \lim_{x\to\frac{3}{2}\pi} \frac{(2\cos^3 x)'}{(1+\sin x)'} = \lim_{x\to\frac{3}{2}\pi} \frac{2\cdot 3\cos^2 x\cdot(\cos x)'}{\cos x} =$$

$$= \lim_{x\to\frac{3}{2}\pi} \frac{6\cos^2 x\cdot(-\sin x)}{\cos x} = \lim_{x\to\frac{3}{2}\pi} (-6\sin x\cos x)$$

$\left((\text{nešto})^n\right)' = n(\text{nešto})^{n-1}\cdot(\text{nešto})'$ $(x^n)' = nx^{n-1}$

$\cos^3 x = (\cos x)^3$

$\frac{0}{0}$

L'Hospitalovo pravilo

$$= \lim_{x\to\frac{3}{2}\pi} \frac{2\cos^3 x}{1+\sin x} = \lim_{x\to\frac{3}{2}\pi} \frac{(2\cos^3 x)'}{(1+\sin x)'} = \lim_{x\to\frac{3}{2}\pi} \frac{2\cdot 3\cos^2 x\cdot(\cos x)'}{\cos x} =$$

$$= \lim_{x\to\frac{3}{2}\pi} \frac{6\cos^2 x\cdot(-\sin x)}{\cos x} = \lim_{x\to\frac{3}{2}\pi} (-6\sin x\cos x) =$$

$$= -6\cdot\sin\frac{3}{2}\pi\cdot\cos\frac{3}{2}\pi$$

$\big((\text{nešto})^n\big)' = n(\text{nešto})^{n-1}\cdot(\text{nešto})'$

$(x^n)' = nx^{n-1}$

$\cos^3 x = (\cos x)^3$

$\frac{0}{0}$ L'Hospitalovo pravilo

$$= \lim_{x\to\frac{3}{2}\pi} \frac{2\cos^3 x}{1+\sin x} = \lim_{x\to\frac{3}{2}\pi} \frac{(2\cos^3 x)'}{(1+\sin x)'} = \lim_{x\to\frac{3}{2}\pi} \frac{2\cdot 3\cos^2 x\cdot(\cos x)'}{\cos x} =$$

$$= \lim_{x\to\frac{3}{2}\pi} \frac{6\cos^2 x\cdot(-\sin x)}{\cos x} = \lim_{x\to\frac{3}{2}\pi} (-6\sin x\cos x) =$$

$$= -6\cdot\sin\frac{3}{2}\pi\cdot\cos\frac{3}{2}\pi = -6\cdot(-1)\cdot 0$$

$\left((\text{nešto})^n\right)' = n(\text{nešto})^{n-1}\cdot(\text{nešto})'$ $\quad (x^n)' = nx^{n-1}$

$\cos^3 x = (\cos x)^3$

$\frac{0}{0}$ L'Hospitalovo pravilo

$$= \lim_{x \to \frac{3}{2}\pi} \frac{2\cos^3 x}{1 + \sin x} = \lim_{x \to \frac{3}{2}\pi} \frac{(2\cos^3 x)'}{(1 + \sin x)'} = \lim_{x \to \frac{3}{2}\pi} \frac{2 \cdot 3\cos^2 x \cdot (\cos x)'}{\cos x} =$$

$$= \lim_{x \to \frac{3}{2}\pi} \frac{6\cos^2 x \cdot (-\sin x)}{\cos x} = \lim_{x \to \frac{3}{2}\pi} (-6\sin x \cos x) =$$

$$= -6 \cdot \sin \frac{3}{2}\pi \cdot \cos \frac{3}{2}\pi = -6 \cdot (-1) \cdot 0 = 0$$

$\left((\text{nešto})^n\right)' = n(\text{nešto})^{n-1} \cdot (\text{nešto})'$ $(x^n)' = nx^{n-1}$

- Dakle, dobili smo

$$\lim_{x \to \frac{3}{2}\pi} \left(\ln (1 + \sin x)^{2 \operatorname{ctg} x} \right) = 0.$$

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- Limes i neprekidna funkcija komutiraju pa slijedi

$$\ln \left(\lim_{x\to\frac{3}{2}\pi} (1+\sin x)^{2\operatorname{ctg} x} \right) = 0.$$

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$$\lim_{x \to \frac{3}{2}\pi} (1 + \sin x)^{2 \operatorname{ctg} x} = e^0 = 1.$$

šesti zadatak

Zadatak 6

Neka je $a > 0$ proizvoljni realni broj. Izračunajte limes

$$\lim_{x \to 0+} x^{\frac{\ln a}{\ln x}}.$$

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Rješenje

- $\lim\limits_{x \to 0+} x =$

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$$\lim_{x \to 0+} x^{\frac{\ln a}{\ln x}}.$$

Rješenje

- $\lim\limits_{x \to 0+} x = 0$

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Rješenje

- $\lim\limits_{x \to 0+} x = 0$
- $\lim\limits_{x \to 0+} \dfrac{\ln a}{\ln x} = \dfrac{\ln a}{-\infty} = 0$
- Radi se o neodređenom obliku 0^0. Međutim, njega ćemo lako riješiti elementarnim transformacijama bez upotrebe L'Hospitalovog pravila.

$$\lim_{x \to 0+} x^{\frac{\ln a}{\ln x}} =$$

0^0

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0^0

$$\lim_{x \to 0+} x^{\frac{\ln a}{\ln x}} = \lim_{x \to 0+}$$

$$a^{\log_a x} = x$$

$$0^0$$

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$$0^0$$

$$\lim_{x \to 0+} x^{\frac{\ln a}{\ln x}} = \lim_{x \to 0+} e^{\ln x^{\frac{\ln a}{\ln x}}}$$

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$$\lim_{x\to 0+} x^{\frac{\ln a}{\ln x}} = \lim_{x\to 0+} e^{\ln x^{\frac{\ln a}{\ln x}}} = \lim_{x\to 0+}$$

$\log_a x^k = k \log_a x$

$a^{\log_a x} = x$

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- Navedeni primjer pokazuje kako neodređeni izraz 0^0 može poprimiti bilo koju pozitivnu realnu vrijednost.

Napomena

- Iako je 0^0 neodređeni izraz, u matematici se svejedno definira da je $0^0 = 1$. Za to postoje opravdani razlozi. Jedan od jednostavnih razloga koje ovdje možemo lako objasniti je binomni teorem

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^k b^{n-k}.$$

  Ako stavimo $a = 0$ i $b = 1$, tada dobivamo da je lijeva strana jednaka $1^n = 1$, a desna je jednaka 0^0. Ako želimo da taj teorem vrijedi za ovaj slučaj, tada moramo definirati da je $0^0 = 1$.

- Riječima rečeno, *nula na nultu jednako je jedan*. U ovom slučaju se ovdje ne radi o neodređenom izrazu.

Napomena

- U slučaju računanja limesa, izraz 0^0 također radi jednostavnosti čitamo *nula na nultu*. Međutim, u ovom kontekstu 0^0 ima potpuno drukčije značenje.
- U kontekstu limesa, 0^0 je preciznije čitati *beskonačno mali broj na beskonačno mali broj*. Drugim riječima, baza i eksponent nisu doslovno jednaki nula, već su to beskonačno male veličine, tj. brojevi koji su po volji jako blizu broja nula.
- Prethodni zadatak pokazuje kako u tom slučaju ne možemo definirati čemu je jednako 0^0 jer je to zaista neodređeni izraz koji može poprimiti bilo koju pozitivnu realnu vrijednost, ovisno o odnosu beskonačno malih veličina u bazi i eksponentu.

sedmi zadatak

Zadatak 7

Odredite jednadžbu tangente na graf funkcije $f(x) = x - x^2$ *koja je paralelna s pravcem* $y = \frac{3}{4}x - 1$.

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Rješenje

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Rješenje

- $p \ldots y = \frac{3}{4}x - 1, \quad t \ldots y - y_0 = k_t(x - x_0)$

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Rješenje

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$$1 - 2x$$

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Zadatak 7

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$$x = \frac{1}{8}$$

$$y_0 =$$

Zadatak 7

Odredite jednadžbu tangente na graf funkcije $f(x) = x - x^2$ *koja je paralelna s pravcem* $y = \frac{3}{4}x - 1$.

Rješenje

- $p \ldots y = \frac{3}{4}x - 1, \quad t \ldots y - y_0 = k_t(x - x_0), \quad k_t = f'(x_0)$
- $t \parallel p \implies k_t = k_p \implies \boxed{k_t = \frac{3}{4}} \qquad \boxed{x_0 = \frac{1}{8}}$

$$\begin{aligned} f'(x) &= k_t \\ (x - x^2)' &= \frac{3}{4} \\ 1 - 2x &= \frac{3}{4} \quad / \cdot 4 \\ 4 - 8x &= 3 \\ -8x &= -1 \\ x &= \frac{1}{8} \end{aligned}$$

$$y_0 = f(x_0)$$

Zadatak 7

Odredite jednadžbu tangente na graf funkcije $f(x) = x - x^2$ koja je paralelna s pravcem $y = \frac{3}{4}x - 1$.

Rješenje

- $p \ldots y = \frac{3}{4}x - 1, \quad t \ldots y - y_0 = k_t(x - x_0), \quad k_t = f'(x_0)$
- $t \parallel p \implies k_t = k_p \implies \boxed{k_t = \frac{3}{4}}$ $\qquad \boxed{x_0 = \frac{1}{8}}$

$$f'(x) = k_t$$
$$(x - x^2)' = \frac{3}{4}$$
$$1 - 2x = \frac{3}{4} \quad / \cdot 4$$
$$4 - 8x = 3$$
$$-8x = -1$$
$$x = \frac{1}{8}$$

$$y_0 = f(x_0) = \frac{1}{8} - \left(\frac{1}{8}\right)^2$$

Zadatak 7

Odredite jednadžbu tangente na graf funkcije $f(x) = x - x^2$ *koja je paralelna s pravcem* $y = \frac{3}{4}x - 1$.

Rješenje

- $p \ldots y = \frac{3}{4}x - 1, \quad t \ldots y - y_0 = k_t(x - x_0), \quad k_t = f'(x_0)$
- $t \parallel p \implies k_t = k_p \implies \boxed{k_t = \frac{3}{4}}$ $\quad \boxed{x_0 = \frac{1}{8}}$

$$
\begin{aligned}
f'(x) &= k_t \\
(x - x^2)' &= \frac{3}{4} \\
1 - 2x &= \frac{3}{4} \quad / \cdot 4 \\
4 - 8x &= 3 \\
-8x &= -1 \\
x &= \frac{1}{8}
\end{aligned}
$$

$$y_0 = f(x_0) = \frac{1}{8} - \left(\frac{1}{8}\right)^2 = \frac{7}{64}$$

Zadatak 7

Odredite jednadžbu tangente na graf funkcije $f(x) = x - x^2$ *koja je paralelna s pravcem* $y = \frac{3}{4}x - 1$.

Rješenje

- $p \dots y = \frac{3}{4}x - 1, \quad t \dots y - y_0 = k_t(x - x_0), \quad k_t = f'(x_0)$
- $t \parallel p \implies k_t = k_p \implies \boxed{k_t = \frac{3}{4}} \qquad \boxed{x_0 = \frac{1}{8}} \qquad y_0 = \frac{7}{64}$

$$f'(x) = k_t$$

$$(x - x^2)' = \frac{3}{4}$$

$$1 - 2x = \frac{3}{4} \quad / \cdot 4$$

$$4 - 8x = 3$$

$$-8x = -1$$

$$x = \frac{1}{8}$$

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Zadatak 7

Odredite jednadžbu tangente na graf funkcije $f(x) = x - x^2$ *koja je paralelna s pravcem* $y = \frac{3}{4}x - 1$.

Rješenje

- $p \ldots y = \frac{3}{4}x - 1, \quad t \ldots y - y_0 = k_t(x - x_0), \quad k_t = f'(x_0)$
- $t \parallel p \implies k_t = k_p \implies \boxed{k_t = \frac{3}{4}} \qquad \boxed{x_0 = \frac{1}{8}} \qquad \boxed{y_0 = \frac{7}{64}}$

$$f'(x) = k_t$$

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Zadatak 7

Odredite jednadžbu tangente na graf funkcije $f(x) = x - x^2$ *koja je paralelna s pravcem* $y = \frac{3}{4}x - 1$.

Rješenje

- $p \ldots y = \frac{3}{4}x - 1, \quad t \ldots y - y_0 = k_t(x - x_0), \quad k_t = f'(x_0)$
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$$(x - x^2)' = \frac{3}{4}$$

$$1 - 2x = \frac{3}{4} \quad / \cdot 4$$

$$4 - 8x = 3$$

$$-8x = -1$$

$$x = \frac{1}{8}$$

$$y_0 = f(x_0) = \frac{1}{8} - \left(\frac{1}{8}\right)^2 = \frac{7}{64}$$

$$y - \frac{7}{64} = \frac{3}{4}\left(x - \frac{1}{8}\right)$$

Zadatak 7

Odredite jednadžbu tangente na graf funkcije $f(x) = x - x^2$ *koja je paralelna s pravcem* $y = \frac{3}{4}x - 1$.

Rješenje

- $p \ldots y = \frac{3}{4}x - 1, \quad t \ldots y - y_0 = k_t(x - x_0), \quad k_t = f'(x_0)$
- $t \parallel p \implies k_t = k_p \implies \boxed{k_t = \frac{3}{4}} \qquad \boxed{x_0 = \frac{1}{8}} \qquad \boxed{y_0 = \frac{7}{64}}$

$$f'(x) = k_t$$

$$(x - x^2)' = \frac{3}{4}$$

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$$y_0 = f(x_0) = \frac{1}{8} - \left(\frac{1}{8}\right)^2 = \frac{7}{64}$$

$$y - \frac{7}{64} = \frac{3}{4}\left(x - \frac{1}{8}\right)$$

$$y =$$

Zadatak 7

Odredite jednadžbu tangente na graf funkcije $f(x) = x - x^2$ *koja je paralelna s pravcem* $y = \frac{3}{4}x - 1$.

Rješenje

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- $t \parallel p \implies k_t = k_p \implies \boxed{k_t = \frac{3}{4}} \qquad \boxed{x_0 = \frac{1}{8}} \qquad \boxed{y_0 = \frac{7}{64}}$

$$f'(x) = k_t$$

$$(x - x^2)' = \frac{3}{4}$$

$$1 - 2x = \frac{3}{4} \quad / \cdot 4$$

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$$-8x = -1$$

$$x = \frac{1}{8}$$

$$y_0 = f(x_0) = \frac{1}{8} - \left(\frac{1}{8}\right)^2 = \frac{7}{64}$$

$$y - \frac{7}{64} = \frac{3}{4}\left(x - \frac{1}{8}\right)$$

$$y = \frac{3}{4}x$$

Zadatak 7

Odredite jednadžbu tangente na graf funkcije $f(x) = x - x^2$ *koja je paralelna s pravcem* $y = \frac{3}{4}x - 1$.

Rješenje

- $p \ldots y = \frac{3}{4}x - 1, \quad t \ldots y - y_0 = k_t(x - x_0), \quad k_t = f'(x_0)$
- $t \parallel p \implies k_t = k_p \implies \boxed{k_t = \frac{3}{4}} \qquad \boxed{x_0 = \frac{1}{8}} \qquad \boxed{y_0 = \frac{7}{64}}$

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$$y - \frac{7}{64} = \frac{3}{4}\left(x - \frac{1}{8}\right)$$

$$y = \frac{3}{4}x - \frac{3}{32}$$

Zadatak 7

Odredite jednadžbu tangente na graf funkcije $f(x) = x - x^2$ *koja je paralelna s pravcem* $y = \frac{3}{4}x - 1$.

Rješenje

- $p \ldots y = \frac{3}{4}x - 1, \quad t \ldots y - y_0 = k_t(x - x_0), \quad k_t = f'(x_0)$
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$$y - \frac{7}{64} = \frac{3}{4}\left(x - \frac{1}{8}\right)$$

$$y = \frac{3}{4}x - \frac{3}{32} + \frac{7}{64}$$

Zadatak 7

Odredite jednadžbu tangente na graf funkcije $f(x) = x - x^2$ *koja je paralelna s pravcem* $y = \frac{3}{4}x - 1$.

Rješenje

- $p \ldots y = \frac{3}{4}x - 1, \quad t \ldots y - y_0 = k_t(x - x_0), \quad k_t = f'(x_0)$
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$$y = \frac{3}{4}x - \frac{3}{32} + \frac{7}{64}$$

$$y =$$

Zadatak 7

Odredite jednadžbu tangente na graf funkcije $f(x) = x - x^2$ *koja je paralelna s pravcem* $y = \frac{3}{4}x - 1$.

Rješenje

- $p \ldots y = \frac{3}{4}x - 1, \quad t \ldots y - y_0 = k_t(x - x_0), \quad k_t = f'(x_0)$
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$$y = \frac{3}{4}x - \frac{3}{32} + \frac{7}{64}$$

$$y = \frac{3}{4}x + \frac{1}{64}$$

Zadatak 7

Odredite jednadžbu tangente na graf funkcije $f(x) = x - x^2$ *koja je paralelna s pravcem* $y = \frac{3}{4}x - 1$.

Rješenje

- $p \ldots y = \frac{3}{4}x - 1, \quad t \ldots y - y_0 = k_t(x - x_0), \quad k_t = f'(x_0)$
- $t \parallel p \implies k_t = k_p \implies \boxed{k_t = \frac{3}{4}} \qquad \boxed{x_0 = \frac{1}{8}} \qquad \boxed{y_0 = \frac{7}{64}}$

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$$y = \frac{3}{4}x - \frac{3}{32} + \frac{7}{64}$$

$$y = \frac{3}{4}x + \frac{1}{64}$$

$$t \ldots y = \frac{3}{4}x + \frac{1}{64}$$

Zadatak 7

Odredite jednadžbu tangente na graf funkcije $f(x) = x - x^2$ *koja je paralelna s pravcem* $y = \frac{3}{4}x - 1$.

Rješenje

- $p \ldots y = \frac{3}{4}x - 1, \quad t \ldots y - y_0 = k_t(x - x_0), \quad k_t = f'(x_0)$
- $t \parallel p \implies k_t = k_p \implies \boxed{k_t = \frac{3}{4}} \qquad \boxed{x_0 = \frac{1}{8}} \qquad \boxed{y_0 = \frac{7}{64}}$

$$f'(x) = k_t$$

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$$1 - 2x = \frac{3}{4} \quad / \cdot 4$$

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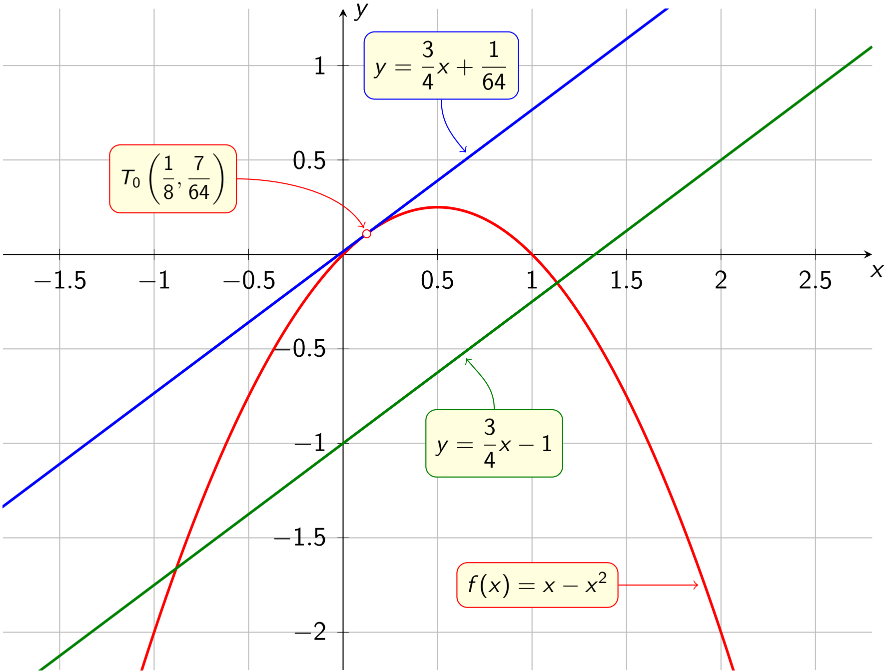
$$y - \frac{7}{64} = \frac{3}{4}\left(x - \frac{1}{8}\right)$$

$$y = \frac{3}{4}x - \frac{3}{32} + \frac{7}{64}$$

$$y = \frac{3}{4}x + \frac{1}{64}$$

$$\boxed{t \ldots y = \frac{3}{4}x + \frac{1}{64}}$$



$y = \frac{3}{4}x + \frac{1}{64}$
$T_0\left(\frac{1}{8}, \frac{7}{64}\right)$
$y = \frac{3}{4}x - 1$
$f(x) = x - x^2$
x
y
−1.5
−1
−0.5
0.5
1
1.5
2
2.5
−2

osmi zadatak

Zadatak 8

Zadana je kružnica $x^2 + y^2 = 10$ *i parabola* $y = 3x^2 - 13x + 13$.

a) *Odredite kut između zadanih krivulja u točki njihovog presjeka s apscisom* 1.

b) *Odredite kut između zadanih krivulja u točki njihovog presjeka s apscisom* 3.

Rješenje

- Ako je $x_0 = 1$, tada iz $y = 3x^2 - 13x + 13$ slijedi

$$y_0 = 3 \cdot 1^2 - 13 \cdot 1 + 13 = 3.$$

Rješenje

- Ako je $x_0 = 1$, tada iz $y = 3x^2 - 13x + 13$ slijedi

$$y_0 = 3 \cdot 1^2 - 13 \cdot 1 + 13 = 3.$$

  Kako je $1^2 + 3^2 = 10$, točka $A(1, 3)$ također leži na kružnici $x^2 + y^2 = 10$.

Rješenje

- Ako je $x_0 = 1$, tada iz $y = 3x^2 - 13x + 13$ slijedi

$$y_0 = 3 \cdot 1^2 - 13 \cdot 1 + 13 = 3.$$

  Kako je $1^2 + 3^2 = 10$, točka $A(1, 3)$ također leži na kružnici $x^2 + y^2 = 10$.

- Ako je $x_0 = 3$, tada iz $y = 3x^2 - 13x + 13$ slijedi

$$y_0 = 3 \cdot 3^2 - 13 \cdot 3 + 13 = 1.$$

Rješenje

- Ako je $x_0 = 1$, tada iz $y = 3x^2 - 13x + 13$ slijedi

$$y_0 = 3 \cdot 1^2 - 13 \cdot 1 + 13 = 3.$$

  Kako je $1^2 + 3^2 = 10$, točka $A(1, 3)$ također leži na kružnici $x^2 + y^2 = 10$.

- Ako je $x_0 = 3$, tada iz $y = 3x^2 - 13x + 13$ slijedi

$$y_0 = 3 \cdot 3^2 - 13 \cdot 3 + 13 = 1.$$

  Kako je $3^2 + 1^2 = 10$, točka $B(3, 1)$ također leži na kružnici $x^2 + y^2 = 10$.

Rješenje

- Ako je $x_0 = 1$, tada iz $y = 3x^2 - 13x + 13$ slijedi

$$y_0 = 3 \cdot 1^2 - 13 \cdot 1 + 13 = 3.$$

  Kako je $1^2 + 3^2 = 10$, točka $A(1,3)$ također leži na kružnici $x^2 + y^2 = 10$.

