# Seminari 10

## Matematičke metode za informatičare

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FOI, Varaždin

# Sadržaj

# prvi zadatak

## Zadatak 1

*Odredite sliku, jezgru, rang i defekt linearnog operatora* $B : \mathbb{R}^5 \to \mathbb{R}^3$ *zadanog matricom*

$$B = \begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{bmatrix}$$

*u paru kanonskih baza. Je li* $B$ *izomorfizam?*

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## Rješenje

Kako je $\dim \mathbb{R}^5 \neq \dim \mathbb{R}^3$, zaključujemo da $\mathbb{R}^5$ i $\mathbb{R}^3$ nisu izomorfni vektorski prostori.

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Dakle, linearni operator $B$ nije izomorfizam.

## Ker $B$

$B : \mathbb{R}^5 \to \mathbb{R}^3$

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$B(x) = \Theta_{\mathbb{R}^3}$

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| $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | |
|---|---|---|---|---|---|

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|---|---|---|---|---|---|
| 1 | 2 | 3 | 1 | 0 | 0 |
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| 1 | 2 | 3 | 1 | 0 | 0 | |
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| 1 | 2 | 3 | 1 | 0 | 0 | $+$ |
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$+$ $+$

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| 1 | 2 | 5 | 2 | 1 | 0 |
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## $\mathrm{Ker}\, B$

$$B : \mathbb{R}^5 \to \mathbb{R}^3$$

$$B(x) = \Theta_{\mathbb{R}^3} \rightsquigarrow BX = O$$

$$\begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

| $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | | |
|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 1 | 0 | 0 | $+$ |
| 1 | 2 | 5 | 2 | 1 | 0 | $/\cdot(-1)\ /\cdot(-1)$ |
| 1 | 2 | 5 | 2 | 1 | 0 | $+$ |
| 0 | 0 | $-2$ | $-1$ | $-1$ | 0 | |
| 1 | 2 | 5 | 2 | 1 | 0 | |
| 0 | 0 | 0 | 0 | 0 | 0 | |

$$Y_{\mathcal{B}} = F_{(\mathcal{A},\mathcal{B})} X_{\mathcal{A}}$$

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| $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | | |
|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 1 | 0 | 0 | $+$ |
| 1 | 2 | 5 | 2 | 1 | 0 | $/\cdot(-1)$ $/\cdot(-1)$ |
| 1 | 2 | 5 | 2 | 1 | 0 | $+$ |
| 0 | 0 | $-2$ | $-1$ | $-1$ | 0 | |
| 1 | 2 | 5 | 2 | 1 | 0 | |
| 0 | 0 | 0 | 0 | 0 | 0 | |
| 0 | 0 | $-2$ | $-1$ | $-1$ | 0 | |

$$Y_{\mathcal{B}} = F_{(\mathcal{A},\mathcal{B})} X_{\mathcal{A}}$$

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| $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | | |
|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 1 | 0 | 0 | $+$ |
| 1 | 2 | 5 | 2 | 1 | 0 | $/\cdot(-1)$ $/\cdot(-1)$ |
| 1 | 2 | 5 | 2 | 1 | 0 | $+$ |
| 0 | 0 | $-2$ | $-1$ | $-1$ | 0 | |
| 1 | 2 | 5 | 2 | 1 | 0 | |
| 0 | 0 | 0 | 0 | 0 | 0 | |
| 0 | 0 | $-2$ | $-1$ | $-1$ | 0 | |
| 1 | 2 | 5 | 2 | 1 | 0 | |

$\boxed{Y_{\mathcal{B}} = F_{(\mathcal{A},\mathcal{B})} X_{\mathcal{A}}}$

$\boxed{\text{Ker } B}$ $B : \mathbb{R}^5 \to \mathbb{R}^3$

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| $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | | |
|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 1 | 0 | 0 | $+$ |
| 1 | 2 | 5 | 2 | 1 | 0 | $/\cdot(-1)$ $/\cdot(-1)$ |
| 1 | 2 | 5 | 2 | 1 | 0 | $+$ |
| 0 | 0 | $-2$ | $-1$ | $-1$ | 0 | |
| 1 | 2 | 5 | 2 | 1 | 0 | |
| 0 | 0 | 0 | 0 | 0 | 0 | |
| 0 | 0 | $-2$ | $-1$ | $-1$ | 0 | |
| 1 | 2 | 5 | 2 | 1 | 0 | |

$$\boxed{Y_{\mathcal{B}} = F_{(\mathcal{A},\mathcal{B})} X_{\mathcal{A}}}$$

Ker $B$

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| $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | | |
|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 1 | 0 | 0 | $+$ |
| 1 | 2 | 5 | 2 | 1 | 0 | $/\cdot(-1)$ $/\cdot(-1)$ |
| 1 | 2 | 5 | 2 | 1 | 0 | $+$ |
| 0 | 0 | $-2$ | $-1$ | $-1$ | 0 | |
| 1 | 2 | 5 | 2 | 1 | 0 | |
| 0 | 0 | 0 | 0 | 0 | 0 | |
| 0 | 0 | $-2$ | $-1$ | $\boxed{-1}$ | 0 | |
| 1 | 2 | 5 | 2 | 1 | 0 | |

$Y_{\mathcal{B}} = F_{(\mathcal{A},\mathcal{B})} X_{\mathcal{A}}$

Ker $B$

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| $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | | |
|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 1 | 0 | 0 | $+$ |
| 1 | 2 | 5 | 2 | 1 | 0 | $/\cdot(-1)$ $/\cdot(-1)$ |
| 1 | 2 | 5 | 2 | 1 | 0 | $+$ |
| 0 | 0 | $-2$ | $-1$ | $-1$ | 0 | |
| 1 | 2 | 5 | 2 | 1 | 0 | |
| 0 | 0 | 0 | 0 | 0 | 0 | |
| 0 | 0 | $-2$ | $-1$ | $\boxed{-1}$ | 0 | $/\cdot 1$ |
| 1 | 2 | 5 | 2 | 1 | 0 | |

$Y_{\mathcal{B}} = F_{(\mathcal{A},\mathcal{B})} X_{\mathcal{A}}$

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| $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | | |
|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 1 | 0 | 0 | $+$ |
| 1 | 2 | 5 | 2 | 1 | 0 | $/\cdot(-1)\ /\cdot(-1)$ |
| 1 | 2 | 5 | 2 | 1 | 0 | $+$ |
| 0 | 0 | $-2$ | $-1$ | $-1$ | 0 | |
| 1 | 2 | 5 | 2 | 1 | 0 | |
| 0 | 0 | 0 | 0 | 0 | 0 | |
| 0 | 0 | $-2$ | $-1$ | $\boxed{-1}$ | 0 | $/\cdot 1$ |
| 1 | 2 | 5 | 2 | 1 | 0 | $+$ |