- Ako je $x_0 = 3$, tada iz $y = 3x^2 - 13x + 13$ slijedi

$$y_0 = 3 \cdot 3^2 - 13 \cdot 3 + 13 = 1.$$

  Kako je $3^2 + 1^2 = 10$, točka $B(3,1)$ također leži na kružnici $x^2 + y^2 = 10$.

- Dakle, trebamo pronaći pod kojim kutovima se sijeku zadane krivulje u točkama $A(1,3)$ i $B(3,1)$.

$x^2 + y^2 = 10$

$$\operatorname{tg} \varphi = \left| \frac{k_1 - k_2}{1 + k_1 k_2} \right|$$

$x^2 + y^2 = 10 \;\Big/ \frac{\mathrm{d}}{\mathrm{d}x}$

$$\operatorname{tg} \varphi = \left| \frac{k_1 - k_2}{1 + k_1 k_2} \right|$$

$$x^2 + y^2 = 10 \;\Big/ \frac{\mathrm{d}}{\mathrm{d}x}$$

$2x$

$$\operatorname{tg}\varphi = \left|\frac{k_1 - k_2}{1 + k_1 k_2}\right|$$

$$x^2 + y^2 = 10 \;\Big/ \frac{\mathrm{d}}{\mathrm{d}x}$$

$$2x +$$

$$\operatorname{tg}\varphi = \left| \frac{k_1 - k_2}{1 + k_1 k_2} \right|$$

$$x^2 + y^2 = 10 \;\Big/ \frac{\mathrm{d}}{\mathrm{d}x}$$

$$2x + 2yy'$$

$$\operatorname{tg} \varphi = \left| \frac{k_1 - k_2}{1 + k_1 k_2} \right|$$

$$x^2 + y^2 = 10 \,\Big/\, \frac{\mathrm{d}}{\mathrm{d}x}$$

$$2x + 2yy'$$

$$y' = \frac{\mathrm{d}y}{\mathrm{d}x}$$

$$\operatorname{tg}\varphi = \left|\frac{k_1 - k_2}{1 + k_1 k_2}\right|$$

$$x^2 + y^2 = 10 \;\Big/\, \frac{\mathrm{d}}{\mathrm{d}x}$$

$$2x + 2yy' = 0$$

$$y' = \frac{\mathrm{d}y}{\mathrm{d}x}$$

$$\operatorname{tg}\varphi = \left|\frac{k_1 - k_2}{1 + k_1 k_2}\right|$$

$$x^2 + y^2 = 10 \;/\; \frac{\mathrm{d}}{\mathrm{d}x}$$

$$2x + 2yy' = 0 \;/ : 2$$

$$y' = \frac{\mathrm{d}y}{\mathrm{d}x}$$

$$\operatorname{tg} \varphi = \left| \frac{k_1 - k_2}{1 + k_1 k_2} \right|$$

$$x^2 + y^2 = 10 \;\Big/ \frac{\mathrm{d}}{\mathrm{d}x}$$

$$2x + 2yy' = 0 \;\Big/ : 2$$

$$x + yy' = 0$$

$$y' = \frac{\mathrm{d}y}{\mathrm{d}x}$$

$$\operatorname{tg}\varphi = \left| \frac{k_1 - k_2}{1 + k_1 k_2} \right|$$

$$x^2 + y^2 = 10 \;/\, \frac{\mathrm{d}}{\mathrm{d}x}$$

$$2x + 2yy' = 0 \;/ : 2$$

$$x + yy' = 0$$

$$yy' =$$

$$y' = \frac{\mathrm{d}y}{\mathrm{d}x}$$

$$\operatorname{tg} \varphi = \left| \frac{k_1 - k_2}{1 + k_1 k_2} \right|$$

$$x^2 + y^2 = 10 \; \Big/ \frac{\mathrm{d}}{\mathrm{d}x}$$

$$2x + 2yy' = 0 \; \Big/ : 2$$

$$x + yy' = 0$$

$$yy' = -x$$

$$y' = \frac{\mathrm{d}y}{\mathrm{d}x}$$

$$\operatorname{tg} \varphi = \left| \frac{k_1 - k_2}{1 + k_1 k_2} \right|$$

$$x^2 + y^2 = 10 \;/ \frac{\mathrm{d}}{\mathrm{d}x}$$

$$2x + 2yy' = 0 \;/ : 2$$

$$x + yy' = 0$$

$$yy' = -x$$

$$y' =$$

$$y' = \frac{\mathrm{d}y}{\mathrm{d}x}$$

$$\operatorname{tg}\varphi = \left|\frac{k_1 - k_2}{1 + k_1 k_2}\right|$$

$$x^2 + y^2 = 10 \;\Big/ \frac{\mathrm{d}}{\mathrm{d}x}$$

$$2x + 2yy' = 0 \;\Big/ : 2$$

$$x + yy' = 0$$

$$yy' = -x$$

$$y' = -\frac{x}{y}$$

$$y' = \frac{\mathrm{d}y}{\mathrm{d}x}$$

$$\operatorname{tg}\varphi = \left| \frac{k_1 - k_2}{1 + k_1 k_2} \right|$$

$$x^2 + y^2 = 10 \;\Big/ \frac{\mathrm{d}}{\mathrm{d}x}$$

$$2x + 2yy' = 0 \;\Big/ : 2$$

$$x + yy' = 0$$

$$yy' = -x$$

$$y' = -\frac{x}{y}$$

$$y' = \frac{\mathrm{d}y}{\mathrm{d}x}$$

$$k_1 = -\frac{x}{y}$$

$$\operatorname{tg}\varphi = \left|\frac{k_1 - k_2}{1 + k_1 k_2}\right|$$

$$x^2 + y^2 = 10 \;\Big/ \frac{\mathrm{d}}{\mathrm{d}x}$$

$$2x + 2yy' = 0 \;\Big/ : 2$$

$$x + yy' = 0$$

$$yy' = -x$$

$$y' = -\frac{x}{y}$$

$$y' = \frac{\mathrm{d}y}{\mathrm{d}x}$$

$$\boxed{k_1 = -\frac{x}{y}}$$

$$\operatorname{tg}\varphi = \left|\frac{k_1 - k_2}{1 + k_1 k_2}\right|$$

$$x^2 + y^2 = 10 \;\Big/ \frac{\mathrm{d}}{\mathrm{d}x}$$

$$2x + 2yy' = 0 \;\Big/ :2$$

$$x + yy' = 0$$

$$yy' = -x$$

$$y' = -\frac{x}{y}$$

$$y' = \frac{\mathrm{d}y}{\mathrm{d}x}$$

$$\boxed{k_1 = -\frac{x}{y}}$$

$$y = 3x^2 - 13x + 13$$

$$\operatorname{tg}\varphi = \left|\frac{k_1 - k_2}{1 + k_1 k_2}\right|$$

$$x^2 + y^2 = 10 \ / \frac{\mathrm{d}}{\mathrm{d}x}$$

$$2x + 2yy' = 0 \ /:2$$

$$x + yy' = 0$$

$$yy' = -x$$

$$y' = -\frac{x}{y}$$

$$y' = \frac{\mathrm{d}y}{\mathrm{d}x}$$

$$\boxed{k_1 = -\frac{x}{y}}$$

$$y = 3x^2 - 13x + 13$$

$$y' =$$

$$\operatorname{tg}\varphi = \left|\frac{k_1 - k_2}{1 + k_1 k_2}\right|$$

$$x^2 + y^2 = 10 \;\Big/ \frac{\mathrm{d}}{\mathrm{d}x}$$

$$2x + 2yy' = 0 \;\Big/ :2$$

$$x + yy' = 0$$

$$yy' = -x$$

$$y' = -\frac{x}{y}$$

$$y' = \frac{\mathrm{d}y}{\mathrm{d}x}$$

$$\boxed{k_1 = -\frac{x}{y}}$$

$$y = 3x^2 - 13x + 13$$

$$y' = 6x - 13$$

$$\operatorname{tg}\varphi = \left|\frac{k_1 - k_2}{1 + k_1 k_2}\right|$$

$$x^2 + y^2 = 10 \;\Big/ \frac{\mathrm{d}}{\mathrm{d}x}$$

$$2x + 2yy' = 0 \;\Big/ : 2$$

$$x + yy' = 0$$

$$yy' = -x$$

$$y' = -\frac{x}{y}$$

$$y' = \frac{\mathrm{d}y}{\mathrm{d}x}$$

$$\boxed{k_1 = -\frac{x}{y}}$$

$$y = 3x^2 - 13x + 13$$

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$$k_2 = 6x - 13$$

$$\operatorname{tg} \varphi = \left| \frac{k_1 - k_2}{1 + k_1 k_2} \right|$$

$$x^2 + y^2 = 10 \;\Big/ \frac{\mathrm{d}}{\mathrm{d}x}$$

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$$x^2 + y^2 = 10 \;/\, \frac{\mathrm{d}}{\mathrm{d}x}$$

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$$\boxed{k_2 = 6x - 13}$$

$$\operatorname{tg}\varphi = \left|\frac{k_1 - k_2}{1 + k_1 k_2}\right|$$

a)

$$\overset{x_0\ y_0}{A(1, 3)}$$

$$x^2 + y^2 = 10 \;\Big/ \frac{\mathrm{d}}{\mathrm{d}x}$$

$$2x + 2yy' = 0 \;\Big/ :2$$

$$x + yy' = 0$$

$$yy' = -x$$

$$y' = -\frac{x}{y}$$

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$$\boxed{k_1 = -\frac{x}{y}}$$

$$y = 3x^2 - 13x + 13$$

$$y' = 6x - 13$$

$$\boxed{k_2 = 6x - 13}$$

$$\operatorname{tg}\varphi = \left|\frac{k_1 - k_2}{1 + k_1 k_2}\right|$$

a)

$$k_1 =$$

$$A(\overset{x_0}{1}, \overset{y_0}{3})$$

$$x^2 + y^2 = 10 \;\Big/ \frac{\mathrm{d}}{\mathrm{d}x}$$

$$2x + 2yy' = 0 \;\Big/ :2$$

$$x + yy' = 0$$

$$yy' = -x$$

$$y' = -\frac{x}{y}$$

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$$y = 3x^2 - 13x + 13$$

$$y' = 6x - 13$$

$$\boxed{k_2 = 6x - 13}$$

$$\operatorname{tg}\varphi = \left|\frac{k_1 - k_2}{1 + k_1 k_2}\right|$$

a)

$$k_1 = -\frac{x_0}{y_0}$$

$$\overset{x_0\ y_0}{A(1, 3)}$$

$$x^2 + y^2 = 10 \;/\; \frac{\mathrm{d}}{\mathrm{d}x}$$

$$2x + 2yy' = 0 \;/ : 2$$

$$x + yy' = 0$$

$$yy' = -x$$

$$y' = -\frac{x}{y}$$

$$y' = \frac{\mathrm{d}y}{\mathrm{d}x}$$

$$\boxed{k_1 = -\frac{x}{y}}$$

$$y = 3x^2 - 13x + 13$$

$$y' = 6x - 13$$

$$\boxed{k_2 = 6x - 13}$$

$$\operatorname{tg} \varphi = \left| \frac{k_1 - k_2}{1 + k_1 k_2} \right|$$

a)

$$k_1 = -\frac{x_0}{y_0} = -\frac{1}{3}$$

$A(1, 3)$ ($x_0 = 1$, $y_0 = 3$)

$$x^2 + y^2 = 10 \;/\frac{\mathrm{d}}{\mathrm{d}x}$$

$$2x + 2yy' = 0 \;/:2$$

$$x + yy' = 0$$

$$yy' = -x$$

$$y' = -\frac{x}{y}$$

$$y' = \frac{\mathrm{d}y}{\mathrm{d}x}$$

$$\boxed{k_1 = -\frac{x}{y}}$$

$$y = 3x^2 - 13x + 13$$

$$y' = 6x - 13$$

$$\boxed{k_2 = 6x - 13}$$

$$\operatorname{tg}\varphi = \left|\frac{k_1 - k_2}{1 + k_1 k_2}\right|$$

a)

$$k_1 = -\frac{x_0}{y_0} = -\frac{1}{3}$$

$$k_2 =$$

$A(1, 3)$ ($x_0 = 1$, $y_0 = 3$)

$$x^2 + y^2 = 10 \;/\, \frac{\mathrm{d}}{\mathrm{d}x}$$

$$2x + 2yy' = 0 \;/:2$$

$$x + yy' = 0$$

$$yy' = -x$$

$$y' = -\frac{x}{y}$$

$$y' = \frac{\mathrm{d}y}{\mathrm{d}x}$$

$$\boxed{k_1 = -\frac{x}{y}}$$

$$y = 3x^2 - 13x + 13$$

$$y' = 6x - 13$$

$$\boxed{k_2 = 6x - 13}$$

$$\operatorname{tg}\varphi = \left|\frac{k_1 - k_2}{1 + k_1 k_2}\right|$$

a)

$$k_1 = -\frac{x_0}{y_0} = -\frac{1}{3}$$

$$k_2 = 6x_0 - 13$$

x_0 y_0

$A(1, 3)$

$$x^2 + y^2 = 10 \;\Big/ \frac{\mathrm{d}}{\mathrm{d}x}$$

$$2x + 2yy' = 0 \;\Big/ :2$$

$$x + yy' = 0$$

$$yy' = -x$$

$$y' = -\frac{x}{y}$$

$$y' = \frac{\mathrm{d}y}{\mathrm{d}x}$$

$$\boxed{k_1 = -\frac{x}{y}}$$

$$y = 3x^2 - 13x + 13$$

$$y' = 6x - 13$$

$$\boxed{k_2 = 6x - 13}$$

$$\operatorname{tg}\varphi = \left| \frac{k_1 - k_2}{1 + k_1 k_2} \right|$$

a)

$$k_1 = -\frac{x_0}{y_0} = -\frac{1}{3}$$

x_0 y_0
$A(1, 3)$

$$k_2 = 6x_0 - 13 = 6 \cdot 1 - 13$$

$$x^2 + y^2 = 10 \;/\frac{\mathrm{d}}{\mathrm{d}x}$$

$$2x + 2yy' = 0 \;/:2$$

$$x + yy' = 0$$

$$yy' = -x$$

$$y' = -\frac{x}{y}$$

$$y' = \frac{\mathrm{d}y}{\mathrm{d}x}$$

$$\boxed{k_1 = -\frac{x}{y}}$$

$$y = 3x^2 - 13x + 13$$

$$y' = 6x - 13$$

$$\boxed{k_2 = 6x - 13}$$

$$\operatorname{tg}\varphi = \left|\frac{k_1 - k_2}{1 + k_1 k_2}\right|$$

a)

$$A(\overset{x_0}{1}, \overset{y_0}{3})$$

$$k_1 = -\frac{x_0}{y_0} = -\frac{1}{3}$$

$$k_2 = 6x_0 - 13 = 6 \cdot 1 - 13 = -7$$

$$x^2 + y^2 = 10 \;/\, \frac{\mathrm{d}}{\mathrm{d}x}$$

$$2x + 2yy' = 0 \;/ :2$$

$$x + yy' = 0$$

$$yy' = -x$$

$$y' = -\frac{x}{y}$$

$$y' = \frac{\mathrm{d}y}{\mathrm{d}x}$$

$$\boxed{k_1 = -\frac{x}{y}}$$

$$y = 3x^2 - 13x + 13$$

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$$\boxed{k_2 = 6x - 13}$$

$$\operatorname{tg}\varphi = \left|\frac{k_1 - k_2}{1 + k_1 k_2}\right|$$

a)

$$k_1 = -\frac{x_0}{y_0} = -\frac{1}{3}$$

x_0 y_0
$A(1, 3)$

$$k_2 = 6x_0 - 13 = 6 \cdot 1 - 13 = -7$$

$$\operatorname{tg}\varphi_1 = \left|\frac{\qquad}{\qquad}\right|$$

$$x^2 + y^2 = 10 \;\Big/ \frac{\mathrm{d}}{\mathrm{d}x}$$

$$2x + 2yy' = 0 \;\Big/ :2$$

$$x + yy' = 0$$

$$yy' = -x$$

$$y' = -\frac{x}{y}$$

$$y' = \frac{\mathrm{d}y}{\mathrm{d}x}$$

$$\boxed{k_1 = -\frac{x}{y}}$$

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$$\boxed{k_2 = 6x - 13}$$

$$\operatorname{tg}\varphi = \left| \frac{k_1 - k_2}{1 + k_1 k_2} \right|$$

a)

$$x_0 \; y_0$$
$$A(1, 3)$$

$$k_1 = -\frac{x_0}{y_0} = -\frac{1}{3}$$

$$k_2 = 6x_0 - 13 = 6 \cdot 1 - 13 = -7$$

$$\operatorname{tg}\varphi_1 = \left| \frac{-\frac{1}{3}}{} \right|$$

$$x^2 + y^2 = 10 \;/\, \frac{\mathrm{d}}{\mathrm{d}x}$$

$$2x + 2yy' = 0 \;/:2$$

$$x + yy' = 0$$

$$yy' = -x$$

$$y' = -\frac{x}{y}$$

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$$y = 3x^2 - 13x + 13$$

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$$\boxed{k_2 = 6x - 13}$$

$$\operatorname{tg}\varphi = \left|\frac{k_1 - k_2}{1 + k_1 k_2}\right|$$

a)

$$k_1 = -\frac{x_0}{y_0} = -\frac{1}{3}$$

$x_0 \; y_0$

$A(1, 3)$

$$k_2 = 6x_0 - 13 = 6 \cdot 1 - 13 = -7$$

$$\operatorname{tg}\varphi_1 = \left|\frac{-\frac{1}{3} - \qquad}{\qquad}\right|$$

$$x^2 + y^2 = 10 \;\Big/ \frac{\mathrm{d}}{\mathrm{d}x}$$

$$2x + 2yy' = 0 \;\Big/ :2$$

$$x + yy' = 0$$

$$yy' = -x$$

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$$\boxed{k_2 = 6x - 13}$$

$$\operatorname{tg}\varphi = \left|\frac{k_1 - k_2}{1 + k_1 k_2}\right|$$

a)

$$x_0 \quad y_0$$
$$A(1, 3)$$

$$k_1 = -\frac{x_0}{y_0} = -\frac{1}{3}$$

$$k_2 = 6x_0 - 13 = 6 \cdot 1 - 13 = -7$$

$$\operatorname{tg}\varphi_1 = \left|\frac{-\frac{1}{3} - (-7)}{}\right|$$

$$x^2 + y^2 = 10 \;/\frac{\mathrm{d}}{\mathrm{d}x}$$

$$2x + 2yy' = 0 \;/:2$$

$$x + yy' = 0$$

$$yy' = -x$$

$$y' = -\frac{x}{y}$$

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$$\operatorname{tg}\varphi = \left|\frac{k_1 - k_2}{1 + k_1 k_2}\right|$$

a)

$$k_1 = -\frac{x_0}{y_0} = -\frac{1}{3}$$

$$A(\overset{x_0}{1}, \overset{y_0}{3})$$

$$k_2 = 6x_0 - 13 = 6 \cdot 1 - 13 = -7$$

$$\operatorname{tg}\varphi_1 = \left|\frac{-\frac{1}{3} - (-7)}{1 + \frac{-1}{3} \cdot (-7)}\right|$$

$$x^2 + y^2 = 10 \;/\frac{\mathrm{d}}{\mathrm{d}x}$$

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$$\operatorname{tg}\varphi = \left|\frac{k_1 - k_2}{1 + k_1 k_2}\right|$$

a)

$$k_1 = -\frac{x_0}{y_0} = -\frac{1}{3}$$

$$A(\overset{x_0}{1}, \overset{y_0}{3})$$

$$k_2 = 6x_0 - 13 = 6 \cdot 1 - 13 = -7$$

$$\operatorname{tg}\varphi_1 = \left|\frac{-\frac{1}{3} - (-7)}{1 + \frac{-1}{3} \cdot (-7)}\right| = \left|\frac{\quad}{\quad}\right|$$

$$x^2 + y^2 = 10 \; \Big/ \frac{\mathrm{d}}{\mathrm{d}x}$$

$$2x + 2yy' = 0 \; \Big/ : 2$$

$$x + yy' = 0$$

$$yy' = -x$$

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$$\operatorname{tg}\varphi = \left| \frac{k_1 - k_2}{1 + k_1 k_2} \right|$$

a)

x_0 y_0
$A(1, 3)$

$$k_1 = -\frac{x_0}{y_0} = -\frac{1}{3}$$

$$k_2 = 6x_0 - 13 = 6 \cdot 1 - 13 = -7$$

$$\operatorname{tg}\varphi_1 = \left| \frac{-\frac{1}{3} - (-7)}{1 + \frac{-1}{3} \cdot (-7)} \right| = \left| \frac{\frac{20}{3}}{} \right|$$

$$x^2 + y^2 = 10 \;/\frac{\mathrm{d}}{\mathrm{d}x}$$

$$2x + 2yy' = 0 \;/:2$$

$$x + yy' = 0$$

$$yy' = -x$$

$$y' = -\frac{x}{y}$$

$$y' = \frac{\mathrm{d}y}{\mathrm{d}x}$$

$$\boxed{k_1 = -\frac{x}{y}}$$

$$y = 3x^2 - 13x + 13$$

$$y' = 6x - 13$$

$$\boxed{k_2 = 6x - 13}$$

$$\operatorname{tg}\varphi = \left|\frac{k_1 - k_2}{1 + k_1 k_2}\right|$$

a)

$$k_1 = -\frac{x_0}{y_0} = -\frac{1}{3}$$

$$A(\overset{x_0}{1}, \overset{y_0}{3})$$

$$k_2 = 6x_0 - 13 = 6 \cdot 1 - 13 = -7$$

$$\operatorname{tg}\varphi_1 = \left|\frac{-\frac{1}{3} - (-7)}{1 + \frac{-1}{3} \cdot (-7)}\right| = \left|\frac{\frac{20}{3}}{\frac{10}{3}}\right|$$

$$x^2 + y^2 = 10 \;\Big/ \frac{\mathrm{d}}{\mathrm{d}x}$$

$$2x + 2yy' = 0 \;\Big/ : 2$$

$$x + yy' = 0$$

$$yy' = -x$$

$$y' = -\frac{x}{y}$$

$$y' = \frac{\mathrm{d}y}{\mathrm{d}x}$$

$$\boxed{k_1 = -\frac{x}{y}}$$

$$y = 3x^2 - 13x + 13$$

$$y' = 6x - 13$$

$$\boxed{k_2 = 6x - 13}$$

$$\operatorname{tg}\varphi = \left|\frac{k_1 - k_2}{1 + k_1 k_2}\right|$$

a)

$$x_0 \; y_0$$
$$A(1, 3)$$

$$k_1 = -\frac{x_0}{y_0} = -\frac{1}{3}$$

$$k_2 = 6x_0 - 13 = 6 \cdot 1 - 13 = -7$$

$$\operatorname{tg}\varphi_1 = \left|\frac{-\frac{1}{3} - (-7)}{1 + \frac{-1}{3} \cdot (-7)}\right| = \left|\frac{\frac{20}{3}}{\frac{10}{3}}\right| = 2$$

$$x^2 + y^2 = 10 \;/\frac{\mathrm{d}}{\mathrm{d}x}$$

$$2x + 2yy' = 0 \;/:2$$

$$x + yy' = 0$$

$$yy' = -x$$

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$$\boxed{k_1 = -\frac{x}{y}}$$

$$y = 3x^2 - 13x + 13$$

$$y' = 6x - 13$$

$$\boxed{k_2 = 6x - 13}$$

$$\operatorname{tg}\varphi = \left|\frac{k_1 - k_2}{1 + k_1 k_2}\right|$$

a)

$$k_1 = -\frac{x_0}{y_0} = -\frac{1}{3}$$

$$A(\overset{x_0}{1}, \overset{y_0}{3})$$

$$k_2 = 6x_0 - 13 = 6 \cdot 1 - 13 = -7$$

$$\operatorname{tg}\varphi_1 = \left|\frac{-\frac{1}{3} - (-7)}{1 + \frac{-1}{3} \cdot (-7)}\right| = \left|\frac{\frac{20}{3}}{\frac{10}{3}}\right| = 2$$

$$\varphi_1 = \operatorname{arctg} 2$$

$$x^2 + y^2 = 10 \;/\, \frac{\mathrm{d}}{\mathrm{d}x}$$

$$2x + 2yy' = 0 \;/ : 2$$

$$x + yy' = 0$$

$$yy' = -x$$

$$y' = -\frac{x}{y}$$

$$y' = \frac{\mathrm{d}y}{\mathrm{d}x}$$

$$\boxed{k_1 = -\frac{x}{y}}$$

$$y = 3x^2 - 13x + 13$$

$$y' = 6x - 13$$

$$\boxed{k_2 = 6x - 13}$$

$$\operatorname{tg}\varphi = \left|\frac{k_1 - k_2}{1 + k_1 k_2}\right|$$

a)

$$k_1 = -\frac{x_0}{y_0} = -\frac{1}{3}$$

$$A(1, 3) \quad (x_0 \; y_0)$$

$$k_2 = 6x_0 - 13 = 6 \cdot 1 - 13 = -7$$

$$\operatorname{tg}\varphi_1 = \left|\frac{-\frac{1}{3} - (-7)}{1 + \frac{-1}{3} \cdot (-7)}\right| = \left|\frac{\frac{20}{3}}{\frac{10}{3}}\right| = 2$$

$$\varphi_1 = \operatorname{arctg} 2 \qquad \varphi_1 = 63^\circ\, 26'\, 6''$$

$$x^2 + y^2 = 10 \,\Big/ \tfrac{\mathrm{d}}{\mathrm{d}x}$$

$$2x + 2yy' = 0 \,\Big/ :2$$

$$x + yy' = 0$$

$$yy' = -x$$

$$y' = -\frac{x}{y}$$

$$y' = \frac{\mathrm{d}y}{\mathrm{d}x}$$

$$k_1 = -\frac{x}{y}$$

$$y = 3x^2 - 13x + 13$$

$$y' = 6x - 13$$

$$k_2 = 6x - 13$$

$$\operatorname{tg}\varphi = \left|\frac{k_1 - k_2}{1 + k_1 k_2}\right|$$

a)

$$x_0 \; y_0$$
$$A(1, 3)$$

$$k_1 = -\frac{x_0}{y_0} = -\frac{1}{3}$$

$$k_2 = 6x_0 - 13 = 6 \cdot 1 - 13 = -7$$

$$\operatorname{tg}\varphi_1 = \left|\frac{-\frac{1}{3} - (-7)}{1 + \frac{-1}{3} \cdot (-7)}\right| = \left|\frac{\frac{20}{3}}{\frac{10}{3}}\right| = 2$$

$$\varphi_1 = \operatorname{arctg} 2$$

$$\varphi_1 = 63^\circ\, 26'\, 6''$$

$$x^2 + y^2 = 10 \,\Big/\, \frac{\mathrm{d}}{\mathrm{d}x}$$

$$2x + 2yy' = 0 \,\big/ :2$$

$$x + yy' = 0$$

$$yy' = -x$$

$$y' = -\frac{x}{y}$$

$$y' = \frac{\mathrm{d}y}{\mathrm{d}x}$$

$$\boxed{k_1 = -\frac{x}{y}}$$

$$y = 3x^2 - 13x + 13$$

$$y' = 6x - 13$$

$$\boxed{k_2 = 6x - 13}$$

$$\operatorname{tg}\varphi = \left|\frac{k_1 - k_2}{1 + k_1 k_2}\right|$$

a)

$$k_1 = -\frac{x_0}{y_0} = -\frac{1}{3}$$

$$A(1, 3) \quad (x_0, y_0)$$

$$k_2 = 6x_0 - 13 = 6 \cdot 1 - 13 = -7$$

$$\operatorname{tg}\varphi_1 = \left|\frac{-\frac{1}{3} - (-7)}{1 + \frac{-1}{3}\cdot(-7)}\right| = \left|\frac{\frac{20}{3}}{\frac{10}{3}}\right| = 2$$

$$\varphi_1 = \operatorname{arctg} 2 \qquad \boxed{\varphi_1 = 63^\circ\, 26'\, 6''}$$

b)

$$B(3, 1) \quad (x_0, y_0)$$

$$x^2 + y^2 = 10 \;/\frac{\mathrm{d}}{\mathrm{d}x}$$

$$2x + 2yy' = 0 \;/:2$$

$$x + yy' = 0$$

$$yy' = -x$$

$$y' = -\frac{x}{y}$$

$$y' = \frac{\mathrm{d}y}{\mathrm{d}x}$$

$$k_1 = -\frac{x}{y}$$

$$y = 3x^2 - 13x + 13$$

$$y' = 6x - 13$$

$$k_2 = 6x - 13$$

$$\operatorname{tg}\varphi = \left|\frac{k_1 - k_2}{1 + k_1 k_2}\right|$$

a)

$$A(\overset{x_0}{1}, \overset{y_0}{3})$$

$$k_1 = -\frac{x_0}{y_0} = -\frac{1}{3}$$

$$k_2 = 6x_0 - 13 = 6 \cdot 1 - 13 = -7$$

$$\operatorname{tg}\varphi_1 = \left|\frac{-\frac{1}{3} - (-7)}{1 + \frac{-1}{3} \cdot (-7)}\right| = \left|\frac{\frac{20}{3}}{\frac{10}{3}}\right| = 2$$

$$\varphi_1 = \operatorname{arctg} 2$$

$$\varphi_1 = 63^\circ\, 26'\, 6''$$

b)

$$B(\overset{x_0}{3}, \overset{y_0}{1})$$

$$k_1 =$$

$$x^2 + y^2 = 10 \;/\frac{\mathrm{d}}{\mathrm{d}x}$$

$$2x + 2yy' = 0 \;/:2$$

$$x + yy' = 0$$

$$yy' = -x$$

$$y' = -\frac{x}{y}$$

$$y' = \frac{\mathrm{d}y}{\mathrm{d}x}$$

$$\boxed{k_1 = -\frac{x}{y}}$$

$$y = 3x^2 - 13x + 13$$

$$y' = 6x - 13$$

$$\boxed{k_2 = 6x - 13}$$

$$\operatorname{tg}\varphi = \left|\frac{k_1 - k_2}{1 + k_1 k_2}\right|$$

a)

$x_0 \; y_0$
$A(1,3)$

$$k_1 = -\frac{x_0}{y_0} = -\frac{1}{3}$$

$$k_2 = 6x_0 - 13 = 6 \cdot 1 - 13 = -7$$

$$\operatorname{tg}\varphi_1 = \left|\frac{-\frac{1}{3} - (-7)}{1 + \frac{-1}{3} \cdot (-7)}\right| = \left|\frac{\frac{20}{3}}{\frac{10}{3}}\right| = 2$$

$$\varphi_1 = \operatorname{arctg} 2 \qquad \boxed{\varphi_1 = 63^\circ\, 26'\, 6''}$$

b)

$x_0 \; y_0$
$B(3,1)$

$$k_1 = -\frac{x_0}{y_0}$$

$$x^2 + y^2 = 10 \;\Big/ \frac{\mathrm{d}}{\mathrm{d}x}$$

$$2x + 2yy' = 0 \;\Big/ :2$$

$$x + yy' = 0$$

$$yy' = -x$$

$$y' = -\frac{x}{y}$$

$$y' = \frac{\mathrm{d}y}{\mathrm{d}x}$$

$$\boxed{k_1 = -\frac{x}{y}}$$

$$y = 3x^2 - 13x + 13$$

$$y' = 6x - 13$$

$$\boxed{k_2 = 6x - 13}$$

$$\operatorname{tg}\varphi = \left|\frac{k_1 - k_2}{1 + k_1 k_2}\right|$$

a)

$$x_0 \; y_0$$
$$A(1, 3)$$

$$k_1 = -\frac{x_0}{y_0} = -\frac{1}{3}$$

$$k_2 = 6x_0 - 13 = 6 \cdot 1 - 13 = -7$$

$$\operatorname{tg}\varphi_1 = \left|\frac{-\frac{1}{3} - (-7)}{1 + \frac{-1}{3} \cdot (-7)}\right| = \left|\frac{\frac{20}{3}}{\frac{10}{3}}\right| = 2$$

$$\varphi_1 = \operatorname{arctg} 2 \qquad \boxed{\varphi_1 = 63^\circ\, 26'\, 6''}$$

b)

$$x_0 \; y_0$$
$$B(3, 1)$$

$$k_1 = -\frac{x_0}{y_0} = -\frac{3}{1}$$

$$x^2 + y^2 = 10 \;\Big/ \frac{\mathrm{d}}{\mathrm{d}x}$$

$$2x + 2yy' = 0 \;\Big/ :2$$

$$x + yy' = 0$$

$$yy' = -x$$

$$y' = -\frac{x}{y}$$

$$y' = \frac{\mathrm{d}y}{\mathrm{d}x}$$

$$\boxed{k_1 = -\frac{x}{y}}$$

$$y = 3x^2 - 13x + 13$$

$$y' = 6x - 13$$

$$\boxed{k_2 = 6x - 13}$$

$$\operatorname{tg}\varphi = \left|\frac{k_1 - k_2}{1 + k_1 k_2}\right|$$

a)

$x_0 \; y_0$

$A(1, 3)$

$$k_1 = -\frac{x_0}{y_0} = -\frac{1}{3}$$

$$k_2 = 6x_0 - 13 = 6 \cdot 1 - 13 = -7$$

$$\operatorname{tg}\varphi_1 = \left|\frac{-\frac{1}{3} - (-7)}{1 + \frac{-1}{3} \cdot (-7)}\right| = \left|\frac{\frac{20}{3}}{\frac{10}{3}}\right| = 2$$

$$\varphi_1 = \operatorname{arctg} 2 \qquad \boxed{\varphi_1 = 63^\circ\, 26'\, 6''}$$

b)

$x_0 \; y_0$

$B(3, 1)$

$$k_1 = -\frac{x_0}{y_0} = -\frac{3}{1} = -3$$

$$x^2 + y^2 = 10 \;\Big/ \frac{\mathrm{d}}{\mathrm{d}x}$$

$$2x + 2yy' = 0 \;/ :2$$

$$x + yy' = 0$$

$$yy' = -x$$

$$y' = -\frac{x}{y}$$

$$y' = \frac{\mathrm{d}y}{\mathrm{d}x}$$

$$\boxed{k_1 = -\frac{x}{y}}$$

$$y = 3x^2 - 13x + 13$$

$$y' = 6x - 13$$

$$\boxed{k_2 = 6x - 13}$$

$$\operatorname{tg}\varphi = \left|\frac{k_1 - k_2}{1 + k_1 k_2}\right|$$

a)

$$x_0 \;\; y_0 \qquad A(1,3)$$

$$k_1 = -\frac{x_0}{y_0} = -\frac{1}{3}$$

$$k_2 = 6x_0 - 13 = 6 \cdot 1 - 13 = -7$$

$$\operatorname{tg}\varphi_1 = \left|\frac{-\frac{1}{3} - (-7)}{1 + \frac{-1}{3}\cdot(-7)}\right| = \left|\frac{\frac{20}{3}}{\frac{10}{3}}\right| = 2$$

$$\varphi_1 = \operatorname{arctg} 2 \qquad \boxed{\varphi_1 = 63^\circ\, 26'\, 6''}$$

b)

$$x_0 \;\; y_0 \qquad B(3,1)$$

$$k_1 = -\frac{x_0}{y_0} = -\frac{3}{1} = -3$$

$$k_2 =$$

$$x^2 + y^2 = 10 \,/\, \frac{\mathrm{d}}{\mathrm{d}x}$$

$$2x + 2yy' = 0 \,/\!:2$$

$$x + yy' = 0$$

$$yy' = -x$$

$$y' = -\frac{x}{y}$$

$$y' = \frac{\mathrm{d}y}{\mathrm{d}x}$$

$$\boxed{k_1 = -\frac{x}{y}}$$

$$y = 3x^2 - 13x + 13$$

$$y' = 6x - 13$$

$$\boxed{k_2 = 6x - 13}$$

$$\operatorname{tg}\varphi = \left| \frac{k_1 - k_2}{1 + k_1 k_2} \right|$$

a)

$$k_1 = -\frac{x_0}{y_0} = -\frac{1}{3}$$

$x_0 \; y_0$

$A(1, 3)$

$$k_2 = 6x_0 - 13 = 6 \cdot 1 - 13 = -7$$

$$\operatorname{tg}\varphi_1 = \left| \frac{-\frac{1}{3} - (-7)}{1 + \frac{-1}{3} \cdot (-7)} \right| = \left| \frac{\frac{20}{3}}{\frac{10}{3}} \right| = 2$$

$$\varphi_1 = \operatorname{arctg} 2 \qquad \boxed{\varphi_1 = 63^\circ\, 26'\, 6''}$$

b)

$$k_1 = -\frac{x_0}{y_0} = -\frac{3}{1} = -3$$

$x_0 \; y_0$

$B(3, 1)$

$$k_2 = 6x_0 - 13$$

$$x^2 + y^2 = 10 \;/\frac{\mathrm{d}}{\mathrm{d}x}$$

$$2x + 2yy' = 0 \;/:2$$

$$x + yy' = 0$$

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$$\operatorname{tg}\varphi = \left|\frac{k_1 - k_2}{1 + k_1 k_2}\right|$$

a)

x_0 y_0
$A(1, 3)$

$$k_1 = -\frac{x_0}{y_0} = -\frac{1}{3}$$

$$k_2 = 6x_0 - 13 = 6 \cdot 1 - 13 = -7$$

$$\operatorname{tg}\varphi_1 = \left|\frac{-\frac{1}{3} - (-7)}{1 + \frac{-1}{3}\cdot(-7)}\right| = \left|\frac{\frac{20}{3}}{\frac{10}{3}}\right| = 2$$

$$\varphi_1 = \operatorname{arctg} 2 \qquad \varphi_1 = 63^\circ\, 26'\, 6''$$

b)

x_0 y_0
$B(3, 1)$

$$k_1 = -\frac{x_0}{y_0} = -\frac{3}{1} = -3$$

$$k_2 = 6x_0 - 13 = 6 \cdot 3 - 13$$

$$x^2 + y^2 = 10 \;/\frac{\mathrm{d}}{\mathrm{d}x}$$

$$2x + 2yy' = 0 \;/:2$$

$$x + yy' = 0$$

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$$\boxed{k_1 = -\frac{x}{y}}$$

$$y = 3x^2 - 13x + 13$$

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$$\operatorname{tg}\varphi = \left|\frac{k_1 - k_2}{1 + k_1 k_2}\right|$$

a)

$x_0 \; y_0$ $A(1, 3)$

$$k_1 = -\frac{x_0}{y_0} = -\frac{1}{3}$$

$$k_2 = 6x_0 - 13 = 6 \cdot 1 - 13 = -7$$

$$\operatorname{tg}\varphi_1 = \left|\frac{-\frac{1}{3} - (-7)}{1 + \frac{-1}{3} \cdot (-7)}\right| = \left|\frac{\frac{20}{3}}{\frac{10}{3}}\right| = 2$$

$$\varphi_1 = \operatorname{arctg} 2 \qquad \boxed{\varphi_1 = 63^\circ\, 26'\, 6''}$$

b)

$x_0 \; y_0$ $B(3, 1)$

$$k_1 = -\frac{x_0}{y_0} = -\frac{3}{1} = -3$$

$$k_2 = 6x_0 - 13 = 6 \cdot 3 - 13 = 5$$

$$x^2 + y^2 = 10 \;/\frac{\mathrm{d}}{\mathrm{d}x}$$

$$2x + 2yy' = 0 \;/:2$$

$$x + yy' = 0$$

$$yy' = -x$$

$$y' = -\frac{x}{y}$$

$$y' = \frac{\mathrm{d}y}{\mathrm{d}x}$$

$$k_1 = -\frac{x}{y}$$

$$y = 3x^2 - 13x + 13$$

$$y' = 6x - 13$$

$$k_2 = 6x - 13$$

$$\operatorname{tg}\varphi = \left|\frac{k_1 - k_2}{1 + k_1 k_2}\right|$$

a)

$$k_1 = -\frac{x_0}{y_0} = -\frac{1}{3}$$

$$A(1,3) \quad (x_0 \; y_0)$$

$$k_2 = 6x_0 - 13 = 6 \cdot 1 - 13 = -7$$

$$\operatorname{tg}\varphi_1 = \left|\frac{-\frac{1}{3} - (-7)}{1 + \frac{-1}{3} \cdot (-7)}\right| = \left|\frac{\frac{20}{3}}{\frac{10}{3}}\right| = 2$$

$$\varphi_1 = \operatorname{arctg} 2 \qquad \varphi_1 = 63^\circ\, 26'\, 6''$$

b)

$$k_1 = -\frac{x_0}{y_0} = -\frac{3}{1} = -3$$

$$B(3,1) \quad (x_0 \; y_0)$$

$$k_2 = 6x_0 - 13 = 6 \cdot 3 - 13 = 5$$

$$\operatorname{tg}\varphi_2 = \left|\frac{\quad}{\quad}\right|$$

$$x^2 + y^2 = 10 \;\Big/ \frac{\mathrm{d}}{\mathrm{d}x}$$

$$2x + 2yy' = 0 \;\Big/ :2$$

$$x + yy' = 0$$

$$yy' = -x$$

$$y' = -\frac{x}{y}$$

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$$\boxed{k_1 = -\frac{x}{y}}$$

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$$y' = 6x - 13$$

$$\boxed{k_2 = 6x - 13}$$

$$\operatorname{tg}\varphi = \left|\frac{k_1 - k_2}{1 + k_1 k_2}\right|$$

a)

$x_0 \; y_0$ $A(1, 3)$

$$k_1 = -\frac{x_0}{y_0} = -\frac{1}{3}$$

$$k_2 = 6x_0 - 13 = 6 \cdot 1 - 13 = -7$$

$$\operatorname{tg}\varphi_1 = \left|\frac{-\frac{1}{3} - (-7)}{1 + \frac{-1}{3} \cdot (-7)}\right| = \left|\frac{\frac{20}{3}}{\frac{10}{3}}\right| = 2$$

$$\varphi_1 = \operatorname{arctg} 2 \qquad \boxed{\varphi_1 = 63^\circ\, 26'\, 6''}$$

b)

$x_0 \; y_0$ $B(3, 1)$

$$k_1 = -\frac{x_0}{y_0} = -\frac{3}{1} = -3$$

$$k_2 = 6x_0 - 13 = 6 \cdot 3 - 13 = 5$$

$$\operatorname{tg}\varphi_2 = \left|\frac{-3}{}\right|$$

$$x^2 + y^2 = 10 \ \Big/ \frac{\mathrm{d}}{\mathrm{d}x}$$

$$2x + 2yy' = 0 \ \Big/ :2$$

$$x + yy' = 0$$

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$$\operatorname{tg}\varphi = \left|\frac{k_1 - k_2}{1 + k_1 k_2}\right|$$

a)

$x_0 \ y_0$ $A(1, 3)$

$$k_1 = -\frac{x_0}{y_0} = -\frac{1}{3}$$

$$k_2 = 6x_0 - 13 = 6 \cdot 1 - 13 = -7$$

$$\operatorname{tg}\varphi_1 = \left|\frac{-\frac{1}{3} - (-7)}{1 + \frac{-1}{3} \cdot (-7)}\right| = \left|\frac{\frac{20}{3}}{\frac{10}{3}}\right| = 2$$

$$\varphi_1 = \operatorname{arctg} 2 \qquad \boxed{\varphi_1 = 63^\circ\, 26'\, 6''}$$

b)

$x_0 \ y_0$ $B(3, 1)$

$$k_1 = -\frac{x_0}{y_0} = -\frac{3}{1} = -3$$

$$k_2 = 6x_0 - 13 = 6 \cdot 3 - 13 = 5$$

$$\operatorname{tg}\varphi_2 = \left|\frac{-3 - }{}\right|$$

$$x^2 + y^2 = 10 \;\Big/ \frac{\mathrm{d}}{\mathrm{d}x}$$

$$2x + 2yy' = 0 \;\Big/ :2$$

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$$yy' = -x$$

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$$y' = 6x - 13$$

$$\boxed{k_2 = 6x - 13}$$

$$\operatorname{tg}\varphi = \left|\frac{k_1 - k_2}{1 + k_1 k_2}\right|$$

a)

$x_0 \; y_0$
$A(1, 3)$

$$k_1 = -\frac{x_0}{y_0} = -\frac{1}{3}$$

$$k_2 = 6x_0 - 13 = 6 \cdot 1 - 13 = -7$$

$$\operatorname{tg}\varphi_1 = \left|\frac{-\frac{1}{3} - (-7)}{1 + \frac{-1}{3} \cdot (-7)}\right| = \left|\frac{\frac{20}{3}}{\frac{10}{3}}\right| = 2$$

$$\varphi_1 = \operatorname{arctg} 2 \qquad \boxed{\varphi_1 = 63^\circ\, 26'\, 6''}$$

b)

$x_0 \; y_0$
$B(3, 1)$

$$k_1 = -\frac{x_0}{y_0} = -\frac{3}{1} = -3$$

$$k_2 = 6x_0 - 13 = 6 \cdot 3 - 13 = 5$$

$$\operatorname{tg}\varphi_2 = \left|\frac{-3 - 5}{}\right|$$

$$x^2 + y^2 = 10 \;\Big/ \frac{\mathrm{d}}{\mathrm{d}x}$$

$$2x + 2yy' = 0 \;\Big/ :2$$

$$x + yy' = 0$$

$$yy' = -x$$

$$y' = -\frac{x}{y}$$

$$y' = \frac{\mathrm{d}y}{\mathrm{d}x}$$

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$$\operatorname{tg}\varphi = \left|\frac{k_1 - k_2}{1 + k_1 k_2}\right|$$

a)

$x_0 \; y_0$
$A(1,3)$

$$k_1 = -\frac{x_0}{y_0} = -\frac{1}{3}$$

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$$\operatorname{tg}\varphi_1 = \left|\frac{-\frac{1}{3} - (-7)}{1 + \frac{-1}{3} \cdot (-7)}\right| = \left|\frac{\frac{20}{3}}{\frac{10}{3}}\right| = 2$$

$$\varphi_1 = \operatorname{arctg} 2 \qquad \boxed{\varphi_1 = 63^\circ\, 26'\, 6''}$$

b)