$Y_{\mathcal{B}} = F_{(\mathcal{A},\mathcal{B})} X_{\mathcal{A}}$

## Ker $B$

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| $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | | |
|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 1 | 0 | 0 | $+$ |
| 1 | 2 | 5 | 2 | 1 | 0 | $/\cdot(-1)\ /\cdot(-1)$ |
| 1 | 2 | 5 | 2 | 1 | 0 | $+$ |
| 0 | 0 | $-2$ | $-1$ | $-1$ | 0 | |
| 1 | 2 | 5 | 2 | 1 | 0 | |
| 0 | 0 | 0 | 0 | 0 | 0 | |
| 0 | 0 | $-2$ | $-1$ | $\boxed{-1}$ | 0 | $/\cdot 1$ |
| 1 | 2 | 5 | 2 | 1 | 0 | $+$ |
| 0 | 0 | $-2$ | $-1$ | $-1$ | 0 | |

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$$\begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

| $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | | |
|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 1 | 0 | 0 | $+$ |
| 1 | 2 | 5 | 2 | 1 | 0 | $/\cdot(-1)$ $/\cdot(-1)$ |
| 1 | 2 | 5 | 2 | 1 | 0 | $+$ |
| 0 | 0 | $-2$ | $-1$ | $-1$ | 0 | |
| 1 | 2 | 5 | 2 | 1 | 0 | |
| 0 | 0 | 0 | 0 | 0 | 0 | |
| 0 | 0 | $-2$ | $-1$ | $\boxed{-1}$ | 0 | $/\cdot 1$ |
| 1 | 2 | 5 | 2 | 1 | 0 | $+$ |
| 0 | 0 | $-2$ | $-1$ | $-1$ | 0 | |
| 1 | | | | | | |

$$Y_{\mathcal{B}} = F_{(\mathcal{A},\mathcal{B})} X_{\mathcal{A}}$$

Ker $B$

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$$\begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

| $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | | |
|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 1 | 0 | 0 | $+$ |
| 1 | 2 | 5 | 2 | 1 | 0 | $/\cdot(-1)$ $/\cdot(-1)$ |
| 1 | 2 | 5 | 2 | 1 | 0 | $+$ |
| 0 | 0 | $-2$ | $-1$ | $-1$ | 0 | |
| 1 | 2 | 5 | 2 | 1 | 0 | |
| 0 | 0 | 0 | 0 | 0 | 0 | |
| 0 | 0 | $-2$ | $-1$ | $\boxed{-1}$ | 0 | $/\cdot 1$ |
| 1 | 2 | 5 | 2 | 1 | 0 | $+$ |
| 0 | 0 | $-2$ | $-1$ | $-1$ | 0 | |
| 1 | 2 | | | | | |

$$Y_{\mathcal{B}} = F_{(\mathcal{A},\mathcal{B})} X_{\mathcal{A}}$$

**Ker $B$**

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| $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | | |
|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 1 | 0 | 0 | + |
| 1 | 2 | 5 | 2 | 1 | 0 | $/\cdot(-1)$ $/\cdot(-1)$ |
| 1 | 2 | 5 | 2 | 1 | 0 | + |
| 0 | 0 | $-2$ | $-1$ | $-1$ | 0 | |
| 1 | 2 | 5 | 2 | 1 | 0 | |
| 0 | 0 | 0 | 0 | 0 | 0 | |
| 0 | 0 | $-2$ | $-1$ | $\boxed{-1}$ | 0 | $/\cdot 1$ |
| 1 | 2 | 5 | 2 | 1 | 0 | + |
| 0 | 0 | $-2$ | $-1$ | $-1$ | 0 | |
| 1 | 2 | 3 | | | | |

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| $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | | |
|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 1 | 0 | 0 | + |
| 1 | 2 | 5 | 2 | 1 | 0 | $/\cdot(-1)$ $/\cdot(-1)$ |
| 1 | 2 | 5 | 2 | 1 | 0 | + |
| 0 | 0 | $-2$ | $-1$ | $-1$ | 0 | |
| 1 | 2 | 5 | 2 | 1 | 0 | |
| 0 | 0 | 0 | 0 | 0 | 0 | |
| 0 | 0 | $-2$ | $-1$ | $\boxed{-1}$ | 0 | $/\cdot 1$ |
| 1 | 2 | 5 | 2 | 1 | 0 | + |
| 0 | 0 | $-2$ | $-1$ | $-1$ | 0 | |
| 1 | 2 | 3 | 1 | | | |

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| $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | | |
|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 1 | 0 | 0 | $+$ |
| 1 | 2 | 5 | 2 | 1 | 0 | $/\cdot(-1)$ $/\cdot(-1)$ |
| 1 | 2 | 5 | 2 | 1 | 0 | $+$ |
| 0 | 0 | $-2$ | $-1$ | $-1$ | 0 | |
| 1 | 2 | 5 | 2 | 1 | 0 | |
| 0 | 0 | 0 | 0 | 0 | 0 | |
| 0 | 0 | $-2$ | $-1$ | $\boxed{-1}$ | 0 | $/\cdot 1$ |
| 1 | 2 | 5 | 2 | 1 | 0 | $+$ |
| 0 | 0 | $-2$ | $-1$ | $-1$ | 0 | |
| 1 | 2 | 3 | 1 | 0 | | |

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| $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | | |
|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 1 | 0 | 0 | $+$ |
| 1 | 2 | 5 | 2 | 1 | 0 | $/\cdot(-1)$ $/\cdot(-1)$ |
| 1 | 2 | 5 | 2 | 1 | 0 | $+$ |
| 0 | 0 | $-2$ | $-1$ | $-1$ | 0 | |
| 1 | 2 | 5 | 2 | 1 | 0 | |
| 0 | 0 | 0 | 0 | 0 | 0 | |
| 0 | 0 | $-2$ | $-1$ | $\boxed{-1}$ | 0 | $/\cdot 1$ |
| 1 | 2 | 5 | 2 | 1 | 0 | $+$ |
| 0 | 0 | $-2$ | $-1$ | $-1$ | 0 | |
| 1 | 2 | 3 | 1 | 0 | 0 | |