$x_0 \; y_0$
$B(3,1)$

$$k_1 = -\frac{x_0}{y_0} = -\frac{3}{1} = -3$$

$$k_2 = 6x_0 - 13 = 6 \cdot 3 - 13 = 5$$

$$\operatorname{tg}\varphi_2 = \left|\frac{-3 - 5}{1 + (-3) \cdot 5}\right|$$

$$x^2 + y^2 = 10 \;\Big/ \frac{\mathrm{d}}{\mathrm{d}x}$$

$$2x + 2yy' = 0 \;\Big/ :2$$

$$x + yy' = 0$$

$$yy' = -x$$

$$y' = -\frac{x}{y}$$

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$$\operatorname{tg}\varphi = \left|\frac{k_1 - k_2}{1 + k_1 k_2}\right|$$

a)

$x_0 \; y_0$

$A(1, 3)$

$$k_1 = -\frac{x_0}{y_0} = -\frac{1}{3}$$

$$k_2 = 6x_0 - 13 = 6 \cdot 1 - 13 = -7$$

$$\operatorname{tg}\varphi_1 = \left|\frac{-\frac{1}{3} - (-7)}{1 + \frac{-1}{3} \cdot (-7)}\right| = \left|\frac{\frac{20}{3}}{\frac{10}{3}}\right| = 2$$

$$\varphi_1 = \operatorname{arctg} 2 \qquad \boxed{\varphi_1 = 63^\circ\, 26'\, 6''}$$

b)

$x_0 \; y_0$

$B(3, 1)$

$$k_1 = -\frac{x_0}{y_0} = -\frac{3}{1} = -3$$

$$k_2 = 6x_0 - 13 = 6 \cdot 3 - 13 = 5$$

$$\operatorname{tg}\varphi_2 = \left|\frac{-3 - 5}{1 + (-3) \cdot 5}\right| = \left|\frac{\quad}{\quad}\right|$$

$$x^2 + y^2 = 10 \;\Big/ \frac{\mathrm{d}}{\mathrm{d}x}$$

$$2x + 2yy' = 0 \;\Big/ :2$$

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$$\operatorname{tg}\varphi = \left|\frac{k_1 - k_2}{1 + k_1 k_2}\right|$$

a)

$x_0 \; y_0$

$A(1, 3)$

$$k_1 = -\frac{x_0}{y_0} = -\frac{1}{3}$$

$$k_2 = 6x_0 - 13 = 6 \cdot 1 - 13 = -7$$

$$\operatorname{tg}\varphi_1 = \left|\frac{-\frac{1}{3} - (-7)}{1 + \frac{-1}{3} \cdot (-7)}\right| = \left|\frac{\frac{20}{3}}{\frac{10}{3}}\right| = 2$$

$$\varphi_1 = \operatorname{arctg} 2 \qquad \boxed{\varphi_1 = 63^\circ\, 26'\, 6''}$$

b)

$x_0 \; y_0$

$B(3, 1)$

$$k_1 = -\frac{x_0}{y_0} = -\frac{3}{1} = -3$$

$$k_2 = 6x_0 - 13 = 6 \cdot 3 - 13 = 5$$

$$\operatorname{tg}\varphi_2 = \left|\frac{-3 - 5}{1 + (-3) \cdot 5}\right| = \left|\frac{-8}{}\right|$$

$$x^2 + y^2 = 10 \;\Big/ \frac{\mathrm{d}}{\mathrm{d}x}$$

$$2x + 2yy' = 0 \;\Big/ :2$$

$$x + yy' = 0$$

$$yy' = -x$$

$$y' = -\frac{x}{y}$$

$$y' = \frac{\mathrm{d}y}{\mathrm{d}x}$$

$$\boxed{k_1 = -\frac{x}{y}}$$

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$$y' = 6x - 13$$

$$\boxed{k_2 = 6x - 13}$$

$$\operatorname{tg}\varphi = \left|\frac{k_1 - k_2}{1 + k_1 k_2}\right|$$

a)

$$k_1 = -\frac{x_0}{y_0} = -\frac{1}{3}$$

$$\overset{x_0\ y_0}{A(1, 3)}$$

$$k_2 = 6x_0 - 13 = 6 \cdot 1 - 13 = -7$$

$$\operatorname{tg}\varphi_1 = \left|\frac{-\frac{1}{3} - (-7)}{1 + \frac{-1}{3} \cdot (-7)}\right| = \left|\frac{\frac{20}{3}}{\frac{10}{3}}\right| = 2$$

$$\varphi_1 = \operatorname{arctg} 2 \qquad \boxed{\varphi_1 = 63^\circ\, 26'\, 6''}$$

b)

$$k_1 = -\frac{x_0}{y_0} = -\frac{3}{1} = -3$$

$$\overset{x_0\ y_0}{B(3, 1)}$$

$$k_2 = 6x_0 - 13 = 6 \cdot 3 - 13 = 5$$

$$\operatorname{tg}\varphi_2 = \left|\frac{-3 - 5}{1 + (-3) \cdot 5}\right| = \left|\frac{-8}{-14}\right|$$

$$x^2 + y^2 = 10 \;\Big/ \frac{\mathrm{d}}{\mathrm{d}x}$$

$$2x + 2yy' = 0 \;\Big/ :2$$

$$x + yy' = 0$$

$$yy' = -x$$

$$y' = -\frac{x}{y}$$

$$y' = \frac{\mathrm{d}y}{\mathrm{d}x}$$

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$$y' = 6x - 13$$

$$\boxed{k_2 = 6x - 13}$$

$$\operatorname{tg}\varphi = \left|\frac{k_1 - k_2}{1 + k_1 k_2}\right|$$

a)

$x_0 \; y_0$

$A(1, 3)$

$$k_1 = -\frac{x_0}{y_0} = -\frac{1}{3}$$

$$k_2 = 6x_0 - 13 = 6 \cdot 1 - 13 = -7$$

$$\operatorname{tg}\varphi_1 = \left|\frac{-\frac{1}{3} - (-7)}{1 + \frac{-1}{3} \cdot (-7)}\right| = \left|\frac{\frac{20}{3}}{\frac{10}{3}}\right| = 2$$

$$\varphi_1 = \operatorname{arctg} 2 \qquad \boxed{\varphi_1 = 63^\circ\, 26'\, 6''}$$

b)

$x_0 \; y_0$

$B(3, 1)$

$$k_1 = -\frac{x_0}{y_0} = -\frac{3}{1} = -3$$

$$k_2 = 6x_0 - 13 = 6 \cdot 3 - 13 = 5$$

$$\operatorname{tg}\varphi_2 = \left|\frac{-3 - 5}{1 + (-3) \cdot 5}\right| = \left|\frac{-8}{-14}\right| = \frac{4}{7}$$

$$x^2 + y^2 = 10 \;/\frac{\mathrm{d}}{\mathrm{d}x}$$

$$2x + 2yy' = 0 \;/:2$$

$$x + yy' = 0$$

$$yy' = -x$$

$$y' = -\frac{x}{y}$$

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$$\operatorname{tg}\varphi = \left|\frac{k_1 - k_2}{1 + k_1 k_2}\right|$$

a)

$x_0 \; y_0$
$A(1, 3)$

$$k_1 = -\frac{x_0}{y_0} = -\frac{1}{3}$$

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$$\operatorname{tg}\varphi_1 = \left|\frac{-\frac{1}{3} - (-7)}{1 + \frac{-1}{3} \cdot (-7)}\right| = \left|\frac{\frac{20}{3}}{\frac{10}{3}}\right| = 2$$

$$\varphi_1 = \operatorname{arctg} 2 \qquad \boxed{\varphi_1 = 63^\circ\, 26'\, 6''}$$

b)

$x_0 \; y_0$
$B(3, 1)$

$$k_1 = -\frac{x_0}{y_0} = -\frac{3}{1} = -3$$

$$k_2 = 6x_0 - 13 = 6 \cdot 3 - 13 = 5$$

$$\operatorname{tg}\varphi_2 = \left|\frac{-3 - 5}{1 + (-3) \cdot 5}\right| = \left|\frac{-8}{-14}\right| = \frac{4}{7}$$

$$\varphi_2 = \operatorname{arctg}\frac{4}{7}$$

$$x^2 + y^2 = 10 \;/\frac{\mathrm{d}}{\mathrm{d}x}$$

$$2x + 2yy' = 0 \;/:2$$

$$x + yy' = 0$$

$$yy' = -x$$

$$y' = -\frac{x}{y}$$

$$y' = \frac{\mathrm{d}y}{\mathrm{d}x}$$

$$\boxed{k_1 = -\frac{x}{y}}$$

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$$\operatorname{tg}\varphi = \left|\frac{k_1 - k_2}{1 + k_1 k_2}\right|$$

a)

$$\overset{x_0\ y_0}{A(1,3)}$$

$$k_1 = -\frac{x_0}{y_0} = -\frac{1}{3}$$

$$k_2 = 6x_0 - 13 = 6\cdot 1 - 13 = -7$$

$$\operatorname{tg}\varphi_1 = \left|\frac{-\frac{1}{3} - (-7)}{1 + \frac{-1}{3}\cdot(-7)}\right| = \left|\frac{\frac{20}{3}}{\frac{10}{3}}\right| = 2$$

$$\varphi_1 = \operatorname{arctg} 2 \qquad \boxed{\varphi_1 = 63^\circ\, 26'\, 6''}$$

b)

$$\overset{x_0\ y_0}{B(3,1)}$$

$$k_1 = -\frac{x_0}{y_0} = -\frac{3}{1} = -3$$

$$k_2 = 6x_0 - 13 = 6\cdot 3 - 13 = 5$$

$$\operatorname{tg}\varphi_2 = \left|\frac{-3 - 5}{1 + (-3)\cdot 5}\right| = \left|\frac{-8}{-14}\right| = \frac{4}{7}$$

$$\varphi_2 = \operatorname{arctg}\frac{4}{7} \qquad \varphi_2 = 29^\circ\, 44'\, 42''$$

$$x^2+y^2=10 \;\Big/ \frac{\mathrm{d}}{\mathrm{d}x}$$

$$2x+2yy'=0 \;\Big/ :2$$

$$x+yy'=0$$

$$yy'=-x$$

$$y'=-\frac{x}{y}$$

$$y'=\frac{\mathrm{d}y}{\mathrm{d}x}$$

$$k_1=-\frac{x}{y}$$

$$y=3x^2-13x+13$$

$$y'=6x-13$$

$$k_2=6x-13$$

$$\operatorname{tg}\varphi=\left|\frac{k_1-k_2}{1+k_1k_2}\right|$$

a)

x_0 y_0
$A(1,3)$

$$k_1=-\frac{x_0}{y_0}=-\frac{1}{3}$$

$$k_2=6x_0-13=6\cdot 1-13=-7$$

$$\operatorname{tg}\varphi_1=\left|\frac{-\frac{1}{3}-(-7)}{1+\frac{-1}{3}\cdot(-7)}\right|=\left|\frac{\frac{20}{3}}{\frac{10}{3}}\right|=2$$

$$\varphi_1=\operatorname{arctg}2 \qquad \varphi_1=63^\circ\,26'\,6''$$

b)

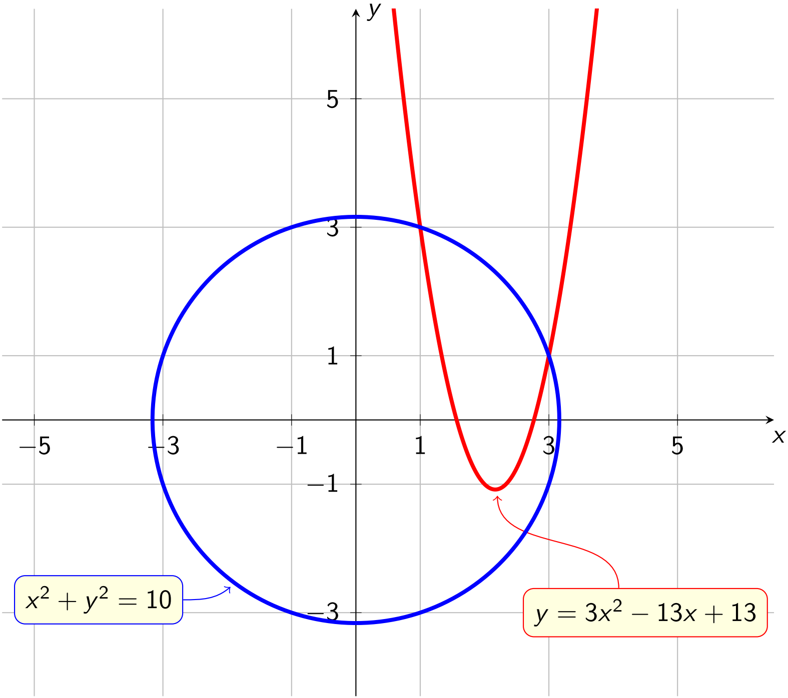
x_0 y_0
$B(3,1)$

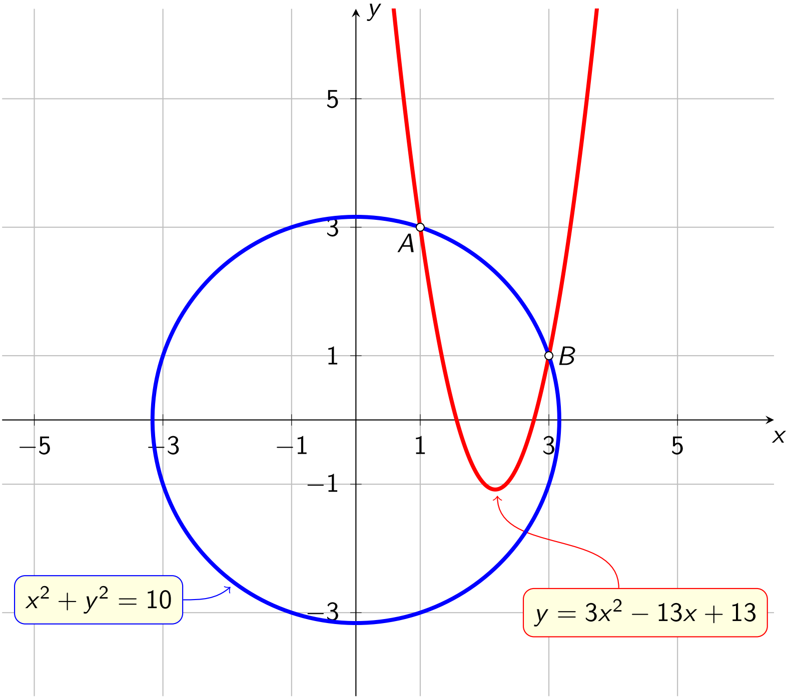
$$k_1=-\frac{x_0}{y_0}=-\frac{3}{1}=-3$$

$$k_2=6x_0-13=6\cdot 3-13=5$$

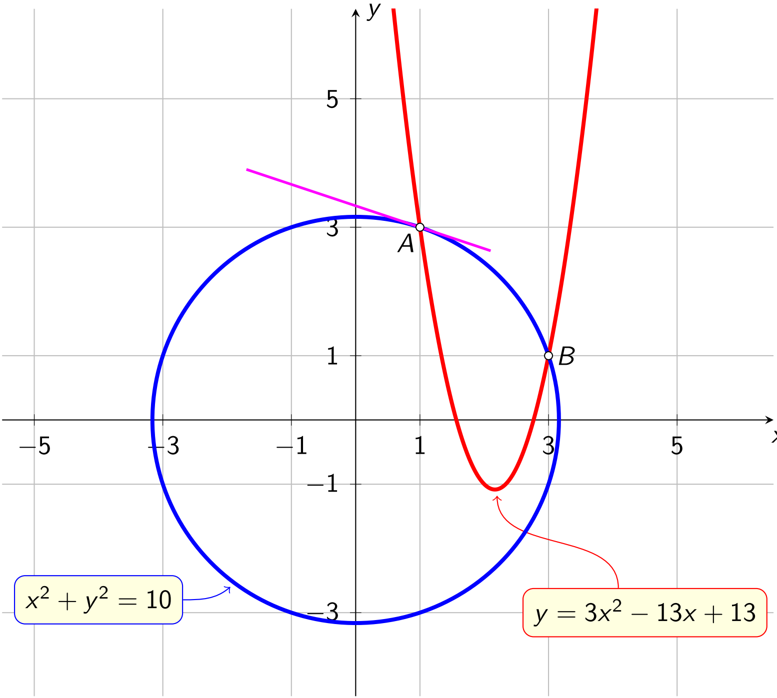
$$\operatorname{tg}\varphi_2=\left|\frac{-3-5}{1+(-3)\cdot 5}\right|=\left|\frac{-8}{-14}\right|=\frac{4}{7}$$

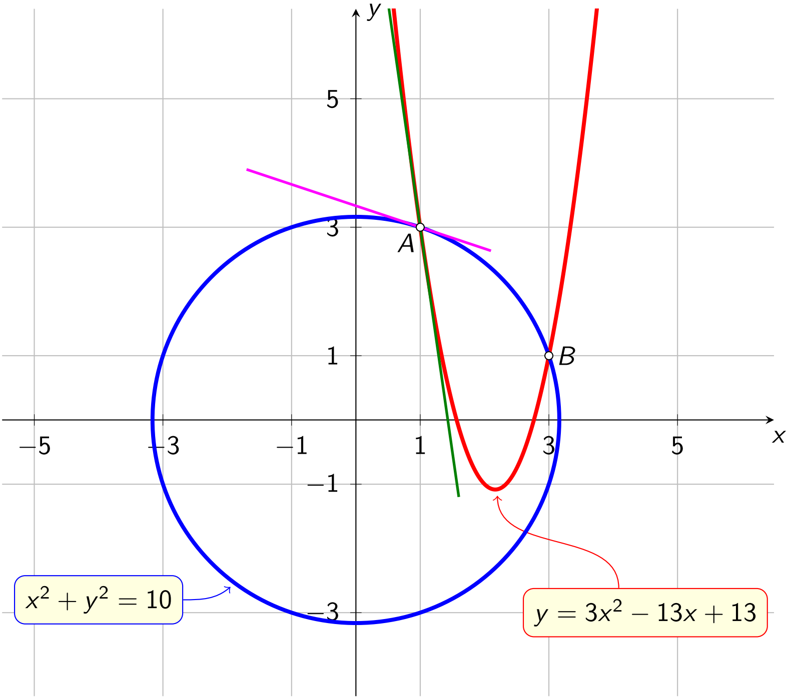
$$\varphi_2=\operatorname{arctg}\frac{4}{7} \qquad \varphi_2=29^\circ\,44'\,42''$$


y
x
5
3
1
−1
−3
−5
−3
−1
1
3
5
$x^2 + y^2 = 10$
$y = 3x^2 - 13x + 13$

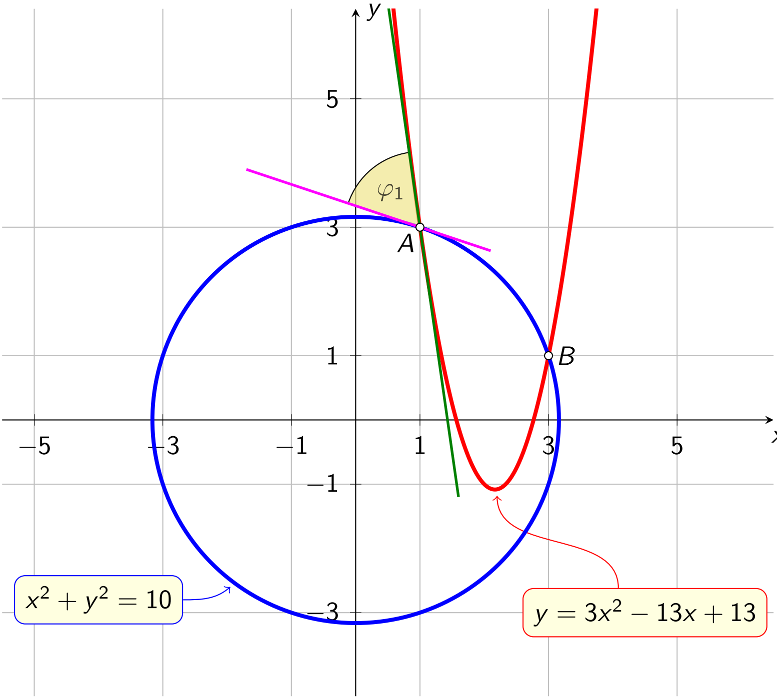


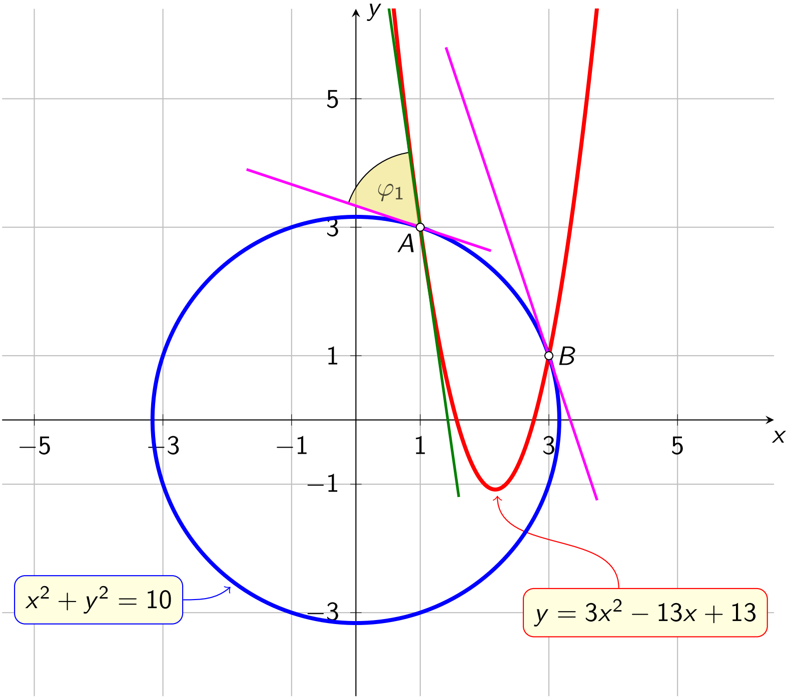
y
x
5
3
1
−1
−3
−5
−3
−1
1
3
5
A
B
$x^2 + y^2 = 10$
$y = 3x^2 - 13x + 13$


y
x
5
3
1
−1
−3
−5
−3
−1
1
3
5
A
B
$x^2 + y^2 = 10$
$y = 3x^2 - 13x + 13$

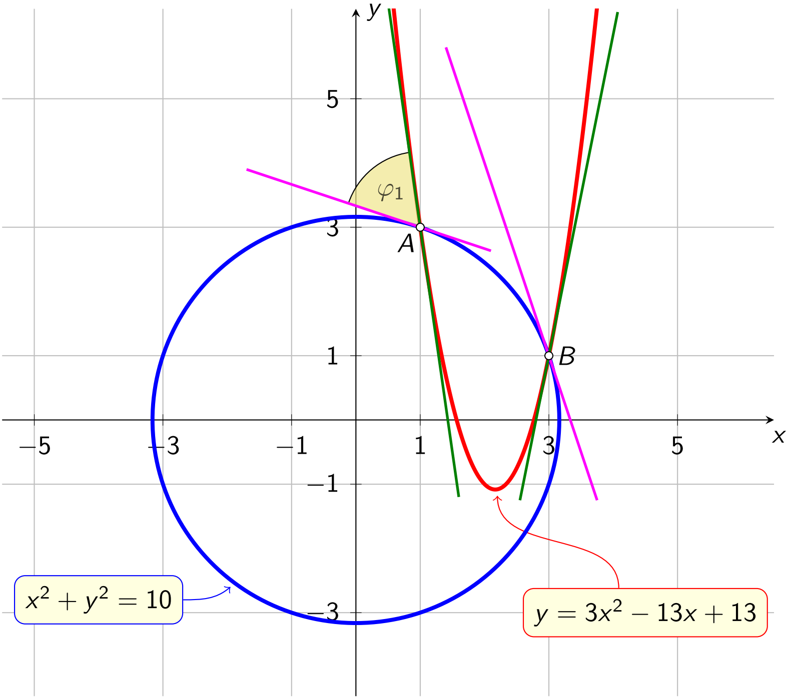


y
x
5
3
1
−1
−3
−5
−3
−1
1
3
5
A
B
$x^2 + y^2 = 10$
$y = 3x^2 - 13x + 13$

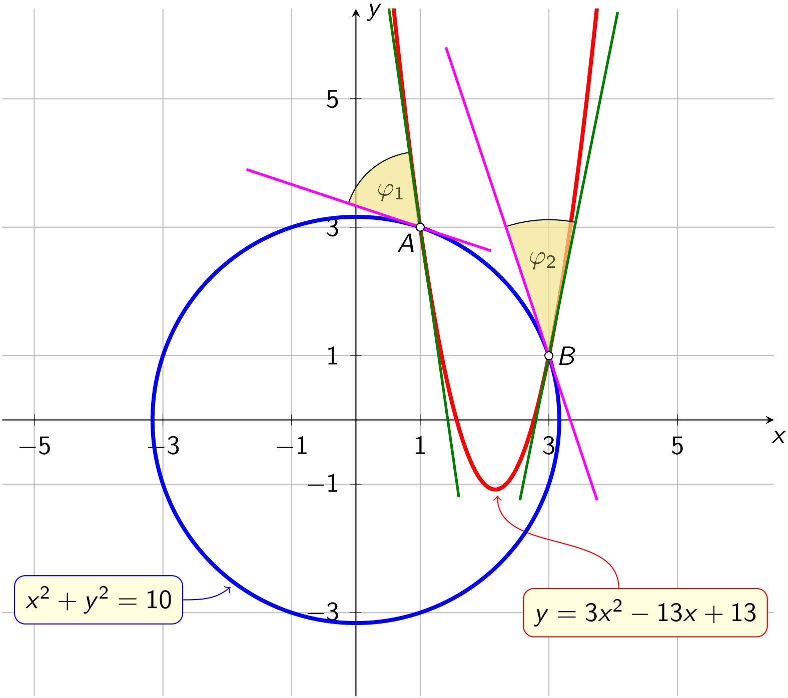

y
5
3
1
−1
−3
x
−5
−3
−1
1
3
5
φ_1
A
B
$x^2 + y^2 = 10$
$y = 3x^2 - 13x + 13$



y
x
5
3
1
−1
−3
−5
−3
−1
1
3
5
φ1
A
B
x2 + y2 = 10
y = 3x2 − 13x + 13



y
x
5
3
1
−1
−3
−5
−3
−1
1
3
5
φ1
A
B
$x^2 + y^2 = 10$
$y = 3x^2 - 13x + 13$



y
x
5
3
1
−1
−3
−5
−3
−1
1
3
5
A
B
φ_1
φ_2
$x^2 + y^2 = 10$
$y = 3x^2 - 13x + 13$

deveti zadatak

Zadatak 9

Odredite intervale monotonosti i ekstreme funkcije $f(x) = x^4 + 4x - 5$.

Zadatak 9

Odredite intervale monotonosti i ekstreme funkcije $f(x) = x^4 + 4x - 5$.

Rješenje

- Domena funkcije f jednaka je $D_f = \mathbb{R}$.

Zadatak 9

Odredite intervale monotonosti i ekstreme funkcije $f(x) = x^4 + 4x - 5$*.*

Rješenje

- Domena funkcije f jednaka je $D_f = \mathbb{R}$.
- Derivacija funkcije f jednaka je $f'(x) = 4x^3 + 4$.

Zadatak 9

Odredite intervale monotonosti i ekstreme funkcije $f(x) = x^4 + 4x - 5$.

Rješenje

- Domena funkcije f jednaka je $D_f = \mathbb{R}$.
- Derivacija funkcije f jednaka je $f'(x) = 4x^3 + 4$.
- Tražimo nultočke derivacije kako bismo dobili stacionarne točke.

$$4x^3 + 4 = 0$$

Zadatak 9

Odredite intervale monotonosti i ekstreme funkcije $f(x) = x^4 + 4x - 5$.

Rješenje

- Domena funkcije f jednaka je $D_f = \mathbb{R}$.
- Derivacija funkcije f jednaka je $f'(x) = 4x^3 + 4$.
- Tražimo nultočke derivacije kako bismo dobili stacionarne točke.

$$4x^3 + 4 = 0 \implies 4x^3 = -4$$

Zadatak 9

Odredite intervale monotonosti i ekstreme funkcije $f(x) = x^4 + 4x - 5$.

Rješenje

- Domena funkcije f jednaka je $D_f = \mathbb{R}$.
- Derivacija funkcije f jednaka je $f'(x) = 4x^3 + 4$.
- Tražimo nultočke derivacije kako bismo dobili stacionarne točke.

$$4x^3 + 4 = 0 \implies 4x^3 = -4 \implies x^3 = -1$$

Zadatak 9

Odredite intervale monotonosti i ekstreme funkcije $f(x) = x^4 + 4x - 5$.

Rješenje

- Domena funkcije f jednaka je $D_f = \mathbb{R}$.
- Derivacija funkcije f jednaka je $f'(x) = 4x^3 + 4$.
- Tražimo nultočke derivacije kako bismo dobili stacionarne točke.

$$4x^3 + 4 = 0 \implies 4x^3 = -4 \implies x^3 = -1 \implies x = -1$$

Zadatak 9

Odredite intervale monotonosti i ekstreme funkcije $f(x) = x^4 + 4x - 5$.

Rješenje

- Domena funkcije f jednaka je $D_f = \mathbb{R}$.
- Derivacija funkcije f jednaka je $f'(x) = 4x^3 + 4$.
- Tražimo nultočke derivacije kako bismo dobili stacionarne točke.

$$4x^3 + 4 = 0 \implies 4x^3 = -4 \implies x^3 = -1 \implies \boxed{x = -1}$$

Zadatak 9

Odredite intervale monotonosti i ekstreme funkcije $f(x) = x^4 + 4x - 5$.

Rješenje

- Domena funkcije f jednaka je $D_f = \mathbb{R}$.
- Derivacija funkcije f jednaka je $f'(x) = 4x^3 + 4$.
- Tražimo nultočke derivacije kako bismo dobili stacionarne točke.

$$4x^3 + 4 = 0 \implies 4x^3 = -4 \implies x^3 = -1 \implies \boxed{x = -1}$$

- $x = -1$ je jedina stacionarna točka funkcije f.

Zadatak 9

Odredite intervale monotonosti i ekstreme funkcije $f(x) = x^4 + 4x - 5$.

Rješenje

- Domena funkcije f jednaka je $D_f = \mathbb{R}$.
- Derivacija funkcije f jednaka je $f'(x) = 4x^3 + 4$.
- Tražimo nultočke derivacije kako bismo dobili stacionarne točke.

$$4x^3 + 4 = 0 \implies 4x^3 = -4 \implies x^3 = -1 \implies \boxed{x = -1}$$

- $x = -1$ je jedina stacionarna točka funkcije f.
- Karakter stacionarne točke možemo ispitati pomoću prve derivacije ili pomoću druge derivacije.

$f'(x) = 4x^3 + 4$ $f(x) = x^4 + 4x - 5$

Ispitivanje karaktera stacionarne točke pomoću prve derivacije

	$-\infty$		-1		$+\infty$
f'					
f					

$f'(x) = 4x^3 + 4$ $f(x) = x^4 + 4x - 5$

Ispitivanje karaktera stacionarne točke pomoću prve derivacije

	$-\infty$		-1		$+\infty$
f'		$-$			
f					

$f'(x) = 4x^3 + 4$ $f(x) = x^4 + 4x - 5$

Ispitivanje karaktera stacionarne točke pomoću prve derivacije

	$-\infty$	-1	$+\infty$
f'	$-$	$+$	
f			

$f'(x) = 4x^3 + 4$ $f(x) = x^4 + 4x - 5$

Ispitivanje karaktera stacionarne točke pomoću prve derivacije

	$-\infty$		-1		$+\infty$
f'		$-$		$+$	
f		$\searrow$			

$f'(x) = 4x^3 + 4$ $f(x) = x^4 + 4x - 5$

Ispitivanje karaktera stacionarne točke pomoću prve derivacije

	$-\infty$		-1		$+\infty$
f'		$-$		$+$	
f		$\searrow$		$\nearrow$	

$f'(x) = 4x^3 + 4$ $\quad$ $f(x) = x^4 + 4x - 5$

Ispitivanje karaktera stacionarne točke pomoću prve derivacije

	$-\infty$		-1		$+\infty$
f'		$-$		$+$	
f		$\searrow$		$\nearrow$	

$$f(-1) = (-1)^4 + 4 \cdot (-1) - 5$$

$f'(x) = 4x^3 + 4$ $f(x) = x^4 + 4x - 5$

Ispitivanje karaktera stacionarne točke pomoću prve derivacije

	$-\infty$		-1		$+\infty$
f'		$-$		$+$	
f		$\searrow$		$\nearrow$	

$$f(-1) = (-1)^4 + 4 \cdot (-1) - 5 = -8$$

$f'(x) = 4x^3 + 4$ $f(x) = x^4 + 4x - 5$

Ispitivanje karaktera stacionarne točke pomoću prve derivacije

	$-\infty$	-1	$+\infty$
f'	$-$	$+$	
f	$\searrow$	$\nearrow$	

$$f(-1) = (-1)^4 + 4 \cdot (-1) - 5 = -8$$

- Funkcija f postiže lokalni minimum u točki $x = -1$ i on iznosi $f(-1) = -8$.

$f'(x) = 4x^3 + 4$ $f(x) = x^4 + 4x - 5$

Ispitivanje karaktera stacionarne točke pomoću prve derivacije

	$-\infty$	-1	$+\infty$
f'	$-$	$+$	
f	$\searrow$	$\nearrow$	

$$f(-1) = (-1)^4 + 4 \cdot (-1) - 5 = -8$$

- Funkcija f postiže lokalni minimum u točki $x = -1$ i on iznosi $f(-1) = -8$. U ovom slučaju taj minimum je ujedno i globalni minimum jer nakon što funkcija prestane padati, nakon toga stalno raste.

$f'(x) = 4x^3 + 4$ $f(x) = x^4 + 4x - 5$

Ispitivanje karaktera stacionarne točke pomoću prve derivacije

	$-\infty$	-1	$+\infty$
f'	$-$	$+$	
f	$\searrow$	$\nearrow$	

$$f(-1) = (-1)^4 + 4 \cdot (-1) - 5 = -8$$

- Funkcija f postiže lokalni minimum u točki $x = -1$ i on iznosi $f(-1) = -8$. U ovom slučaju taj minimum je ujedno i globalni minimum jer nakon što funkcija prestane padati, nakon toga stalno raste.
- Funkcija f strogo pada na intervalu $\langle -\infty, -1 \rangle$, a strogo raste na intervalu $\langle -1, +\infty \rangle$.

$f'(x) = 4x^3 + 4$

$f(x) = x^4 + 4x - 5$

Ispitivanje karaktera stacionarne točke pomoću druge derivacije

$f'(x) = 4x^3 + 4$ $f(x) = x^4 + 4x - 5$

Ispitivanje karaktera stacionarne točke pomoću druge derivacije

- $f''(x) = 12x^2$

$f'(x) = 4x^3 + 4$ $f(x) = x^4 + 4x - 5$

Ispitivanje karaktera stacionarne točke pomoću druge derivacije

- $f''(x) = 12x^2, \quad f''(-1) = 12 \cdot (-1)^2$

$f'(x) = 4x^3 + 4$ $f(x) = x^4 + 4x - 5$

Ispitivanje karaktera stacionarne točke pomoću druge derivacije

- $f''(x) = 12x^2, \quad f''(-1) = 12 \cdot (-1)^2 = 12$

$f'(x) = 4x^3 + 4$ $f(x) = x^4 + 4x - 5$

Ispitivanje karaktera stacionarne točke pomoću druge derivacije

- $f''(x) = 12x^2, \quad f''(-1) = 12 \cdot (-1)^2 = 12 > 0$

$f'(x) = 4x^3 + 4$ $f(x) = x^4 + 4x - 5$

Ispitivanje karaktera stacionarne točke pomoću druge derivacije

- $f''(x) = 12x^2, \quad f''(-1) = 12 \cdot (-1)^2 = 12 > 0$
- Kako je $f'(-1) = 0$ i $f''(-1) > 0$,

$f'(x) = 4x^3 + 4$ $\quad$ $f(x) = x^4 + 4x - 5$

Ispitivanje karaktera stacionarne točke pomoću druge derivacije

- $f''(x) = 12x^2, \quad f''(-1) = 12 \cdot (-1)^2 = 12 > 0$
- Kako je $f'(-1) = 0$ i $f''(-1) > 0$, zaključujemo da u točki $x = -1$ funkcija f postiže lokalni minimum

$f'(x) = 4x^3 + 4$ $f(x) = x^4 + 4x - 5$

Ispitivanje karaktera stacionarne točke pomoću druge derivacije

- $f''(x) = 12x^2, \quad f''(-1) = 12 \cdot (-1)^2 = 12 > 0$
- Kako je $f'(-1) = 0$ i $f''(-1) > 0$, zaključujemo da u točki $x = -1$ funkcija f postiže lokalni minimum koji je jednak

$$f(-1) =$$

$f'(x) = 4x^3 + 4$ $f(x) = x^4 + 4x - 5$

Ispitivanje karaktera stacionarne točke pomoću druge derivacije

- $f''(x) = 12x^2, \quad f''(-1) = 12 \cdot (-1)^2 = 12 > 0$
- Kako je $f'(-1) = 0$ i $f''(-1) > 0$, zaključujemo da u točki $x = -1$ funkcija f postiže lokalni minimum koji je jednak

$$f(-1) = (-1)^4 + 4 \cdot (-1) - 5$$

$f'(x) = 4x^3 + 4$ $f(x) = x^4 + 4x - 5$

Ispitivanje karaktera stacionarne točke pomoću druge derivacije

- $f''(x) = 12x^2, \quad f''(-1) = 12 \cdot (-1)^2 = 12 > 0$
- Kako je $f'(-1) = 0$ i $f''(-1) > 0$, zaključujemo da u točki $x = -1$ funkcija f postiže lokalni minimum koji je jednak

$$f(-1) = (-1)^4 + 4 \cdot (-1) - 5 = -8.$$

$f'(x) = 4x^3 + 4$ $f(x) = x^4 + 4x - 5$

Ispitivanje karaktera stacionarne točke pomoću druge derivacije

- $f''(x) = 12x^2, \quad f''(-1) = 12 \cdot (-1)^2 = 12 > 0$
- Kako je $f'(-1) = 0$ i $f''(-1) > 0$, zaključujemo da u točki $x = -1$ funkcija f postiže lokalni minimum koji je jednak
$$f(-1) = (-1)^4 + 4 \cdot (-1) - 5 = -8.$$
- U ovom slučaju nije odmah jasno radi li se o globalnom minimumu kao što je to bilo jasno iz prethodne tablice preko prve derivacije.

$f'(x) = 4x^3 + 4$ $f(x) = x^4 + 4x - 5$

Ispitivanje karaktera stacionarne točke pomoću druge derivacije

- $f''(x) = 12x^2, \quad f''(-1) = 12 \cdot (-1)^2 = 12 > 0$
- Kako je $f'(-1) = 0$ i $f''(-1) > 0$, zaključujemo da u točki $x = -1$ funkcija f postiže lokalni minimum koji je jednak

$$f(-1) = (-1)^4 + 4 \cdot (-1) - 5 = -8.$$

- U ovom slučaju nije odmah jasno radi li se o globalnom minimumu kao što je to bilo jasno iz prethodne tablice preko prve derivacije.
- Isto tako, ako želimo dobiti intervale monotonosti, onda to moramo raditi preko prve derivacije.

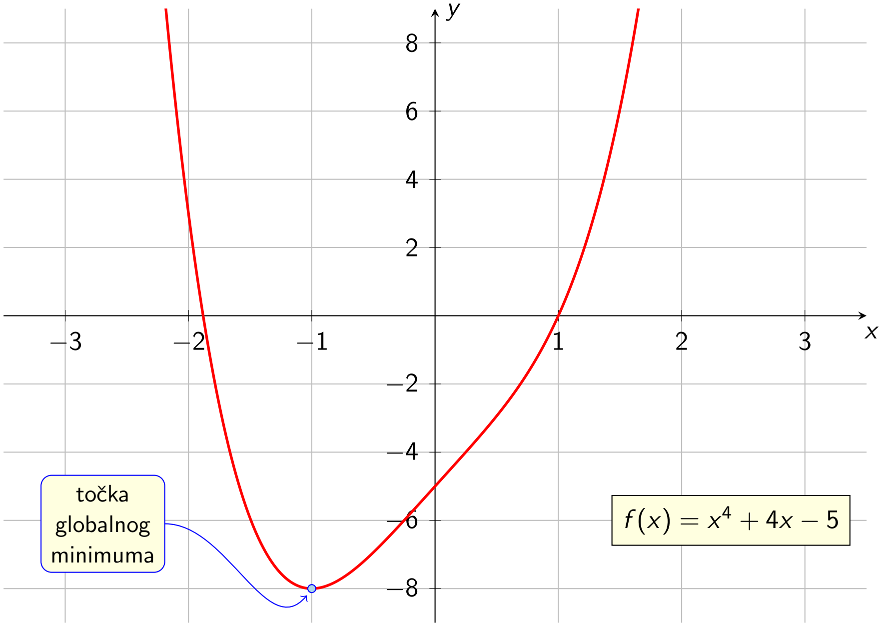
$f'(x) = 4x^3 + 4$ $f(x) = x^4 + 4x - 5$

Ispitivanje karaktera stacionarne točke pomoću druge derivacije

- $f''(x) = 12x^2, \quad f''(-1) = 12 \cdot (-1)^2 = 12 > 0$
- Kako je $f'(-1) = 0$ i $f''(-1) > 0$, zaključujemo da u točki $x = -1$ funkcija f postiže lokalni minimum koji je jednak

$$f(-1) = (-1)^4 + 4 \cdot (-1) - 5 = -8.$$

- U ovom slučaju nije odmah jasno radi li se o globalnom minimumu kao što je to bilo jasno iz prethodne tablice preko prve derivacije.
- Isto tako, ako želimo dobiti intervale monotonosti, onda to moramo raditi preko prve derivacije. Preko druge derivacije ne možemo dobiti intervale monotonosti funkcije, nego intervale konveksnosti i konkavnosti (tema idućih seminara).


y
8
6
4
2
−3
−2
−1
1
2
3
x
−2
−4
−6
−8
točka globalnog minimuma
$f(x) = x^4 + 4x - 5$

deseti zadatak

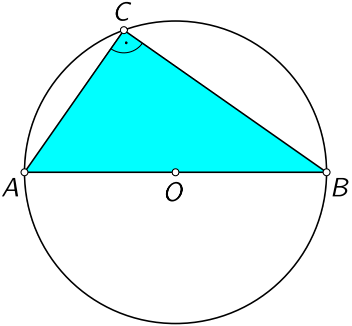
Zadatak 10

U kružnicu polumjera 2 upisan je pravokutni trokut. Odredite duljine stranica trokuta tako da njegova površina bude maksimalna.

Zadatak 10

U kružnicu polumjera 2 upisan je pravokutni trokut. Odredite duljine stranica trokuta tako da njegova površina bude maksimalna.

Rješenje



Zadatak 10

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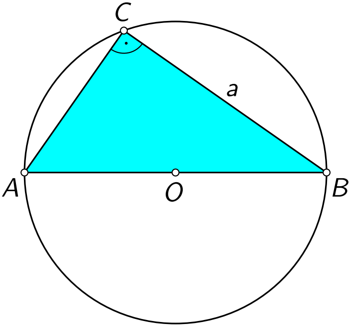
Rješenje



Zadatak 10

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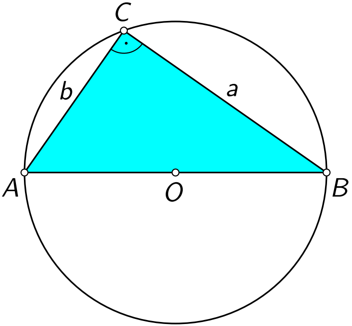
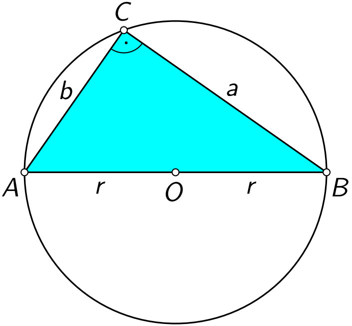
Rješenje



Zadatak 10

U kružnicu polumjera 2 upisan je pravokutni trokut. Odredite duljine stranica trokuta tako da njegova površina bude maksimalna.

Rješenje

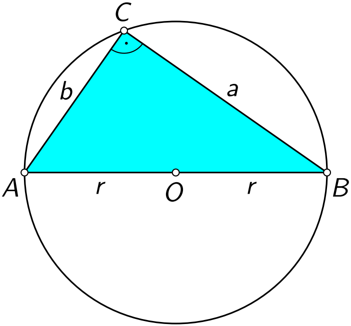


$$|OA| = |OB| = r = 2$$

Zadatak 10

U kružnicu polumjera 2 upisan je pravokutni trokut. Odredite duljine stranica trokuta tako da njegova površina bude maksimalna.

Rješenje



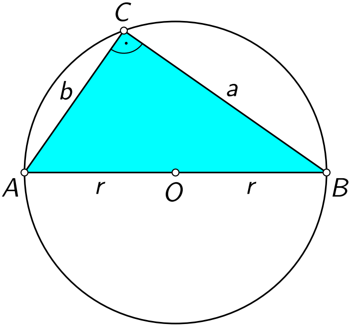
$$|OA| = |OB| = r = 2$$

$$c = 2r = 4$$

Zadatak 10

U kružnicu polumjera 2 upisan je pravokutni trokut. Odredite duljine stranica trokuta tako da njegova površina bude maksimalna.

Rješenje



$$|OA| = |OB| = r = 2$$

$$c = 2r = 4$$

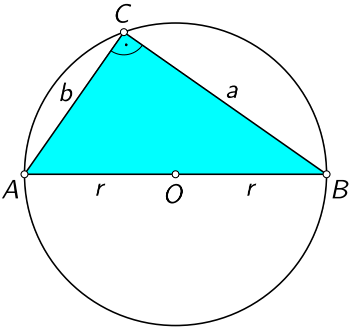
Talesov teorem

Zadatak 10

U kružnicu polumjera 2 upisan je pravokutni trokut. Odredite duljine stranica trokuta tako da njegova površina bude maksimalna.