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Ker $B$

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$$\begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$B(x) = \Theta_{\mathbb{R}^3} \rightsquigarrow BX = O$

$$\begin{array}{ccccc|cl}
x_1 & x_2 & x_3 & x_4 & x_5 & & \\
\hline
1 & 2 & 3 & 1 & 0 & 0 & + \\
1 & 2 & 5 & 2 & 1 & 0 & /\cdot(-1)\ /\cdot(-1) \\
1 & 2 & 5 & 2 & 1 & 0 & + \\
\hline
0 & 0 & -2 & -1 & -1 & 0 & \\
1 & 2 & 5 & 2 & 1 & 0 & \\
0 & 0 & 0 & 0 & 0 & 0 & \\
\hline
0 & 0 & -2 & -1 & \boxed{-1} & 0 & /\cdot 1 \\
1 & 2 & 5 & 2 & 1 & 0 & + \\
\hline
0 & 0 & -2 & -1 & -1 & 0 & \\
1 & 2 & 3 & 1 & 0 & 0 &
\end{array}$$

$$-2x_3 - x_4 - x_5 = 0$$

$$Y_{\mathcal{B}} = F_{(\mathcal{A},\mathcal{B})} X_{\mathcal{A}}$$

## Ker $B$

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$$\begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

| $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | | |
|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 1 | 0 | 0 | $+$ |
| 1 | 2 | 5 | 2 | 1 | 0 | $/\cdot(-1)$ $/\cdot(-1)$ |
| 1 | 2 | 5 | 2 | 1 | 0 | $+$ |
| 0 | 0 | $-2$ | $-1$ | $-1$ | 0 | |
| 1 | 2 | 5 | 2 | 1 | 0 | |
| 0 | 0 | 0 | 0 | 0 | 0 | |
| 0 | 0 | $-2$ | $-1$ | $\boxed{-1}$ | 0 | $/\cdot 1$ |
| 1 | 2 | 5 | 2 | 1 | 0 | $+$ |
| 0 | 0 | $-2$ | $-1$ | $-1$ | 0 | |
| 1 | 2 | 3 | 1 | 0 | 0 | |

$$-2x_3 - x_4 - x_5 = 0$$
$$x_1 + 2x_2 + 3x_3 + x_4 = 0$$

$$Y_{\mathcal{B}} = F_{(\mathcal{A},\mathcal{B})} X_{\mathcal{A}}$$

Ker $B$

$B:\mathbb{R}^5\to\mathbb{R}^3$

$$B(x)=\Theta_{\mathbb{R}^3} \rightsquigarrow BX=O$$

$$\begin{bmatrix}1&2&3&1&0\\1&2&5&2&1\\1&2&5&2&1\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\\x_4\\x_5\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

| $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | | |
|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 1 | 0 | 0 | + |
| 1 | 2 | 5 | 2 | 1 | 0 | $/\cdot(-1)$ $/\cdot(-1)$ |
| 1 | 2 | 5 | 2 | 1 | 0 | + |
| 0 | 0 | $-2$ | $-1$ | $-1$ | 0 | |
| 1 | 2 | 5 | 2 | 1 | 0 | |
| 0 | 0 | 0 | 0 | 0 | 0 | |
| 0 | 0 | $-2$ | $-1$ | $\boxed{-1}$ | 0 | $/\cdot 1$ |
| 1 | 2 | 5 | 2 | 1 | 0 | + |
| 0 | 0 | $-2$ | $-1$ | $-1$ | 0 | |
| 1 | 2 | 3 | 1 | 0 | 0 | |

$$\left.\begin{aligned}-2x_3-x_4-x_5&=0\\x_1+2x_2+3x_3+x_4&=0\end{aligned}\right\}$$

$$\boxed{Y_{\mathcal{B}}=F_{(\mathcal{A},\mathcal{B})}X_{\mathcal{A}}}$$

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| $x_1$ | $\boxed{x_2}$ | $\boxed{x_3}$ | $\boxed{x_4}$ | $x_5$ | | |
|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 1 | 0 | 0 | $+$ |
| 1 | 2 | 5 | 2 | 1 | 0 | $/\cdot(-1)$ $/\cdot(-1)$ |
| 1 | 2 | 5 | 2 | 1 | 0 | $+$ |
| 0 | 0 | $-2$ | $-1$ | $-1$ | 0 | |
| 1 | 2 | 5 | 2 | 1 | 0 | |
| 0 | 0 | 0 | 0 | 0 | 0 | |
| 0 | 0 | $-2$ | $-1$ | $\boxed{-1}$ | 0 | $/\cdot 1$ |
| 1 | 2 | 5 | 2 | 1 | 0 | $+$ |
| 0 | 0 | $-2$ | $-1$ | $-1$ | 0 | |
| 1 | 2 | 3 | 1 | 0 | 0 | |

$$\left.\begin{aligned} -2x_3 - x_4 - x_5 &= 0 \\ x_1 + 2x_2 + 3x_3 + x_4 &= 0 \end{aligned}\right\}$$

$$\boxed{Y_{\mathcal{B}} = F_{(\mathcal{A},\mathcal{B})} X_{\mathcal{A}}}$$

Ker $B$

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| $x_1$ | $\boxed{x_2}$ | $\boxed{x_3}$ | $\boxed{x_4}$ | $x_5$ | | |
|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 1 | 0 | 0 | + |
| 1 | 2 | 5 | 2 | 1 | 0 | $/\cdot(-1)$ $/\cdot(-1)$ |
| 1 | 2 | 5 | 2 | 1 | 0 | + |
| 0 | 0 | $-2$ | $-1$ | $-1$ | 0 | |
| 1 | 2 | 5 | 2 | 1 | 0 | |
| 0 | 0 | 0 | 0 | 0 | 0 | |
| 0 | 0 | $-2$ | $-1$ | $\boxed{-1}$ | 0 | $/\cdot 1$ |
| 1 | 2 | 5 | 2 | 1 | 0 | + |
| 0 | 0 | $-2$ | $-1$ | $-1$ | 0 | |
| 1 | 2 | 3 | 1 | 0 | 0 | |

$$\left.\begin{aligned} -2x_3 - x_4 - x_5 &= 0 \\ x_1 + 2x_2 + 3x_3 + x_4 &= 0 \end{aligned}\right\}$$

$$x_5 = -2x_3 - x_4$$

$Y_{\mathcal{B}} = F_{(\mathcal{A},\mathcal{B})} X_{\mathcal{A}}$

Ker $B$

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$$\begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