Rješenje

$$c = 4$$



$$|OA| = |OB| = r = 2$$

$$c = 2r = 4$$

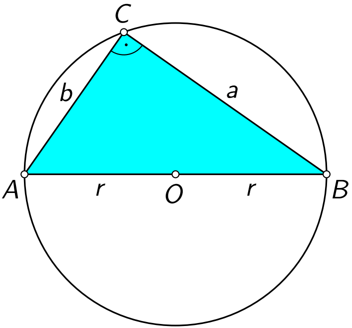
Talesov teorem

Zadatak 10

U kružnicu polumjera 2 upisan je pravokutni trokut. Odredite duljine stranica trokuta tako da njegova površina bude maksimalna.

Rješenje

$$\boxed{c = 4}$$



$$|OA| = |OB| = r = 2$$

$$c = 2r = 4$$

Talesov teorem

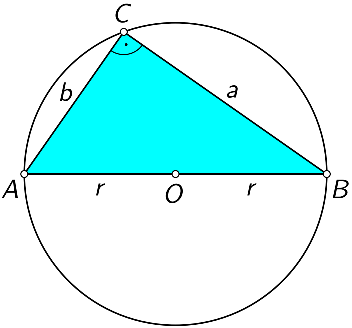
Zadatak 10

U kružnicu polumjera 2 upisan je pravokutni trokut. Odredite duljine stranica trokuta tako da njegova površina bude maksimalna.

Rješenje

$$\boxed{c = 4}$$

$$a^2 + b^2 = c^2$$



$$|OA| = |OB| = r = 2$$

$$c = 2r = 4$$

Talesov teorem

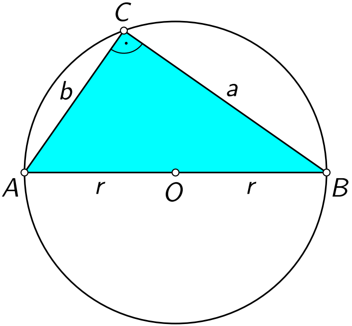
Zadatak 10

U kružnicu polumjera 2 upisan je pravokutni trokut. Odredite duljine stranica trokuta tako da njegova površina bude maksimalna.

Rješenje

$$\boxed{c = 4}$$

$$a^2 + b^2 = c^2$$ ← Pitagorin teorem



$$|OA| = |OB| = r = 2$$

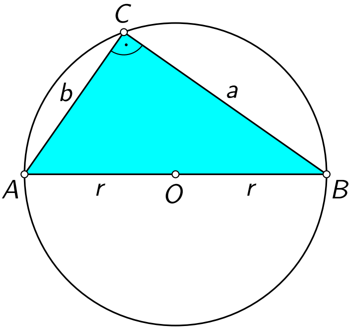
$$c = 2r = 4$$ ← Talesov teorem

Zadatak 10

U kružnicu polumjera 2 upisan je pravokutni trokut. Odredite duljine stranica trokuta tako da njegova površina bude maksimalna.

Rješenje

$$\boxed{c = 4}$$



$$|OA| = |OB| = r = 2$$

$$c = 2r = 4$$

Talesov teorem

$$a^2 + b^2 = c^2$$

Pitagorin teorem

$$a^2 + b^2 = 16$$

Zadatak 10

U kružnicu polumjera 2 upisan je pravokutni trokut. Odredite duljine stranica trokuta tako da njegova površina bude maksimalna.

Rješenje

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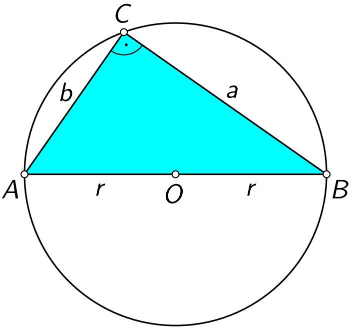

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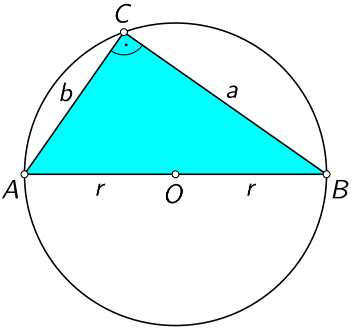

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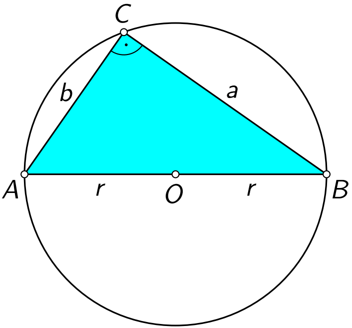

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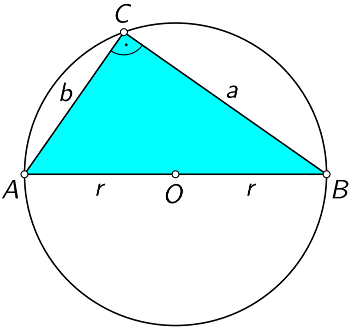

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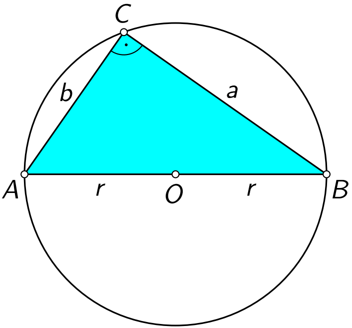

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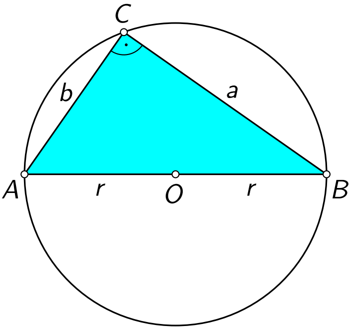

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$$P = \frac{1}{2}ab$$

Zadatak 10

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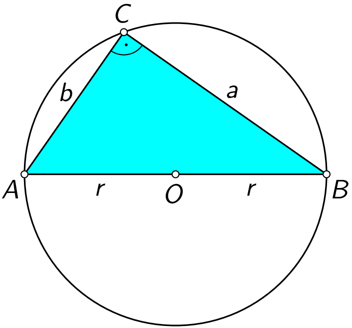

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$$P = \frac{1}{2}ab$$ ← površina pravokutnog trokuta

Zadatak 10

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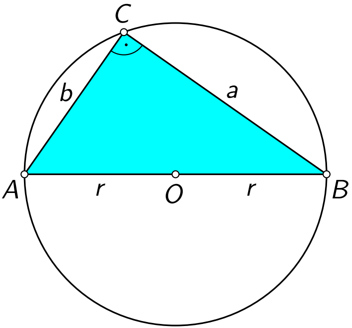

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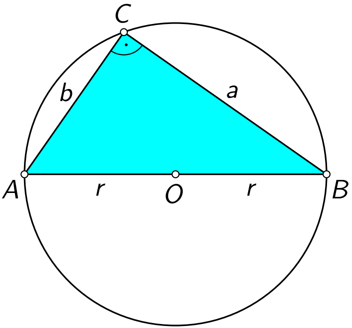

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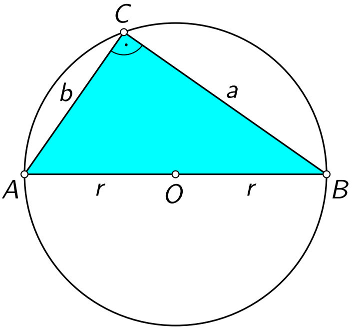

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$$P = \frac{1}{2}ab$$ ← površina pravokutnog trokuta

$$P = \frac{1}{2}a\sqrt{16 - a^2}$$

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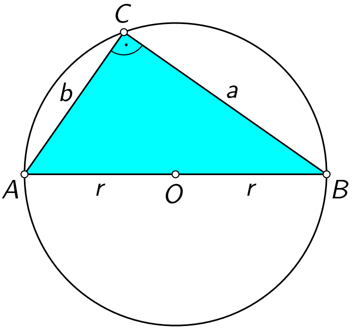

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$$P = \frac{1}{2}ab$$ ← površina pravokutnog trokuta

$$P = \frac{1}{2}a\sqrt{16 - a^2}$$

$$P(a) = \frac{1}{2}a\sqrt{16 - a^2}$$

Zadatak 10

U kružnicu polumjera 2 upisan je pravokutni trokut. Odredite duljine stranica trokuta tako da njegova površina bude maksimalna.

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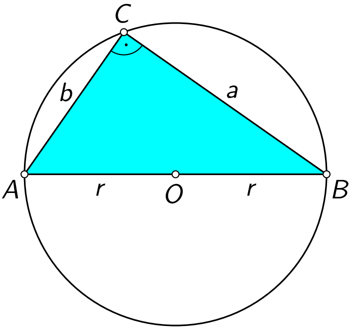

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$$P(a) = \frac{1}{2}a\sqrt{16 - a^2}$$

- Tražimo maksimum (ukoliko postoji) funkcije jedne varijable
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$$P(a) = \frac{1}{2} a \sqrt{16 - a^2}.$$
- Najprije odredimo derivaciju funkcije P.

$P'(a) =$

- Tražimo maksimum (ukoliko postoji) funkcije jedne varijable

$$P(a) = \frac{1}{2}a\sqrt{16 - a^2}.$$

- Najprije odredimo derivaciju funkcije P.

$P'(a) =$

$$(uv)'(x) = u'(x) \cdot v(x) + u(x) \cdot v'(x)$$

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$$P'(a) = \left(\frac{1}{2}a\right)'$$

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$$P'(a) = \left(\frac{1}{2}a\right)' \cdot \sqrt{16 - a^2} +$$

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$$P'(a) = \left(\frac{1}{2}a\right)' \cdot \sqrt{16 - a^2} + \frac{1}{2}a \cdot \left(\sqrt{16 - a^2}\right)' =$$

$$=$$

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$$= \frac{1}{2}\sqrt{16 - a^2} + \frac{1}{2}a \cdot$$

$$\left(\sqrt{\text{nešto}}\right)' = \frac{1}{2\sqrt{\text{nešto}}} \cdot (\text{nešto})'$$

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$$=$$

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$$= \frac{1}{2} \sqrt{16 - a^2} + \frac{1}{2} a \cdot \frac{}{\phantom{2\sqrt{16 - a^2}}}$$

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$$= \frac{1}{2}\sqrt{16 - a^2} + \frac{1}{2}a \cdot \frac{}{2\sqrt{16 - a^2}}$$

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$$= \frac{1}{2}\sqrt{16 - a^2} + \frac{1}{2}a \cdot \frac{1}{2\sqrt{16 - a^2}} \cdot (16 - a^2)' =$$

$$= \frac{1}{2}\sqrt{16 - a^2} + \frac{1}{2}a \cdot \frac{-2a}{2\sqrt{16 - a^2}}$$

- Tražimo maksimum (ukoliko postoji) funkcije jedne varijable

$$P(a) = \frac{1}{2} a \sqrt{16 - a^2}.$$

- Najprije odredimo derivaciju funkcije P.

$$P'(a) = \left(\frac{1}{2} a\right)' \cdot \sqrt{16 - a^2} + \frac{1}{2} a \cdot \left(\sqrt{16 - a^2}\right)' =$$

$$= \frac{1}{2}\sqrt{16 - a^2} + \frac{1}{2} a \cdot \frac{1}{2\sqrt{16 - a^2}} \cdot (16 - a^2)' =$$

$$= \frac{1}{2}\sqrt{16 - a^2} + \frac{1}{2} a \cdot \frac{-2a}{2\sqrt{16 - a^2}} =$$

- Tražimo maksimum (ukoliko postoji) funkcije jedne varijable

$$P(a) = \frac{1}{2}a\sqrt{16 - a^2}.$$

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$$P'(a) = \left(\frac{1}{2}a\right)' \cdot \sqrt{16 - a^2} + \frac{1}{2}a \cdot \left(\sqrt{16 - a^2}\right)' =$$

$$= \frac{1}{2}\sqrt{16 - a^2} + \frac{1}{2}a \cdot \frac{1}{2\sqrt{16 - a^2}} \cdot (16 - a^2)' =$$

$$= \frac{1}{2}\sqrt{16 - a^2} + \frac{1}{2}a \cdot \frac{-2a}{2\sqrt{16 - a^2}} = \frac{1}{2}\sqrt{16 - a^2}$$

- Tražimo maksimum (ukoliko postoji) funkcije jedne varijable

$$P(a) = \frac{1}{2}a\sqrt{16 - a^2}.$$

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$$P'(a) = \left(\frac{1}{2}a\right)' \cdot \sqrt{16 - a^2} + \frac{1}{2}a \cdot \left(\sqrt{16 - a^2}\right)' =$$

$$= \frac{1}{2}\sqrt{16 - a^2} + \frac{1}{2}a \cdot \frac{1}{2\sqrt{16 - a^2}} \cdot (16 - a^2)' =$$

$$= \frac{1}{2}\sqrt{16 - a^2} + \frac{1}{2}a \cdot \frac{-2a}{2\sqrt{16 - a^2}} = \frac{1}{2}\sqrt{16 - a^2} - \underline{\hspace{3cm}}$$

- Tražimo maksimum (ukoliko postoji) funkcije jedne varijable

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$$= \frac{1}{2}\sqrt{16 - a^2} + \frac{1}{2}a \cdot \frac{1}{2\sqrt{16 - a^2}} \cdot (16 - a^2)' =$$

$$= \frac{1}{2}\sqrt{16 - a^2} + \frac{1}{2}a \cdot \frac{-2a}{2\sqrt{16 - a^2}} = \frac{1}{2}\sqrt{16 - a^2} - \frac{}{2\sqrt{16 - a^2}}$$

- Tražimo maksimum (ukoliko postoji) funkcije jedne varijable

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$$= \frac{1}{2}\sqrt{16 - a^2} + \frac{1}{2}a \cdot \frac{1}{2\sqrt{16 - a^2}} \cdot (16 - a^2)' =$$

$$= \frac{1}{2}\sqrt{16 - a^2} + \frac{1}{2}a \cdot \frac{-2a}{2\sqrt{16 - a^2}} = \frac{1}{2}\sqrt{16 - a^2} - \frac{a^2}{2\sqrt{16 - a^2}}$$

- Tražimo maksimum (ukoliko postoji) funkcije jedne varijable
$$P(a) = \frac{1}{2}a\sqrt{16 - a^2}.$$
- Najprije odredimo derivaciju funkcije P.

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$$= \frac{1}{2}\sqrt{16 - a^2} + \frac{1}{2}a \cdot \frac{1}{2\sqrt{16 - a^2}} \cdot (16 - a^2)' =$$

$$= \frac{1}{2}\sqrt{16 - a^2} + \frac{1}{2}a \cdot \frac{-2a}{2\sqrt{16 - a^2}} = \frac{1}{2}\sqrt{16 - a^2} - \frac{a^2}{2\sqrt{16 - a^2}} =$$

$$= \frac{}{}$$

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$$= \frac{\quad}{2\sqrt{16 - a^2}}$$

- Tražimo maksimum (ukoliko postoji) funkcije jedne varijable

$$P(a) = \frac{1}{2}a\sqrt{16-a^2}.$$

- Najprije odredimo derivaciju funkcije P.

$$P'(a) = \left(\frac{1}{2}a\right)' \cdot \sqrt{16-a^2} + \frac{1}{2}a \cdot \left(\sqrt{16-a^2}\right)' =$$

$$= \frac{1}{2}\sqrt{16-a^2} + \frac{1}{2}a \cdot \frac{1}{2\sqrt{16-a^2}} \cdot (16-a^2)' =$$

$$= \frac{1}{2}\sqrt{16-a^2} + \frac{1}{2}a \cdot \frac{-2a}{2\sqrt{16-a^2}} = \frac{1}{2}\sqrt{16-a^2} - \frac{a^2}{2\sqrt{16-a^2}} =$$

$$= \frac{\sqrt{16-a^2}^{\,2}}{2\sqrt{16-a^2}}$$

- Tražimo maksimum (ukoliko postoji) funkcije jedne varijable
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- Najprije odredimo derivaciju funkcije P.

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$$= \frac{1}{2}\sqrt{16 - a^2} + \frac{1}{2}a \cdot \frac{1}{2\sqrt{16 - a^2}} \cdot (16 - a^2)' =$$

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$$= \frac{\sqrt{16-a^2}^{\,2} - a^2}{2\sqrt{16-a^2}} = \underline{\hspace{3cm}}$$

- Tražimo maksimum (ukoliko postoji) funkcije jedne varijable

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$$= \frac{1}{2}\sqrt{16 - a^2} + \frac{1}{2}a \cdot \frac{1}{2\sqrt{16 - a^2}} \cdot (16 - a^2)' =$$

$$= \frac{1}{2}\sqrt{16 - a^2} + \frac{1}{2}a \cdot \frac{-2a}{2\sqrt{16 - a^2}} = \frac{1}{2}\sqrt{16 - a^2} - \frac{a^2}{2\sqrt{16 - a^2}} =$$

$$= \frac{\sqrt{16 - a^2}^{\,2} - a^2}{2\sqrt{16 - a^2}} = \frac{}{2\sqrt{16 - a^2}}$$

- Tražimo maksimum (ukoliko postoji) funkcije jedne varijable

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$$= \frac{1}{2}\sqrt{16 - a^2} + \frac{1}{2}a \cdot \frac{1}{2\sqrt{16 - a^2}} \cdot (16 - a^2)' =$$

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$$= \frac{\sqrt{16 - a^2}^{\,2} - a^2}{2\sqrt{16 - a^2}} = \frac{16 - a^2}{2\sqrt{16 - a^2}}$$

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$$= \frac{1}{2}\sqrt{16 - a^2} + \frac{1}{2}a \cdot \frac{-2a}{2\sqrt{16 - a^2}} = \frac{1}{2}\sqrt{16 - a^2} - \frac{a^2}{2\sqrt{16 - a^2}} =$$

$$= \frac{\sqrt{16 - a^2}^{\,2} - a^2}{2\sqrt{16 - a^2}} = \frac{16 - a^2 - a^2}{2\sqrt{16 - a^2}}$$

- Tražimo maksimum (ukoliko postoji) funkcije jedne varijable

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$$= \frac{1}{2}\sqrt{16 - a^2} + \frac{1}{2}a \cdot \frac{1}{2\sqrt{16 - a^2}} \cdot (16 - a^2)' =$$

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$$= \frac{\sqrt{16 - a^2}^{\,2} - a^2}{2\sqrt{16 - a^2}} = \frac{16 - a^2 - a^2}{2\sqrt{16 - a^2}} = \frac{16 - 2a^2}{2\sqrt{16 - a^2}}$$

- Tražimo maksimum (ukoliko postoji) funkcije jedne varijable

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$$= \frac{\sqrt{16 - a^2}^{\,2} - a^2}{2\sqrt{16 - a^2}} = \frac{16 - a^2 - a^2}{2\sqrt{16 - a^2}} = \frac{16 - 2a^2}{2\sqrt{16 - a^2}} = \frac{8 - a^2}{\sqrt{16 - a^2}}$$

$P'(a) = 0$

$$P'(a) = \frac{8 - a^2}{\sqrt{16 - a^2}}$$

$$P(a) = \frac{1}{2}a\sqrt{16 - a^2}$$

$$P'(a) = 0$$

$$\frac{8 - a^2}{\sqrt{16 - a^2}} = 0$$

$$\boxed{P'(a) = \frac{8 - a^2}{\sqrt{16 - a^2}}}$$

$$\boxed{P(a) = \frac{1}{2} a \sqrt{16 - a^2}}$$

$$P'(a) = 0$$

$$\frac{8 - a^2}{\sqrt{16 - a^2}} = 0$$

$$8 - a^2 = 0$$

$$P'(a) = \frac{8 - a^2}{\sqrt{16 - a^2}}$$

$$P(a) = \frac{1}{2}a\sqrt{16 - a^2}$$

$$
\begin{aligned}
P'(a) &= 0 \\
\frac{8 - a^2}{\sqrt{16 - a^2}} &= 0 \\
8 - a^2 &= 0 \\
a^2 &= 8
\end{aligned}
$$

$$P'(a) = \frac{8 - a^2}{\sqrt{16 - a^2}}$$

$$P(a) = \frac{1}{2}a\sqrt{16 - a^2}$$

$$P'(a) = \frac{8 - a^2}{\sqrt{16 - a^2}}$$

$$P(a) = \frac{1}{2}a\sqrt{16 - a^2}$$

$$\begin{aligned} P'(a) &= 0 \\ \frac{8 - a^2}{\sqrt{16 - a^2}} &= 0 \\ 8 - a^2 &= 0 \\ a^2 &= 8 \\ a &= \pm 2\sqrt{2} \end{aligned}$$

$$P'(a) = \frac{8 - a^2}{\sqrt{16 - a^2}}$$

$$P(a) = \frac{1}{2}a\sqrt{16 - a^2}$$

$$P'(a) = 0$$

$$\frac{8 - a^2}{\sqrt{16 - a^2}} = 0$$

$$8 - a^2 = 0$$

$$a^2 = 8$$

$$a = \pm 2\sqrt{2}$$ ← $a > 0$

$$P'(a) = \frac{8 - a^2}{\sqrt{16 - a^2}}$$

$$P(a) = \frac{1}{2}a\sqrt{16 - a^2}$$

$$\begin{aligned}
P'(a) &= 0 \\
\frac{8 - a^2}{\sqrt{16 - a^2}} &= 0 \\
8 - a^2 &= 0 \\
a^2 &= 8 \\
a &= \pm 2\sqrt{2} \quad \leftarrow \boxed{a > 0} \\
a &= 2\sqrt{2}
\end{aligned}$$

$$P'(a) = 0$$

$$\frac{8 - a^2}{\sqrt{16 - a^2}} = 0$$

$$8 - a^2 = 0$$

$$a^2 = 8$$

$$a = \pm 2\sqrt{2} \quad \leftarrow \boxed{a > 0}$$

$$\boxed{a = 2\sqrt{2}}$$

$$\boxed{P'(a) = \frac{8 - a^2}{\sqrt{16 - a^2}}}$$

$$\boxed{P(a) = \frac{1}{2} a \sqrt{16 - a^2}}$$

$$P'(a) = \frac{8 - a^2}{\sqrt{16 - a^2}}$$

$$P(a) = \frac{1}{2}a\sqrt{16 - a^2}$$

$$P'(a) = 0$$

$$\frac{8 - a^2}{\sqrt{16 - a^2}} = 0$$

$$8 - a^2 = 0$$

$$a^2 = 8$$

$$a = \pm 2\sqrt{2} \quad \leftarrow \quad a > 0$$

$$\boxed{a = 2\sqrt{2}}$$

	0	$2\sqrt{2}$	4
P'			
P			

$$P'(a) = \frac{8-a^2}{\sqrt{16-a^2}}$$

$$P(a) = \frac{1}{2}a\sqrt{16-a^2}$$

$$P'(a) = 0$$

$$\frac{8-a^2}{\sqrt{16-a^2}} = 0$$

$$8 - a^2 = 0$$

$$a^2 = 8$$

$$a = \pm 2\sqrt{2} \quad \leftarrow \quad a > 0$$

$$\boxed{a = 2\sqrt{2}}$$

$$16 - a^2 \geqslant 0$$

	0	$2\sqrt{2}$	4
P'			
P			

$$P'(a) = 0$$

$$\frac{8 - a^2}{\sqrt{16 - a^2}} = 0$$

$$8 - a^2 = 0$$

$$a^2 = 8$$

$$a = \pm 2\sqrt{2} \quad \longleftarrow \quad \boxed{a > 0}$$

$$\boxed{a = 2\sqrt{2}}$$

$$\boxed{P'(a) = \frac{8 - a^2}{\sqrt{16 - a^2}}}$$

$$\boxed{P(a) = \frac{1}{2} a \sqrt{16 - a^2}}$$

$$16 - a^2 \geqslant 0 \iff a^2 \leqslant 16$$

	0	$2\sqrt{2}$	4
P'			
P			

$$P'(a) = 0$$

$$\frac{8-a^2}{\sqrt{16-a^2}} = 0$$

$$8 - a^2 = 0$$

$$a^2 = 8$$

$$a = \pm 2\sqrt{2} \quad \leftarrow \quad a > 0$$

$$\boxed{a = 2\sqrt{2}}$$

$$\boxed{P'(a) = \frac{8-a^2}{\sqrt{16-a^2}}}$$

$$\boxed{P(a) = \frac{1}{2}a\sqrt{16-a^2}}$$

$$16 - a^2 \geqslant 0 \iff a^2 \leqslant 16 \iff a \in [-4, 4]$$

	0	$2\sqrt{2}$	4
P'			
P			

$$P'(a) = 0$$

$$\frac{8-a^2}{\sqrt{16-a^2}} = 0$$

$$8 - a^2 = 0$$

$$a^2 = 8$$

$$a = \pm 2\sqrt{2} \quad \leftarrow \quad a > 0$$

$$\boxed{a = 2\sqrt{2}}$$

$$P'(a) = \frac{8-a^2}{\sqrt{16-a^2}}$$

$$P(a) = \frac{1}{2}a\sqrt{16-a^2}$$

$$16 - a^2 \geqslant 0 \;\Leftrightarrow\; a^2 \leqslant 16 \;\Leftrightarrow\; a \in [-4, 4]$$