| $x_1$ | $\boxed{x_2}$ | $\boxed{x_3}$ | $\boxed{x_4}$ | $x_5$ | | |
|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 1 | 0 | 0 | $+$ |
| 1 | 2 | 5 | 2 | 1 | 0 | $/\cdot(-1)$ $/\cdot(-1)$ |
| 1 | 2 | 5 | 2 | 1 | 0 | $+$ |
| 0 | 0 | $-2$ | $-1$ | $-1$ | 0 | |
| 1 | 2 | 5 | 2 | 1 | 0 | |
| 0 | 0 | 0 | 0 | 0 | 0 | |
| 0 | 0 | $-2$ | $-1$ | $\boxed{-1}$ | 0 | $/\cdot 1$ |
| 1 | 2 | 5 | 2 | 1 | 0 | $+$ |
| 0 | 0 | $-2$ | $-1$ | $-1$ | 0 | |
| 1 | 2 | 3 | 1 | 0 | 0 | |

$$\left.\begin{aligned} -2x_3 - x_4 - x_5 &= 0 \\ x_1 + 2x_2 + 3x_3 + x_4 &= 0 \end{aligned}\right\}$$

$$\begin{aligned} x_5 &= -2x_3 - x_4 \\ x_1 &= -2x_2 - 3x_3 - x_4 \end{aligned}$$

$Y_{\mathcal{B}} = F_{(\mathcal{A},\mathcal{B})} X_{\mathcal{A}}$

Ker $B$

$B : \mathbb{R}^5 \to \mathbb{R}^3$

$$x_5 = -2x_3 - x_4$$
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$$= x_2 \cdot ($$

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Ker $B$

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$$\mathcal{B}_{\operatorname{Ker} B} = \{(-2, 1, 0, 0, 0),\ (-3, 0, 1, 0, -2),\ (-1, 0, 0, 1, -1)\}$$

## $\operatorname{Ker} B$

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$$\mathcal{B}_{\operatorname{Ker} B} = \big\{(-2, 1, 0, 0, 0),\ (-3, 0, 1, 0, -2),\ (-1, 0, 0, 1, -1)\big\}$$

$$d(B) = 3$$

## Ker $B$

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$$(-2x_2 - 3x_3 - x_4,\ x_2,\ x_3,\ x_4,\ -2x_3 - x_4) =$$
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$$\mathcal{B}_{\operatorname{Ker} B} = \left\{ (-2, 1, 0, 0, 0),\ (-3, 0, 1, 0, -2),\ (-1, 0, 0, 1, -1) \right\}$$

$$d(B) = 3 \longrightarrow B \text{ nije injekcija}$$

## Im $B$

$B : \mathbb{R}^5 \to \mathbb{R}^3$

## Im $B$

$B : \mathbb{R}^5 \to \mathbb{R}^3$

$$B = \begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{bmatrix}$$

$\operatorname{Im} B$

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$$B = \begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{bmatrix}$$

$$r(B) + d(B) = \dim \mathbb{R}^5$$

## $\boxed{\operatorname{Im} B}$

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$$r(B) + d(B) = \dim \mathbb{R}^5$$

$$r(B) + 3 = 5$$

## $\operatorname{Im} B$

$$B : \mathbb{R}^5 \to \mathbb{R}^3$$

$$B = \begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{bmatrix}$$

$$\begin{aligned} r(B) + d(B) &= \dim \mathbb{R}^5 \\ r(B) + 3 &= 5 \\ r(B) &= 2 \end{aligned}$$

## Im $B$

$$B : \mathbb{R}^5 \to \mathbb{R}^3$$

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$$\begin{aligned} r(B) + d(B) &= \dim \mathbb{R}^5 \\ r(B) + 3 &= 5 \\ r(B) &= 2 \end{aligned}$$

$$r(B) \neq \dim \mathbb{R}^3$$

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$$r(B) + d(B) = \dim \mathbb{R}^5$$
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$r(B) \neq \dim \mathbb{R}^3 \longrightarrow B$ nije surjekcija

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$r(B) \neq \dim \mathbb{R}^3 \longrightarrow B$ nije surjekcija

$$\begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{bmatrix} / \cdot (-1)$$

**Im $B$**

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$$\begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{bmatrix} \begin{matrix} / \cdot (-1) \\ + \\ \phantom{1} \end{matrix}$$

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Im $B$

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Ako je $\dim U = \dim V$, je li linearni operator $f : U \to V$ izomorfizam?

# Problem svojstvenih vrijednosti

# Glavne minore

Neka je $A \in M_n(F)$ pri čemu je $F$ polje.

- **Glavna podmatrica** reda $r$ matrice $A$ je svaka podmatrica $A_{i_1,i_2,\ldots,i_r}$ koja se sastoji od onih elemenata matrice $A$ koji se nalaze na presjeku $r$ redaka i $r$ stupaca s istim indeksima $i_1, i_2, \ldots, i_r$.
- Glavnih podmatrica reda $r$ matrice $A$ ima ukupno $\binom{n}{r}$.
- **Glavna minora** $\Delta_{i_1,i_2,\ldots,i_r}$ reda $r$ matrice $A$ je determinanta pripadne glavne podmatrice, tj. $\Delta_{i_1,i_2,\ldots,i_r} = \det A_{i_1,i_2,\ldots,i_r}$.

# Karakteristični polinom

- $k_A^{(1)}(\lambda) = \det\left(A - \lambda I\right)$

$$k_A^{(1)}(\lambda) = (-1)^n \lambda^n + a_1 \lambda^{n-1} + a_2 \lambda^{n-2} + \ldots + a_{n-1}\lambda + a_n$$

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- $c_1 = -\operatorname{tr} A, \quad c_n = (-1)^n \det A$

# Problem svojstvenih vrijednosti

- $V \leftsquigarrow$ konačnodimenzionalni vektorski prostor nad poljem $F$
- $\mathcal{B} \leftsquigarrow$ neka baza za vektorski prostor $V$

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$$f : V \to V$$

$$f(x) = \lambda x \rightsquigarrow F_{\mathcal{B}} X_{\mathcal{B}} = \lambda X_{\mathcal{B}}$$

$$\boxed{\left(F_{\mathcal{B}} - \lambda I\right) X_{\mathcal{B}} = O}$$

# Problem svojstvenih vrijednosti

- $V \leftsquigarrow$ konačnodimenzionalni vektorski prostor nad poljem $F$
- $\mathcal{B} \leftsquigarrow$ neka baza za vektorski prostor $V$

$$f : V \to V$$

$$f(x) = \lambda x \rightsquigarrow F_{\mathcal{B}} X_{\mathcal{B}} = \lambda X_{\mathcal{B}}$$

$$\boxed{\big(F_{\mathcal{B}} - \lambda I\big) X_{\mathcal{B}} = O}$$

$$\boxed{S(\lambda) = \big\{x \in V : f(x) = \lambda x\big\}}$$

# drugi zadatak

## Zadatak 2

*Zadana je matrica* $A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$.

a) *Odredite svojstvene vrijednosti matrice* $A$.

b) *Odredite svojstvene potprostore matrice* $A$.

c) *Odredite minimalni polinom matrice* $A$.

d) *Izrazite* $A^{-1}$ *pomoću potencija matrice* $A$.

## Rješenje

a) $k_A(\lambda) = \lambda^3 + c_1\lambda^2 + c_2\lambda + c_3$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

## Rješenje

a) $$k_A(\lambda) = \lambda^3 + c_1\lambda^2 + c_2\lambda + c_3$$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$c_1 =$

## Rješenje

a) $$k_A(\lambda) = \lambda^3 + c_1\lambda^2 + c_2\lambda + c_3$$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$c_1 = (-1)^1 \cdot \left(\Delta_1 + \Delta_2 + \Delta_3\right)$$

## Rješenje

a) $k_A(\lambda) = \lambda^3 + c_1\lambda^2 + c_2\lambda + c_3$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$c_1 = (-1)^1 \cdot \left(\Delta_1 + \Delta_2 + \Delta_3\right) = -($$

## Rješenje

a) $k_A(\lambda) = \lambda^3 + c_1\lambda^2 + c_2\lambda + c_3$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix} \quad 1.$$

1.

$$c_1 = (-1)^1 \cdot \left(\Delta_1 + \Delta_2 + \Delta_3\right) = -(4$$

## Rješenje

a) $$k_A(\lambda) = \lambda^3 + c_1\lambda^2 + c_2\lambda + c_3$$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$c_1 = (-1)^1 \cdot \left(\Delta_1 + \Delta_2 + \Delta_3\right) = -(4 +$$

## Rješenje

a) $k_A(\lambda) = \lambda^3 + c_1\lambda^2 + c_2\lambda + c_3$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

2.

2.

$$c_1 = (-1)^1 \cdot \left(\Delta_1 + \Delta_2 + \Delta_3\right) = -(4 + 5$$

## Rješenje

a) $$k_A(\lambda) = \lambda^3 + c_1\lambda^2 + c_2\lambda + c_3 \qquad A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$c_1 = (-1)^1 \cdot \left(\Delta_1 + \Delta_2 + \Delta_3\right) = -(4 + 5 +$$

## Rješenje

a) $k_A(\lambda) = \lambda^3 + c_1\lambda^2 + c_2\lambda + c_3$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix} \leftarrow 3.$$

$$\begin{array}{c} \uparrow \\ 3. \end{array}$$

$$c_1 = (-1)^1 \cdot \left(\Delta_1 + \Delta_2 + \Delta_3\right) = -(4 + 5 + 2$$

## Rješenje

a) $$k_A(\lambda) = \lambda^3 + c_1\lambda^2 + c_2\lambda + c_3 \qquad A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$c_1 = (-1)^1 \cdot \big(\Delta_1 + \Delta_2 + \Delta_3\big) = -(4 + 5 + 2)$$

## Rješenje

a) $k_A(\lambda) = \lambda^3 + c_1\lambda^2 + c_2\lambda + c_3$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$c_1 = (-1)^1 \cdot \left(\Delta_1 + \Delta_2 + \Delta_3\right) = -(4 + 5 + 2) = -11$$

## Rješenje

a) $k_A(\lambda) = \lambda^3 + c_1\lambda^2 + c_2\lambda + c_3$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$c_1 = (-1)^1 \cdot \left(\Delta_1 + \Delta_2 + \Delta_3\right) = -(4+5+2) = -11$$

$$c_2 =$$

## Rješenje

a) $k_A(\lambda) = \lambda^3 + c_1\lambda^2 + c_2\lambda + c_3$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$c_1 = (-1)^1 \cdot \left(\Delta_1 + \Delta_2 + \Delta_3\right) = -(4 + 5 + 2) = -11$$

$$c_2 = (-1)^2 \cdot \left(\Delta_{1,2} + \Delta_{1,3} + \Delta_{2,3}\right)$$

## Rješenje

a) $k_A(\lambda) = \lambda^3 + c_1\lambda^2 + c_2\lambda + c_3$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$c_1 = (-1)^1 \cdot \left(\Delta_1 + \Delta_2 + \Delta_3\right) = -(4+5+2) = -11$$

$$c_2 = (-1)^2 \cdot \left(\Delta_{1,2} + \Delta_{1,3} + \Delta_{2,3}\right) =$$

## Rješenje

a) $k_A(\lambda) = \lambda^3 + c_1\lambda^2 + c_2\lambda + c_3$

$$A = \begin{array}{cc} \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix} & \begin{array}{l} \leftarrow 1. \\ \leftarrow 2. \\ \\ \end{array} \\ \begin{array}{ccc} \uparrow & \uparrow & \\ 1. & 2. & \end{array} & \end{array}$$

$$c_1 = (-1)^1 \cdot \left(\Delta_1 + \Delta_2 + \Delta_3\right) = -(4 + 5 + 2) = -11$$

$$c_2 = (-1)^2 \cdot \left(\Delta_{1,2} + \Delta_{1,3} + \Delta_{2,3}\right) = \begin{vmatrix} 4 & 1 \\ 2 & 5 \end{vmatrix}$$

## Rješenje

a) $k_A(\lambda) = \lambda^3 + c_1\lambda^2 + c_2\lambda + c_3$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$c_1 = (-1)^1 \cdot \big(\Delta_1 + \Delta_2 + \Delta_3\big) = -(4+5+2) = -11$$

$$c_2 = (-1)^2 \cdot \big(\Delta_{1,2} + \Delta_{1,3} + \Delta_{2,3}\big) = \begin{vmatrix} 4 & 1 \\ 2 & 5 \end{vmatrix} +$$

## Rješenje

a) $$k_A(\lambda) = \lambda^3 + c_1\lambda^2 + c_2\lambda + c_3$$

$$A = \begin{array}{cc} \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix} & \begin{array}{l} 1. \\ \\ 3. \end{array} \\ \begin{array}{ccc} 1. & \quad & 3. \end{array} & \end{array}$$

$$c_1 = (-1)^1 \cdot \left(\Delta_1 + \Delta_2 + \Delta_3\right) = -(4+5+2) = -11$$

$$c_2 = (-1)^2 \cdot \left(\Delta_{1,2} + \Delta_{1,3} + \Delta_{2,3}\right) = \begin{vmatrix} 4 & 1 \\ 2 & 5 \end{vmatrix} + \begin{vmatrix} 4 & -1 \\ 1 & 2 \end{vmatrix}$$