	0	$2\sqrt{2}$	4
P'			
P			

$$P'(a) = 0$$

$$\frac{8-a^2}{\sqrt{16-a^2}} = 0$$

$$8 - a^2 = 0$$

$$a^2 = 8$$

$$a = \pm 2\sqrt{2} \quad \leftarrow \quad a > 0$$

$$\boxed{a = 2\sqrt{2}}$$

$$P'(a) = \frac{8-a^2}{\sqrt{16-a^2}}$$

$$P(a) = \frac{1}{2}a\sqrt{16-a^2}$$

$$16 - a^2 \geqslant 0 \;\Leftrightarrow\; a^2 \leqslant 16 \;\Leftrightarrow\; a \in [-4, 4]$$

	0	$2\sqrt{2}$	4
P'			
P			

$$P'(a) = 0$$

$$\frac{8-a^2}{\sqrt{16-a^2}} = 0$$

$$8 - a^2 = 0$$

$$a^2 = 8$$

$$a = \pm 2\sqrt{2} \quad \leftarrow \quad a > 0$$

$$\boxed{a = 2\sqrt{2}}$$

$$P'(a) = \frac{8-a^2}{\sqrt{16-a^2}}$$

$$P(a) = \frac{1}{2}a\sqrt{16-a^2}$$

$$16 - a^2 \geqslant 0 \iff a^2 \leqslant 16 \iff a \in [-4, 4]$$

	0		$2\sqrt{2}$		4
P'		$+$			
P					

$$P'(a) = 0$$

$$\frac{8-a^2}{\sqrt{16-a^2}} = 0$$

$$8 - a^2 = 0$$

$$a^2 = 8$$

$$a = \pm 2\sqrt{2}$$

$a > 0$

$$\boxed{a = 2\sqrt{2}}$$

$$P'(a) = \frac{8-a^2}{\sqrt{16-a^2}}$$

$$P(a) = \frac{1}{2}a\sqrt{16-a^2}$$

$$16 - a^2 \geqslant 0 \iff a^2 \leqslant 16 \iff a \in [-4, 4]$$

	0		$2\sqrt{2}$		4
P'		$+$		$-$	
P					

$$P'(a) = 0$$

$$\frac{8 - a^2}{\sqrt{16 - a^2}} = 0$$

$$8 - a^2 = 0$$

$$a^2 = 8$$

$$a = \pm 2\sqrt{2} \quad \leftarrow \quad a > 0$$

$$\boxed{a = 2\sqrt{2}}$$

$$\boxed{P'(a) = \frac{8 - a^2}{\sqrt{16 - a^2}}}$$

$$\boxed{P(a) = \frac{1}{2} a \sqrt{16 - a^2}}$$

$$16 - a^2 \geqslant 0 \iff a^2 \leqslant 16 \iff a \in [-4, 4]$$

	0		$2\sqrt{2}$		4
P'		$+$		$-$	
P		$\nearrow$			

$$P'(a) = 0$$

$$\frac{8 - a^2}{\sqrt{16 - a^2}} = 0$$

$$8 - a^2 = 0$$

$$a^2 = 8$$

$$a = \pm 2\sqrt{2}$$ ⟵ $a > 0$

$$\boxed{a = 2\sqrt{2}}$$

$$P'(a) = \frac{8 - a^2}{\sqrt{16 - a^2}}$$

$$P(a) = \frac{1}{2} a \sqrt{16 - a^2}$$

$$16 - a^2 \geqslant 0 \iff a^2 \leqslant 16 \iff a \in [-4, 4]$$

	0		$2\sqrt{2}$		4
P'		$+$		$-$	
P		$\nearrow$		$\searrow$	

$$P'(a) = 0$$

$$\frac{8 - a^2}{\sqrt{16 - a^2}} = 0$$

$$8 - a^2 = 0$$

$$a^2 = 8$$

$$a = \pm 2\sqrt{2} \quad \leftarrow \quad a > 0$$

$$\boxed{a = 2\sqrt{2}}$$

$$\boxed{P'(a) = \frac{8 - a^2}{\sqrt{16 - a^2}}} \qquad \boxed{P(a) = \frac{1}{2} a \sqrt{16 - a^2}}$$

$$16 - a^2 \geqslant 0 \iff a^2 \leqslant 16 \iff a \in [-4, 4]$$

	0		$2\sqrt{2}$		4
P'		$+$		$-$	
P		$\nearrow$		$\searrow$	

✏ Funkcija P u točki $a = 2\sqrt{2}$ postiže globalni maksimum 4 na segmentu $[0, 4]$.

$$P'(a) = 0$$

$$\frac{8-a^2}{\sqrt{16-a^2}} = 0$$

$$8 - a^2 = 0$$

$$a^2 = 8$$

$$a = \pm 2\sqrt{2}$$

$a > 0$

$$\boxed{a = 2\sqrt{2}}$$

$$P'(a) = \frac{8-a^2}{\sqrt{16-a^2}}$$

$$P(a) = \frac{1}{2}a\sqrt{16-a^2}$$

$$16 - a^2 \geqslant 0 \quad \Leftrightarrow \quad a^2 \leqslant 16 \quad \Leftrightarrow \quad a \in [-4, 4]$$

	0	$2\sqrt{2}$	4
P'	$+$	$-$	
P	$\nearrow$	$\searrow$	

✏ Funkcija P u točki $a = 2\sqrt{2}$ postiže globalni maksimum 4 na segmentu $[0, 4]$.

$$P\left(2\sqrt{2}\right) =$$

$$P'(a) = \frac{8-a^2}{\sqrt{16-a^2}}$$

$$P(a) = \frac{1}{2}a\sqrt{16-a^2}$$

$$16 - a^2 \geqslant 0 \iff a^2 \leqslant 16 \iff a \in [-4, 4]$$

$$P'(a) = 0$$

$$\frac{8-a^2}{\sqrt{16-a^2}} = 0$$

$$8 - a^2 = 0$$

$$a^2 = 8$$

$$a = \pm 2\sqrt{2} \quad (a > 0)$$

$$\boxed{a = 2\sqrt{2}}$$

	0		$2\sqrt{2}$		4
P'		$+$		$-$	
P		$\nearrow$		$\searrow$	

✏ Funkcija P u točki $a = 2\sqrt{2}$ postiže globalni maksimum 4 na segmentu $[0, 4]$.

$$P\left(2\sqrt{2}\right) = \frac{1}{2} \cdot 2\sqrt{2}$$

$$P'(a) = \frac{8 - a^2}{\sqrt{16 - a^2}}$$

$$P(a) = \frac{1}{2} a \sqrt{16 - a^2}$$

$$16 - a^2 \geqslant 0 \;\Leftrightarrow\; a^2 \leqslant 16 \;\Leftrightarrow\; a \in [-4, 4]$$

$$P'(a) = 0$$

$$\frac{8 - a^2}{\sqrt{16 - a^2}} = 0$$

$$8 - a^2 = 0$$

$$a^2 = 8$$

$$a = \pm 2\sqrt{2}$$

$a > 0$

$$\boxed{a = 2\sqrt{2}}$$

	0		$2\sqrt{2}$		4
P'		$+$		$-$	
P		$\nearrow$		$\searrow$	

✏ Funkcija P u točki $a = 2\sqrt{2}$ postiže globalni maksimum 4 na segmentu $[0, 4]$.

$$P\left(2\sqrt{2}\right) = \frac{1}{2} \cdot 2\sqrt{2} \cdot \sqrt{16 - \left(2\sqrt{2}\right)^2}$$

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P'		$+$		$-$	
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$$P(2\sqrt{2}) = \frac{1}{2} \cdot 2\sqrt{2} \cdot \sqrt{16 - \left(2\sqrt{2}\right)^2} = \sqrt{2} \cdot \sqrt{8} = \sqrt{16}$$

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$$a = \pm 2\sqrt{2} \quad \leftarrow \quad a > 0$$

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	0		$2\sqrt{2}$		4
P'		$+$		$-$	
P		$\nearrow$		$\searrow$	

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	0		$2\sqrt{2}$		4
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✏ Funkcija P u točki $a = 2\sqrt{2}$ postiže globalni maksimum 4 na segmentu $[0, 4]$.

$$P_{\max} = 4$$

$$P(2\sqrt{2}) = \frac{1}{2} \cdot 2\sqrt{2} \cdot \sqrt{16 - \left(2\sqrt{2}\right)^2} = \sqrt{2} \cdot \sqrt{8} = \sqrt{16} = 4$$

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$$a = \pm 2\sqrt{2}$$
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$a > 0$

	0		$2\sqrt{2}$		4
P'		$+$		$-$	
P		$\nearrow$		$\searrow$	

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$$P_{\max} = 4$$

$$P(2\sqrt{2}) = \frac{1}{2} \cdot 2\sqrt{2} \cdot \sqrt{16 - (2\sqrt{2})^2} = \sqrt{2} \cdot \sqrt{8} = \sqrt{16} = 4$$

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$$16 - a^2 \geqslant 0 \;\Leftrightarrow\; a^2 \leqslant 16 \;\Leftrightarrow\; a \in [-4, 4]$$

	0	$2\sqrt{2}$	4
P'	$+$	$-$	
P	$\nearrow$	$\searrow$	

✏ Funkcija P u točki $a = 2\sqrt{2}$ postiže globalni maksimum 4 na segmentu $[0, 4]$.

$$\boxed{P_{\text{max}} = 4}$$

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$$b = \sqrt{16 - a^2}$$

$$P'(a) = \frac{8-a^2}{\sqrt{16-a^2}}$$

$$P(a) = \frac{1}{2}a\sqrt{16-a^2}$$

$$P'(a) = 0$$

$$\frac{8-a^2}{\sqrt{16-a^2}} = 0$$

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$$a^2 = 8$$

$$a = \pm 2\sqrt{2} \quad \leftarrow \quad a > 0$$

$$\boxed{a = 2\sqrt{2}}$$

$$16 - a^2 \geqslant 0 \;\Leftrightarrow\; a^2 \leqslant 16 \;\Leftrightarrow\; a \in [-4, 4]$$

	0	$2\sqrt{2}$	4
P'	$+$	$-$	
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$$16 - a^2 \geqslant 0 \iff a^2 \leqslant 16 \iff a \in [-4, 4]$$

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$$a = \pm 2\sqrt{2} \quad \leftarrow \boxed{a > 0}$$

$$\boxed{a = 2\sqrt{2}}$$

$$b = 2\sqrt{2}$$

$$\boxed{P'(a) = \frac{8-a^2}{\sqrt{16-a^2}}}$$

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$$16 - a^2 \geqslant 0 \;\Leftrightarrow\; a^2 \leqslant 16 \;\Leftrightarrow\; a \in [-4, 4]$$

	0		$2\sqrt{2}$		4
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$$8 - a^2 = 0$$

$$a^2 = 8$$

$$a = \pm 2\sqrt{2} \quad \leftarrow \quad a > 0$$

$$\boxed{a = 2\sqrt{2}}$$

$$c = 4 \qquad \boxed{b = 2\sqrt{2}}$$

	0		$2\sqrt{2}$		4
P'		$+$		$-$	
P		$\nearrow$		$\searrow$	

✏ Funkcija P u točki $a = 2\sqrt{2}$ postiže globalni maksimum 4 na segmentu $[0, 4]$.

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$$a^2 = 8$$

$$a = \pm 2\sqrt{2} \quad \leftarrow \quad a > 0$$

$$a = 2\sqrt{2}$$

$$b = 2\sqrt{2}$$

$$c = 4$$

	0		$2\sqrt{2}$		4
P'		$+$		$-$	
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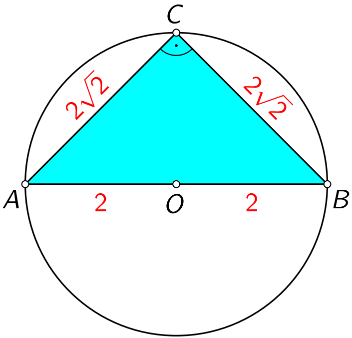


✏ Pravokutni trokut maksimalne površine upisan u kružnicu polumjera 2 jest jednakokračni pravokutni trokut čije su duljine kateta jednake $2\sqrt{2}$, a duljina hipotenuze je jednaka 4.

✏ Površina takvog trokuta jednaka je 4, tj. po iznosu (bez mjernih jedinica) je jednaka duljini hipotenuze.

jedanaesti zadatak

Zadatak 11

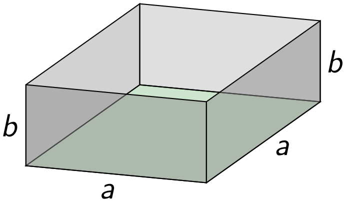
Odredite dimenzije otvorenog bazena s kvadratnim dnom volumena $32\,m^3$ *tako da za oblaganje njegovih bočnih dijelova i dna bude potrebna najmanja količina materijala.*

Zadatak 11

Odredite dimenzije otvorenog bazena s kvadratnim dnom volumena $32\,m^3$ tako da za oblaganje njegovih bočnih dijelova i dna bude potrebna najmanja količina materijala.

Rješenje

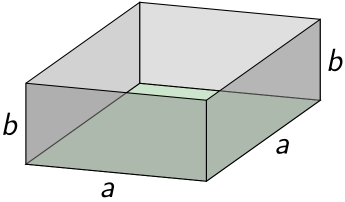
- Bazen ima oblik kvadra čija baza je kvadrat pri čemu taj kvadar nema gornju bazu (gornja strana je otvorena).



Zadatak 11

Odredite dimenzije otvorenog bazena s kvadratnim dnom volumena $32\,m^3$ tako da za oblaganje njegovih bočnih dijelova i dna bude potrebna najmanja količina materijala.

Rješenje

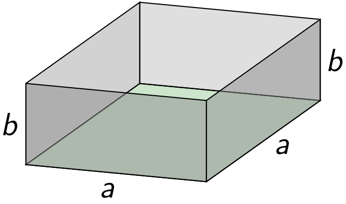


- Bazen ima oblik kvadra čija baza je kvadrat pri čemu taj kvadar nema gornju bazu (gornja strana je otvorena).
- Uz zadani volumen kvadra tražimo njegove dimenzije tako da mu oplošje bude minimalno (u tom slučaju potrošit će se najmanja količina materijala za izgradnju bazena).

Zadatak 11

Odredite dimenzije otvorenog bazena s kvadratnim dnom volumena $32\,m^3$ tako da za oblaganje njegovih bočnih dijelova i dna bude potrebna najmanja količina materijala.

Rješenje



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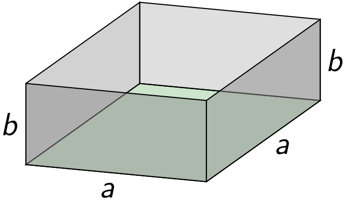
- Bazen je omeđen s jednim kvadratom duljine stranice a i četiri pravokutnika s duljinama stranica a i b.

Zadatak 11

Odredite dimenzije otvorenog bazena s kvadratnim dnom volumena $32\, m^3$ tako da za oblaganje njegovih bočnih dijelova i dna bude potrebna najmanja količina materijala.

Rješenje



- Bazen ima oblik kvadra čija baza je kvadrat pri čemu taj kvadar nema gornju bazu (gornja strana je otvorena).
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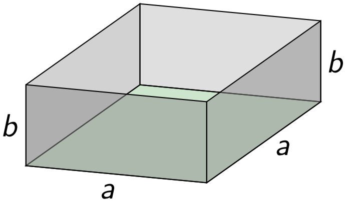
- Bazen je omeđen s jednim kvadratom duljine stranice a i četiri pravokutnika s duljinama stranica a i b. Stoga je njegov volumen jednak $V = a^2 b$,

Zadatak 11

Odredite dimenzije otvorenog bazena s kvadratnim dnom volumena $32\,m^3$ tako da za oblaganje njegovih bočnih dijelova i dna bude potrebna najmanja količina materijala.

Rješenje



- Bazen ima oblik kvadra čija baza je kvadrat pri čemu taj kvadar nema gornju bazu (gornja strana je otvorena).
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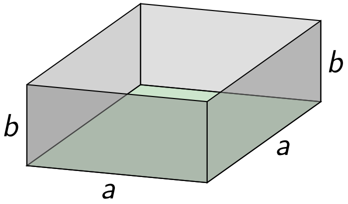
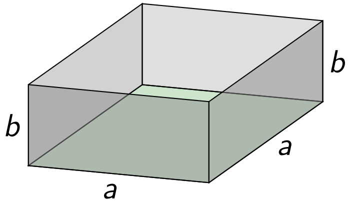
- Bazen je omeđen s jednim kvadratom duljine stranice a i četiri pravokutnika s duljinama stranica a i b. Stoga je njegov volumen jednak $V = a^2 b$, a oplošje je jednako $O = a^2 + 4ab$.