## Rješenje

a) $k_A(\lambda) = \lambda^3 + c_1\lambda^2 + c_2\lambda + c_3$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$c_1 = (-1)^1 \cdot \left(\Delta_1 + \Delta_2 + \Delta_3\right) = -(4+5+2) = -11$$

$$c_2 = (-1)^2 \cdot \left(\Delta_{1,2} + \Delta_{1,3} + \Delta_{2,3}\right) = \begin{vmatrix} 4 & 1 \\ 2 & 5 \end{vmatrix} + \begin{vmatrix} 4 & -1 \\ 1 & 2 \end{vmatrix} +$$

## Rješenje

a) $k_A(\lambda) = \lambda^3 + c_1\lambda^2 + c_2\lambda + c_3$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix} \begin{array}{l} \\ \leftarrow 2. \\ \leftarrow 3. \end{array}$$

$$\begin{array}{ccc} & \uparrow & \uparrow \\ & 2. & 3. \end{array}$$

$$c_1 = (-1)^1 \cdot \left(\Delta_1 + \Delta_2 + \Delta_3\right) = -(4 + 5 + 2) = -11$$

$$c_2 = (-1)^2 \cdot \left(\Delta_{1,2} + \Delta_{1,3} + \Delta_{2,3}\right) = \begin{vmatrix} 4 & 1 \\ 2 & 5 \end{vmatrix} + \begin{vmatrix} 4 & -1 \\ 1 & 2 \end{vmatrix} + \begin{vmatrix} 5 & -2 \\ 1 & 2 \end{vmatrix}$$

## Rješenje

a) $k_A(\lambda) = \lambda^3 + c_1\lambda^2 + c_2\lambda + c_3$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$c_1 = (-1)^1 \cdot \left(\Delta_1 + \Delta_2 + \Delta_3\right) = -(4+5+2) = -11$$

$$c_2 = (-1)^2 \cdot \left(\Delta_{1,2} + \Delta_{1,3} + \Delta_{2,3}\right) = \begin{vmatrix} 4 & 1 \\ 2 & 5 \end{vmatrix} + \begin{vmatrix} 4 & -1 \\ 1 & 2 \end{vmatrix} + \begin{vmatrix} 5 & -2 \\ 1 & 2 \end{vmatrix} =$$

## Rješenje

a) $k_A(\lambda) = \lambda^3 + c_1\lambda^2 + c_2\lambda + c_3$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$c_1 = (-1)^1 \cdot \left(\Delta_1 + \Delta_2 + \Delta_3\right) = -(4+5+2) = -11$$

$$c_2 = (-1)^2 \cdot \left(\Delta_{1,2} + \Delta_{1,3} + \Delta_{2,3}\right) = \begin{vmatrix} 4 & 1 \\ 2 & 5 \end{vmatrix} + \begin{vmatrix} 4 & -1 \\ 1 & 2 \end{vmatrix} + \begin{vmatrix} 5 & -2 \\ 1 & 2 \end{vmatrix} =$$

$$= 18$$

## Rješenje

a) $k_A(\lambda) = \lambda^3 + c_1\lambda^2 + c_2\lambda + c_3$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$c_1 = (-1)^1 \cdot \left(\Delta_1 + \Delta_2 + \Delta_3\right) = -(4+5+2) = -11$$

$$c_2 = (-1)^2 \cdot \left(\Delta_{1,2} + \Delta_{1,3} + \Delta_{2,3}\right) = \begin{vmatrix} 4 & 1 \\ 2 & 5 \end{vmatrix} + \begin{vmatrix} 4 & -1 \\ 1 & 2 \end{vmatrix} + \begin{vmatrix} 5 & -2 \\ 1 & 2 \end{vmatrix} =$$

$$= 18 + 9$$

## Rješenje

a) $k_A(\lambda) = \lambda^3 + c_1\lambda^2 + c_2\lambda + c_3$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$c_1 = (-1)^1 \cdot \left(\Delta_1 + \Delta_2 + \Delta_3\right) = -(4+5+2) = -11$$

$$c_2 = (-1)^2 \cdot \left(\Delta_{1,2} + \Delta_{1,3} + \Delta_{2,3}\right) = \begin{vmatrix} 4 & 1 \\ 2 & 5 \end{vmatrix} + \begin{vmatrix} 4 & -1 \\ 1 & 2 \end{vmatrix} + \begin{vmatrix} 5 & -2 \\ 1 & 2 \end{vmatrix} =$$

$$= 18 + 9 + 12$$

## Rješenje

a) $k_A(\lambda) = \lambda^3 + c_1\lambda^2 + c_2\lambda + c_3$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$c_1 = (-1)^1 \cdot \big(\Delta_1 + \Delta_2 + \Delta_3\big) = -(4+5+2) = -11$$

$$c_2 = (-1)^2 \cdot \big(\Delta_{1,2} + \Delta_{1,3} + \Delta_{2,3}\big) = \begin{vmatrix} 4 & 1 \\ 2 & 5 \end{vmatrix} + \begin{vmatrix} 4 & -1 \\ 1 & 2 \end{vmatrix} + \begin{vmatrix} 5 & -2 \\ 1 & 2 \end{vmatrix} =$$

$$= 18 + 9 + 12 = 39$$

## Rješenje

a) $k_A(\lambda) = \lambda^3 + c_1\lambda^2 + c_2\lambda + c_3$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$c_1 = (-1)^1 \cdot \left(\Delta_1 + \Delta_2 + \Delta_3\right) = -(4+5+2) = -11$$

$$c_2 = (-1)^2 \cdot \left(\Delta_{1,2} + \Delta_{1,3} + \Delta_{2,3}\right) = \begin{vmatrix} 4 & 1 \\ 2 & 5 \end{vmatrix} + \begin{vmatrix} 4 & -1 \\ 1 & 2 \end{vmatrix} + \begin{vmatrix} 5 & -2 \\ 1 & 2 \end{vmatrix} =$$

$$= 18 + 9 + 12 = 39$$

$$c_3 =$$

## Rješenje

a) $k_A(\lambda) = \lambda^3 + c_1\lambda^2 + c_2\lambda + c_3$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$c_1 = (-1)^1 \cdot \left(\Delta_1 + \Delta_2 + \Delta_3\right) = -(4 + 5 + 2) = -11$$

$$c_2 = (-1)^2 \cdot \left(\Delta_{1,2} + \Delta_{1,3} + \Delta_{2,3}\right) = \begin{vmatrix} 4 & 1 \\ 2 & 5 \end{vmatrix} + \begin{vmatrix} 4 & -1 \\ 1 & 2 \end{vmatrix} + \begin{vmatrix} 5 & -2 \\ 1 & 2 \end{vmatrix} =$$

$$= 18 + 9 + 12 = 39$$

$$c_3 = (-1)^3 \cdot \Delta_{1,2,3}$$

## Rješenje

a) $k_A(\lambda) = \lambda^3 + c_1\lambda^2 + c_2\lambda + c_3$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$c_1 = (-1)^1 \cdot \left(\Delta_1 + \Delta_2 + \Delta_3\right) = -(4+5+2) = -11$$

$$c_2 = (-1)^2 \cdot \left(\Delta_{1,2} + \Delta_{1,3} + \Delta_{2,3}\right) = \begin{vmatrix} 4 & 1 \\ 2 & 5 \end{vmatrix} + \begin{vmatrix} 4 & -1 \\ 1 & 2 \end{vmatrix} + \begin{vmatrix} 5 & -2 \\ 1 & 2 \end{vmatrix} =$$

$$= 18 + 9 + 12 = 39$$

$$c_3 = (-1)^3 \cdot \Delta_{1,2,3} = -1 \cdot$$

## Rješenje

a) $$k_A(\lambda) = \lambda^3 + c_1\lambda^2 + c_2\lambda + c_3 \qquad A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$c_1 = (-1)^1 \cdot \left(\Delta_1 + \Delta_2 + \Delta_3\right) = -(4+5+2) = -11$$

$$c_2 = (-1)^2 \cdot \left(\Delta_{1,2} + \Delta_{1,3} + \Delta_{2,3}\right) = \begin{vmatrix} 4 & 1 \\ 2 & 5 \end{vmatrix} + \begin{vmatrix} 4 & -1 \\ 1 & 2 \end{vmatrix} + \begin{vmatrix} 5 & -2 \\ 1 & 2 \end{vmatrix} =$$

$$= 18 + 9 + 12 = 39$$

$$c_3 = (-1)^3 \cdot \Delta_{1,2,3} = -1 \cdot \begin{vmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{vmatrix}$$

## Rješenje

a) $k_A(\lambda) = \lambda^3 + c_1\lambda^2 + c_2\lambda + c_3$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$c_1 = (-1)^1 \cdot \left(\Delta_1 + \Delta_2 + \Delta_3\right) = -(4+5+2) = -11$$

$$c_2 = (-1)^2 \cdot \left(\Delta_{1,2} + \Delta_{1,3} + \Delta_{2,3}\right) = \begin{vmatrix} 4 & 1 \\ 2 & 5 \end{vmatrix} + \begin{vmatrix} 4 & -1 \\ 1 & 2 \end{vmatrix} + \begin{vmatrix} 5 & -2 \\ 1 & 2 \end{vmatrix} =$$

$$= 18 + 9 + 12 = 39$$

$$c_3 = (-1)^3 \cdot \Delta_{1,2,3} = -1 \cdot \begin{vmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{vmatrix} = -1 \cdot 45$$

## Rješenje

a) $k_A(\lambda) = \lambda^3 + c_1\lambda^2 + c_2\lambda + c_3$ $\qquad A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$

$$c_1 = (-1)^1 \cdot \left(\Delta_1 + \Delta_2 + \Delta_3\right) = -(4+5+2) = -11$$

$$c_2 = (-1)^2 \cdot \left(\Delta_{1,2} + \Delta_{1,3} + \Delta_{2,3}\right) = \begin{vmatrix} 4 & 1 \\ 2 & 5 \end{vmatrix} + \begin{vmatrix} 4 & -1 \\ 1 & 2 \end{vmatrix} + \begin{vmatrix} 5 & -2 \\ 1 & 2 \end{vmatrix} =$$

$$= 18 + 9 + 12 = 39$$

$$c_3 = (-1)^3 \cdot \Delta_{1,2,3} = -1 \cdot \begin{vmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{vmatrix} = -1 \cdot 45 = -45$$

## Rješenje

a)

$$k_A(\lambda) = \lambda^3 + c_1\lambda^2 + c_2\lambda + c_3 \qquad A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$

$$c_1 = (-1)^1 \cdot \left(\Delta_1 + \Delta_2 + \Delta_3\right) = -(4+5+2) = -11$$

$$c_2 = (-1)^2 \cdot \left(\Delta_{1,2} + \Delta_{1,3} + \Delta_{2,3}\right) = \begin{vmatrix} 4 & 1 \\ 2 & 5 \end{vmatrix} + \begin{vmatrix} 4 & -1 \\ 1 & 2 \end{vmatrix} + \begin{vmatrix} 5 & -2 \\ 1 & 2 \end{vmatrix} =$$

$$= 18 + 9 + 12 = 39$$

$$c_3 = (-1)^3 \cdot \Delta_{1,2,3} = -1 \cdot \begin{vmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{vmatrix} = -1 \cdot 45 = -45$$

$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$

$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$

$$\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$$

$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$

$$\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$$

$1, -1,$

$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$

$$\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$$

$1, -1, 3, -3,$

$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$

$$\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$$

$1, -1, 3, -3, 5, -5,$

$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$

$$\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$$

$1, -1, 3, -3, 5, -5, 9, -9,$

$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$

$$\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$$

$1, -1, 3, -3, 5, -5, 9, -9, 15, -15,$

$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$

$$\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$$

$$1, -1, 3, -3, 5, -5, 9, -9, 15, -15, 45, -45$$

$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$

$$\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$$

$$1, -1, 3, -3, 5, -5, 9, -9, 15, -15, 45, -45$$

$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$

$$\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$$

$$1, -1, 3, -3, 5, -5, 9, -9, 15, -15, 45, -45$$

| | | | | |
|---|---|---|---|---|
| | 1 | | | |
| | | | | |

$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$

$$\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$$

$$1, -1, 3, -3, 5, -5, 9, -9, 15, -15, 45, -45$$

| | | | | |
|---|---|---|---|---|
| | $1$ | $-11$ | | |
| | | | | |

$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$

$$\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$$

$$1, -1, 3, -3, 5, -5, 9, -9, 15, -15, 45, -45$$

| | | | | |
|---|---|---|---|---|
| | 1 | $-11$ | 39 | |
| | | | | |

$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$

$$\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$$

$$1, -1, 3, -3, 5, -5, 9, -9, 15, -15, 45, -45$$

| | 1 | $-11$ | 39 | $-45$ |
|---|---|---|---|---|
| | | | | |

$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$

$$\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$$

$$1, -1, 3, -3, 5, -5, 9, -9, 15, -15, 45, -45$$

$$\begin{array}{c|c|c|c|c}
 & 1 & -11 & 39 & -45 \\
\hline
3 & & & &
\end{array}$$