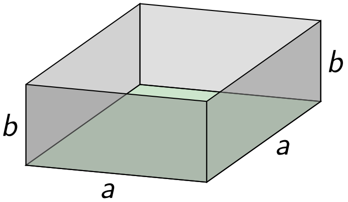

b
b
a
a

$V = a^2 b$



$V = a^2 b$

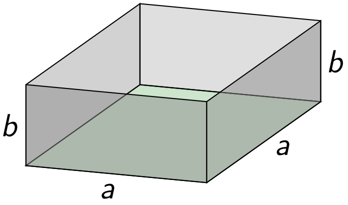
$32 = a^2 b$



$$V = a^2 b$$

$$32 = a^2 b$$

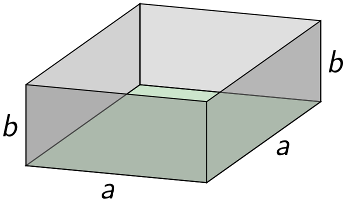
$$b = \frac{32}{a^2}$$



$$V = a^2 b$$

$$32 = a^2 b$$

$$\boxed{b = \frac{32}{a^2}}$$

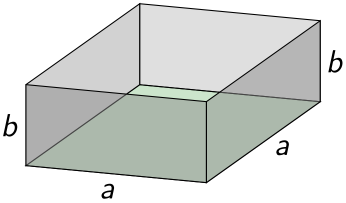


$V = a^2 b$

$O = a^2 + 4ab$

$32 = a^2 b$

$$\boxed{b = \frac{32}{a^2}}$$



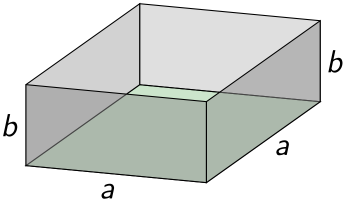
$$V = a^2 b$$

$$32 = a^2 b$$

$$\boxed{b = \frac{32}{a^2}}$$

$$O = a^2 + 4ab$$

$$O = a^2 + 4a \cdot$$



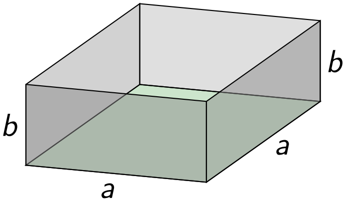
$$V = a^2 b$$

$$32 = a^2 b$$

$$\boxed{b = \frac{32}{a^2}}$$

$$O = a^2 + 4ab$$

$$O = a^2 + 4a \cdot \frac{32}{a^2}$$



$V = a^2 b$

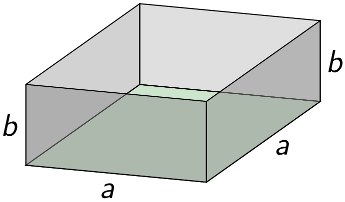
$32 = a^2 b$

$$\boxed{b = \frac{32}{a^2}}$$

$O = a^2 + 4ab$

$O = a^2 + 4a \cdot \frac{32}{a^2}$

$O = a^2 +$



$$V = a^2 b$$

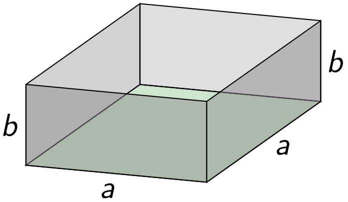
$$32 = a^2 b$$

$$\boxed{b = \frac{32}{a^2}}$$

$$O = a^2 + 4ab$$

$$O = a^2 + 4a \cdot \frac{32}{a^2}$$

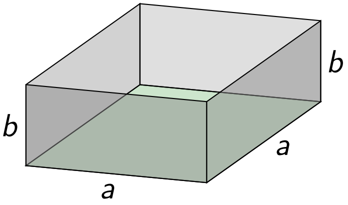
$$O = a^2 + \frac{128}{a}$$



$$V = a^2 b \qquad O = a^2 + 4ab$$

$$32 = a^2 b \qquad O = a^2 + 4a \cdot \frac{32}{a^2}$$

$$\boxed{b = \frac{32}{a^2}} \qquad O = a^2 + \frac{128}{a}$$



$$O(a) = a^2 + 128a^{-1}$$

$$V = a^2 b$$

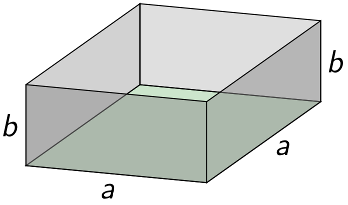
$$32 = a^2 b$$

$$\boxed{b = \frac{32}{a^2}}$$

$$O = a^2 + 4ab$$

$$O = a^2 + 4a \cdot \frac{32}{a^2}$$

$$O = a^2 + \frac{128}{a}$$

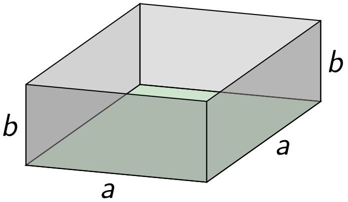


$$\boxed{O(a) = a^2 + 128a^{-1}}$$

$$V = a^2 b \qquad O = a^2 + 4ab$$

$$32 = a^2 b \qquad O = a^2 + 4a \cdot \frac{32}{a^2}$$

$$\boxed{b = \frac{32}{a^2}} \qquad O = a^2 + \frac{128}{a}$$



$$\boxed{O(a) = a^2 + 128a^{-1}}$$

$$O'(a) =$$

$$V = a^2 b$$

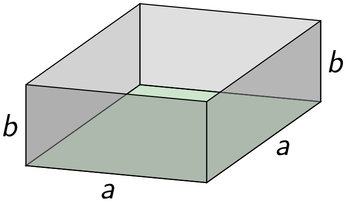
$$32 = a^2 b$$

$$\boxed{b = \frac{32}{a^2}}$$

$$O = a^2 + 4ab$$

$$O = a^2 + 4a \cdot \frac{32}{a^2}$$

$$O = a^2 + \frac{128}{a}$$



$$\boxed{O(a) = a^2 + 128a^{-1}}$$

$$O'(a) = 2a - 128a^{-2}$$

$V = a^2 b$

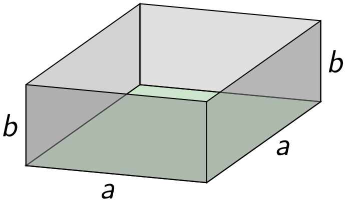
$32 = a^2 b$

$$b = \frac{32}{a^2}$$

$O = a^2 + 4ab$

$O = a^2 + 4a \cdot \frac{32}{a^2}$

$O = a^2 + \frac{128}{a}$



$$O(a) = a^2 + 128a^{-1}$$

$O'(a) = 2a - 128a^{-2}$

$2a - 128a^{-2} = 0$

$V = a^2 b$

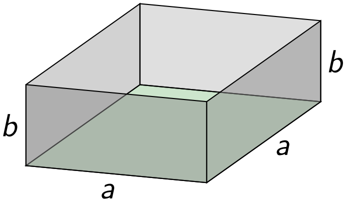
$32 = a^2 b$

$$\boxed{b = \frac{32}{a^2}}$$

$O = a^2 + 4ab$

$O = a^2 + 4a \cdot \frac{32}{a^2}$

$O = a^2 + \frac{128}{a}$



$$\boxed{O(a) = a^2 + 128a^{-1}}$$

$O'(a) = 2a - 128a^{-2}$

$2a - 128a^{-2} = 0 \ / \cdot a^2$

$V = a^2 b$

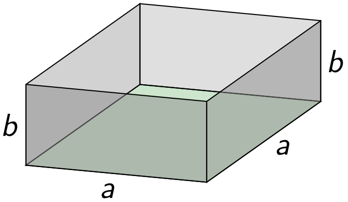
$32 = a^2 b$

$$\boxed{b = \frac{32}{a^2}}$$

$O = a^2 + 4ab$

$O = a^2 + 4a \cdot \frac{32}{a^2}$

$O = a^2 + \frac{128}{a}$



$$\boxed{O(a) = a^2 + 128a^{-1}}$$

$O'(a) = 2a - 128a^{-2}$

$2a - 128a^{-2} = 0 \;/\cdot a^2$

$2a^3 - 128 = 0$

$V = a^2 b$

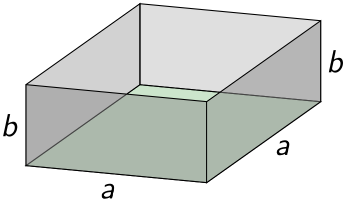
$32 = a^2 b$

$$\boxed{b = \frac{32}{a^2}}$$

$O = a^2 + 4ab$

$O = a^2 + 4a \cdot \frac{32}{a^2}$

$O = a^2 + \frac{128}{a}$



$$\boxed{O(a) = a^2 + 128a^{-1}}$$

$O'(a) = 2a - 128a^{-2}$

$2a - 128a^{-2} = 0 \quad / \cdot a^2$

$2a^3 - 128 = 0$

$2a^3 = 128$

$V = a^2 b$

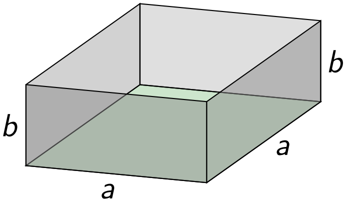
$32 = a^2 b$

$$\boxed{b = \frac{32}{a^2}}$$

$O = a^2 + 4ab$

$O = a^2 + 4a \cdot \frac{32}{a^2}$

$O = a^2 + \frac{128}{a}$



$$\boxed{O(a) = a^2 + 128a^{-1}}$$

$O'(a) = 2a - 128a^{-2}$

$2a - 128a^{-2} = 0 \quad / \cdot a^2$

$2a^3 - 128 = 0$

$2a^3 = 128$

$a^3 = 64$

$V = a^2 b$

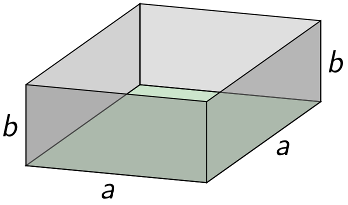
$32 = a^2 b$

$$b = \frac{32}{a^2}$$

$O = a^2 + 4ab$

$O = a^2 + 4a \cdot \frac{32}{a^2}$

$O = a^2 + \frac{128}{a}$



$$O(a) = a^2 + 128a^{-1}$$

$O'(a) = 2a - 128a^{-2}$

$2a - 128a^{-2} = 0 \;/\cdot a^2$

$2a^3 - 128 = 0$

$2a^3 = 128$

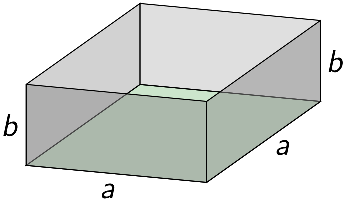
$a^3 = 64$

$a = \sqrt[3]{64}$

$$V = a^2 b \qquad O = a^2 + 4ab$$

$$32 = a^2 b \qquad O = a^2 + 4a \cdot \frac{32}{a^2}$$

$$\boxed{b = \frac{32}{a^2}} \qquad O = a^2 + \frac{128}{a}$$



$$\boxed{O(a) = a^2 + 128a^{-1}}$$

$$O'(a) = 2a - 128a^{-2}$$

$$2a - 128a^{-2} = 0 \ /\cdot a^2$$

$$2a^3 - 128 = 0$$

$$2a^3 = 128$$

$$a^3 = 64$$

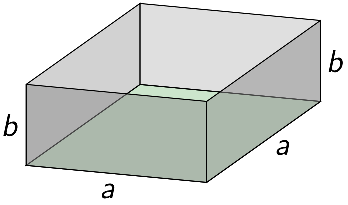
$$a = \sqrt[3]{64}$$

$$a = 4$$

$$V = a^2 b \qquad O = a^2 + 4ab$$

$$32 = a^2 b \qquad O = a^2 + 4a \cdot \frac{32}{a^2}$$

$$\boxed{b = \frac{32}{a^2}} \qquad O = a^2 + \frac{128}{a}$$



$$\boxed{O(a) = a^2 + 128a^{-1}}$$

$$O'(a) = 2a - 128a^{-2}$$

$$2a - 128a^{-2} = 0 \ /\cdot a^2$$

$$2a^3 - 128 = 0$$

$$2a^3 = 128$$

$$a^3 = 64$$

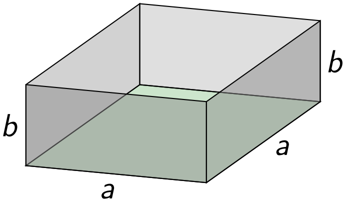
$$a = \sqrt[3]{64}$$

$$\boxed{\boxed{a = 4}}$$

$$V = a^2 b \qquad O = a^2 + 4ab$$

$$32 = a^2 b \qquad O = a^2 + 4a \cdot \frac{32}{a^2}$$

$$\boxed{b = \frac{32}{a^2}} \qquad O = a^2 + \frac{128}{a}$$



$$\boxed{O(a) = a^2 + 128a^{-1}}$$

$$O'(a) = 2a - 128a^{-2}$$

$$2a - 128a^{-2} = 0 \quad / \cdot a^2$$

$$2a^3 - 128 = 0$$

$$2a^3 = 128$$

$$a^3 = 64$$

$$a = \sqrt[3]{64}$$

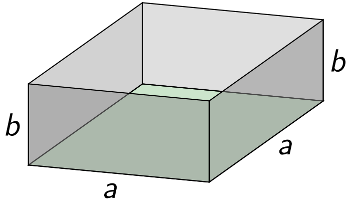
$$\boxed{\boxed{a = 4}}$$

	0	4	$+\infty$
O'			
O			

$$V = a^2 b \qquad O = a^2 + 4ab$$

$$32 = a^2 b \qquad O = a^2 + 4a \cdot \frac{32}{a^2}$$

$$\boxed{b = \frac{32}{a^2}} \qquad O = a^2 + \frac{128}{a}$$



$$\boxed{O(a) = a^2 + 128a^{-1}}$$

$$O'(a) = 2a - 128a^{-2}$$

$$2a - 128a^{-2} = 0 \;/\cdot a^2$$

$$2a^3 - 128 = 0$$

$$2a^3 = 128$$

$$a^3 = 64$$

$$a = \sqrt[3]{64}$$

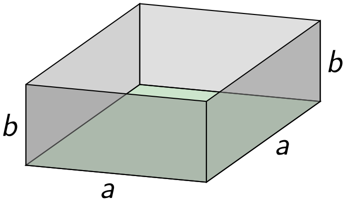
$$\boxed{a = 4}$$

	0		4		$+\infty$
O'		$-$			
O					

$$V = a^2 b \qquad O = a^2 + 4ab$$

$$32 = a^2 b \qquad O = a^2 + 4a \cdot \frac{32}{a^2}$$

$$\boxed{b = \frac{32}{a^2}} \qquad O = a^2 + \frac{128}{a}$$



$$\boxed{O(a) = a^2 + 128a^{-1}}$$

$$O'(a) = 2a - 128a^{-2}$$

$$2a - 128a^{-2} = 0 \quad / \cdot a^2$$

$$2a^3 - 128 = 0$$

$$2a^3 = 128$$

$$a^3 = 64$$

$$a = \sqrt[3]{64}$$

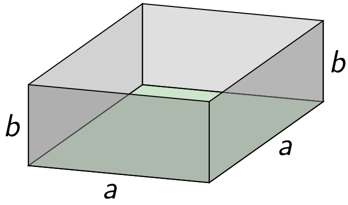
$$\boxed{a = 4}$$

	0		4		$+\infty$
O'		$-$		$+$	
O					

$$V = a^2 b \qquad O = a^2 + 4ab$$

$$32 = a^2 b \qquad O = a^2 + 4a \cdot \frac{32}{a^2}$$

$$\boxed{b = \frac{32}{a^2}} \qquad O = a^2 + \frac{128}{a}$$



$$\boxed{O(a) = a^2 + 128a^{-1}}$$

$$O'(a) = 2a - 128a^{-2}$$

$$2a - 128a^{-2} = 0 \;/ \cdot a^2$$

$$2a^3 - 128 = 0$$

$$2a^3 = 128$$

$$a^3 = 64$$

$$a = \sqrt[3]{64}$$

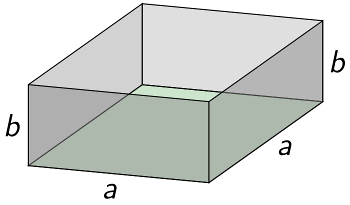
$$\boxed{a = 4}$$

	0		4		$+\infty$
O'		$-$		$+$	
O		$\searrow$			

$$V = a^2 b \qquad O = a^2 + 4ab$$

$$32 = a^2 b \qquad O = a^2 + 4a \cdot \frac{32}{a^2}$$

$$\boxed{b = \frac{32}{a^2}} \qquad O = a^2 + \frac{128}{a}$$



$$\boxed{O(a) = a^2 + 128a^{-1}}$$

$$O'(a) = 2a - 128a^{-2}$$

$$2a - 128a^{-2} = 0 \quad / \cdot a^2$$

$$2a^3 - 128 = 0$$

$$2a^3 = 128$$

$$a^3 = 64$$

$$a = \sqrt[3]{64}$$

$$\boxed{\boxed{a = 4}}$$

	0		4		$+\infty$
O'		$-$		$+$	
O		↘		↗	

$$V = a^2 b$$

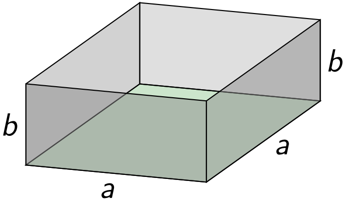
$$32 = a^2 b$$

$$\boxed{b = \frac{32}{a^2}}$$

$$O = a^2 + 4ab$$

$$O = a^2 + 4a \cdot \frac{32}{a^2}$$

$$O = a^2 + \frac{128}{a}$$



$$\boxed{O(a) = a^2 + 128a^{-1}}$$

$$O'(a) = 2a - 128a^{-2}$$

$$2a - 128a^{-2} = 0 \quad / \cdot a^2$$

$$2a^3 - 128 = 0$$

$$2a^3 = 128$$

$$a^3 = 64$$

$$a = \sqrt[3]{64}$$

$$\boxed{\boxed{a = 4}}$$

	0		4		$+\infty$
O'		$-$		$+$	
O		↘		↗	

$$O(4) =$$

$V = a^2 b$

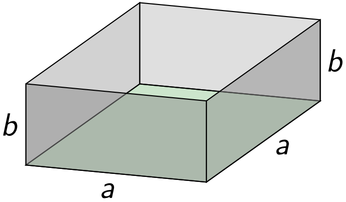
$32 = a^2 b$

$$\boxed{b = \frac{32}{a^2}}$$

$O = a^2 + 4ab$

$O = a^2 + 4a \cdot \frac{32}{a^2}$

$O = a^2 + \frac{128}{a}$



$$\boxed{O(a) = a^2 + 128a^{-1}}$$

$O'(a) = 2a - 128a^{-2}$

$2a - 128a^{-2} = 0 \; / \cdot a^2$

$2a^3 - 128 = 0$

$2a^3 = 128$

$a^3 = 64$

$a = \sqrt[3]{64}$

$$\boxed{\boxed{a = 4}}$$

	0		4		$+\infty$
O'		$-$		$+$	
O		$\searrow$		$\nearrow$	

$O(4) = 4^2 + 128 \cdot 4^{-1}$

$$V = a^2 b$$

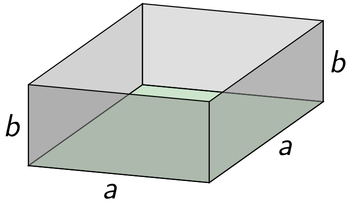
$$32 = a^2 b$$

$$\boxed{b = \frac{32}{a^2}}$$

$$O = a^2 + 4ab$$

$$O = a^2 + 4a \cdot \frac{32}{a^2}$$

$$O = a^2 + \frac{128}{a}$$



$$\boxed{O(a) = a^2 + 128a^{-1}}$$

$$O'(a) = 2a - 128a^{-2}$$

$$2a - 128a^{-2} = 0 \quad / \cdot a^2$$

$$2a^3 - 128 = 0$$

$$2a^3 = 128$$

$$a^3 = 64$$

$$a = \sqrt[3]{64}$$

$$\boxed{a = 4}$$

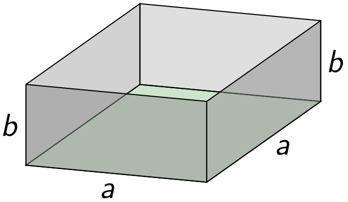
	0		4		$+\infty$
O'		$-$		$+$	
O		$\searrow$		$\nearrow$	

$$O(4) = 4^2 + 128 \cdot 4^{-1} = 48$$

$$V = a^2 b \qquad O = a^2 + 4ab$$

$$32 = a^2 b \qquad O = a^2 + 4a \cdot \frac{32}{a^2}$$

$$\boxed{b = \frac{32}{a^2}} \qquad O = a^2 + \frac{128}{a}$$



$$O_{\min} = 48$$

$$\boxed{O(a) = a^2 + 128a^{-1}}$$

$$O'(a) = 2a - 128a^{-2}$$

	0		4		$+\infty$
O'		$-$		$+$	
O		$\searrow$		$\nearrow$	

$$2a - 128a^{-2} = 0 \quad / \cdot a^2$$

$$2a^3 - 128 = 0$$

$$2a^3 = 128$$

$$a^3 = 64$$

$$a = \sqrt[3]{64}$$

$$\boxed{a = 4}$$

$$O(4) = 4^2 + 128 \cdot 4^{-1} = 48$$

$$V = a^2 b$$

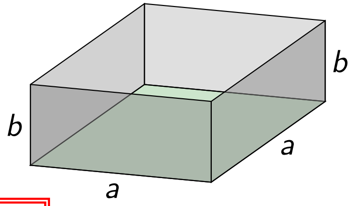
$$32 = a^2 b$$

$$\boxed{b = \frac{32}{a^2}}$$

$$O = a^2 + 4ab$$

$$O = a^2 + 4a \cdot \frac{32}{a^2}$$

$$O = a^2 + \frac{128}{a}$$



$$\boxed{O(a) = a^2 + 128a^{-1}}$$

$$O'(a) = 2a - 128a^{-2}$$

$$2a - 128a^{-2} = 0 \;/\cdot a^2$$

$$2a^3 - 128 = 0$$

$$2a^3 = 128$$

$$a^3 = 64$$

$$a = \sqrt[3]{64}$$

$$\boxed{a = 4}$$

$$\boxed{O_{\min} = 48}$$

	0		4		$+\infty$
O'		$-$		$+$	
O		↘		↗	

$$O(4) = 4^2 + 128 \cdot 4^{-1} = 48$$

$V = a^2 b$

$32 = a^2 b$

$$\boxed{b = \frac{32}{a^2}}$$

$O = a^2 + 4ab$

$O = a^2 + 4a \cdot \frac{32}{a^2}$

$O = a^2 + \frac{128}{a}$

$$\boxed{O(a) = a^2 + 128a^{-1}}$$

$O'(a) = 2a - 128a^{-2}$

$2a - 128a^{-2} = 0 \quad / \cdot a^2$

$2a^3 - 128 = 0$

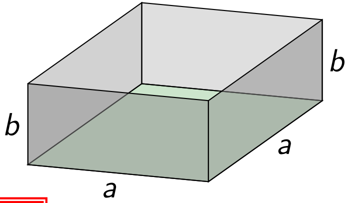
$2a^3 = 128$

$a^3 = 64$

$a = \sqrt[3]{64}$

$$\boxed{a = 4}$$

$b = \frac{32}{4^2}$



$$\boxed{O_{\min} = 48}$$

	0		4		$+\infty$
O'		$-$		$+$	
O		↘		↗	

$O(4) = 4^2 + 128 \cdot 4^{-1} = 48$

$$V = a^2 b \qquad O = a^2 + 4ab$$

$$32 = a^2 b \qquad O = a^2 + 4a \cdot \frac{32}{a^2}$$

$$\boxed{b = \frac{32}{a^2}} \qquad O = a^2 + \frac{128}{a}$$

$$\boxed{O(a) = a^2 + 128a^{-1}}$$

$$O'(a) = 2a - 128a^{-2}$$

$$2a - 128a^{-2} = 0 \quad / \cdot a^2$$

$$2a^3 - 128 = 0$$

$$2a^3 = 128$$

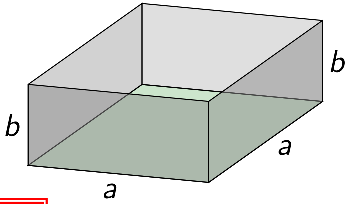
$$a^3 = 64$$

$$a = \sqrt[3]{64}$$

$$\boxed{a = 4}$$

$$b = \frac{32}{4^2}$$

$$b = \frac{32}{16}$$



$$\boxed{O_{\min} = 48}$$

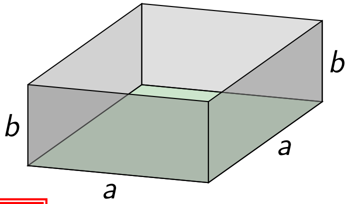
	0		4		$+\infty$
O'		$-$		$+$	
O		↘		↗	

$$O(4) = 4^2 + 128 \cdot 4^{-1} = 48$$

$$V = a^2 b \qquad O = a^2 + 4ab$$

$$32 = a^2 b \qquad O = a^2 + 4a \cdot \frac{32}{a^2}$$

$$\boxed{b = \frac{32}{a^2}} \qquad O = a^2 + \frac{128}{a}$$



$$\boxed{O_{\min} = 48}$$

$$\boxed{O(a) = a^2 + 128a^{-1}}$$

$$O'(a) = 2a - 128a^{-2}$$

$$2a - 128a^{-2} = 0 \quad / \cdot a^2$$

$$2a^3 - 128 = 0$$

$$2a^3 = 128$$

$$a^3 = 64$$

$$a = \sqrt[3]{64}$$

$$\boxed{a = 4}$$

$$b = \frac{32}{4^2}$$

$$b = \frac{32}{16}$$

$$b = 2$$

	0		4		$+\infty$
O'		$-$		$+$	
O		↘		↗	

$$O(4) = 4^2 + 128 \cdot 4^{-1} = 48$$

$$V = a^2 b$$

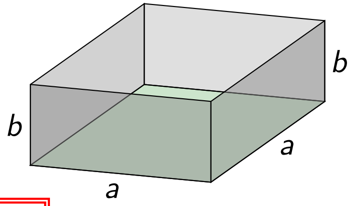
$$32 = a^2 b$$

$$b = \frac{32}{a^2}$$

$$O = a^2 + 4ab$$

$$O = a^2 + 4a \cdot \frac{32}{a^2}$$

$$O = a^2 + \frac{128}{a}$$



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$$b = 2$$

$$O_{\min} = 48$$

	0		4		$+\infty$
O'		$-$		$+$	
O		↘		↗	

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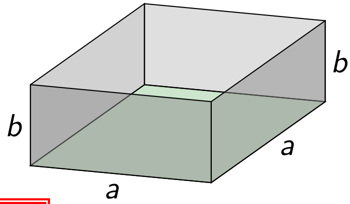

$$\boxed{O_{\min} = 48}$$

$$\boxed{O(a) = a^2 + 128a^{-1}}$$

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$$2a - 128a^{-2} = 0 \quad / \cdot a^2$$

$$2a^3 - 128 = 0$$

$$2a^3 = 128$$

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$$a = \sqrt[3]{64}$$

$$\boxed{a = 4}$$

$$b = \frac{32}{4^2}$$

$$b = \frac{32}{16}$$

$$\boxed{b = 2}$$

	0	4	$+\infty$
O'	$-$	$+$	
O	↘	↗	

$$O(4) = 4^2 + 128 \cdot 4^{-1} = 48$$

✏ Uz zadani volumen bazena od $32\,\text{m}^3$ i kvadratnim dnom, minimalno oplošje iznosi $48\,\text{m}^2$, a postiže se ako je $a = 4\,\text{m}$ i $b = 2\,\text{m}$.