$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$

$$\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$$

$$1, -1, 3, -3, 5, -5, 9, -9, 15, -15, 45, -45$$

| | 1 | $-11$ | 39 | $-45$ |
|---|---|---|---|---|
| 3 | 1 | | | |

$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$

$$\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$$

$$1, -1, 3, -3, 5, -5, 9, -9, 15, -15, 45, -45$$

$$\begin{array}{c|c|c|c|c} & 1 & -11 & 39 & -45 \\ \hline 3 & 1 & -8 & & \end{array} \qquad 3 \cdot 1 + (-11) = -8$$

$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$

$$\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$$

$$1, -1, 3, -3, 5, -5, 9, -9, 15, -15, 45, -45$$

| | 1 | $-11$ | 39 | $-45$ |
|---|---|---|---|---|
| 3 | 1 | $-8$ | 15 | |

$$3 \cdot (-8) + 39 = 15$$

$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$

$$\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$$

$$1, -1, 3, -3, 5, -5, 9, -9, 15, -15, 45, -45$$

$$\begin{array}{c|c|c|c|c} & 1 & -11 & 39 & -45 \\ \hline 3 & 1 & -8 & 15 & 0 \end{array} \qquad 3 \cdot 15 + (-45) = 0$$

$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$

$$\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$$

$$1, -1, 3, -3, 5, -5, 9, -9, 15, -15, 45, -45$$

| | 1 | $-11$ | 39 | $-45$ |
|---|---|---|---|---|
| 3 | 1 | $-8$ | 15 | 0 |

$$(\lambda - 3)$$

$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$

$$\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$$

$$1, -1, 3, -3, 5, -5, 9, -9, 15, -15, 45, -45$$

| | | | | |
|---|---|---|---|---|
| | 1 | $-11$ | 39 | $-45$ |
| 3 | 1 | $-8$ | 15 | 0 |

$$(\lambda - 3)(\qquad\qquad\qquad)$$

$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$

$$\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$$

$$1, -1, 3, -3, 5, -5, 9, -9, 15, -15, 45, -45$$

| | 1 | $-11$ | 39 | $-45$ |
|---|---|---|---|---|
| 3 | 1 | $-8$ | 15 | 0 |

$$(\lambda - 3)\big(\lambda^2 \qquad\qquad\qquad \big)$$

$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$

$$\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$$

$$1, -1, 3, -3, 5, -5, 9, -9, 15, -15, 45, -45$$

| | 1 | −11 | 39 | −45 |
|---|---|---|---|---|
| 3 | 1 | −8 | 15 | 0 |

$$(\lambda - 3)\big(\lambda^2 - 8\lambda \qquad\quad \big)$$

$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$

$$\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$$

$$1, -1, 3, -3, 5, -5, 9, -9, 15, -15, 45, -45$$

| | 1 | −11 | 39 | −45 |
|---|---|---|---|---|
| 3 | 1 | −8 | 15 | 0 |

$$(\lambda - 3)\left(\lambda^2 - 8\lambda + 15\right)$$

$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$

$$\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$$

$$1, -1, 3, -3, 5, -5, 9, -9, 15, -15, 45, -45$$

| | 1 | −11 | 39 | −45 |
|---|---|---|---|---|
| 3 | 1 | −8 | 15 | 0 |

$$(\lambda - 3)\left(\lambda^2 - 8\lambda + 15\right) = 0$$

$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$

$$\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$$

$$1, -1, 3, -3, 5, -5, 9, -9, 15, -15, 45, -45$$

| | 1 | −11 | 39 | −45 |
|---|---|---|---|---|
| 3 | 1 | −8 | 15 | 0 |

$$(\lambda - 3)\left(\lambda^2 - 8\lambda + 15\right) = 0$$

$$\lambda_1 = 3$$

$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$

$$\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$$

$$1, -1, 3, -3, 5, -5, 9, -9, 15, -15, 45, -45$$

| | 1 | $-11$ | 39 | $-45$ |
|---|---|---|---|---|
| 3 | 1 | $-8$ | 15 | 0 |

$$(\lambda - 3)\left(\lambda^2 - 8\lambda + 15\right) = 0$$

$$\boxed{\lambda_1 = 3}$$

$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$

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$$1, -1, 3, -3, 5, -5, 9, -9, 15, -15, 45, -45$$

| | 1 | $-11$ | 39 | $-45$ |
|---|---|---|---|---|
| 3 | 1 | $-8$ | 15 | 0 |

$$(\lambda - 3)\left(\lambda^2 - 8\lambda + 15\right) = 0$$

$$\boxed{\lambda_1 = 3} \qquad \lambda^2 - 8\lambda + 15 = 0$$

$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$

$$\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$$

$$1, -1, 3, -3, 5, -5, 9, -9, 15, -15, 45, -45$$

| | 1 | $-11$ | 39 | $-45$ |
|---|---|---|---|---|
| 3 | 1 | $-8$ | 15 | 0 |

$$(\lambda - 3)\left(\lambda^2 - 8\lambda + 15\right) = 0$$

$$\boxed{\lambda_1 = 3} \qquad \lambda^2 - 8\lambda + 15 = 0$$

$$\lambda_{2,3} = \frac{-(-8) \pm \sqrt{(-8)^2 - 4 \cdot 1 \cdot 15}}{2 \cdot 1}$$

$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$

$$\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$$

$$1, -1, 3, -3, 5, -5, 9, -9, 15, -15, 45, -45$$

| | 1 | −11 | 39 | −45 |
|---|---|---|---|---|
| 3 | 1 | −8 | 15 | 0 |

$$(\lambda - 3)\left(\lambda^2 - 8\lambda + 15\right) = 0$$

$$\boxed{\lambda_1 = 3} \qquad \lambda^2 - 8\lambda + 15 = 0$$

$$\lambda_{2,3} = \frac{-(-8) \pm \sqrt{(-8)^2 - 4 \cdot 1 \cdot 15}}{2 \cdot 1}$$

$$\lambda_{2,3} = \frac{8 \pm 2}{2}$$

$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$

$$\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$$

$$1, -1, 3, -3, 5, -5, 9, -9, 15, -15, 45, -45$$

| | 1 | $-11$ | 39 | $-45$ |
|---|---|---|---|---|
| 3 | 1 | $-8$ | 15 | 0 |

$$(\lambda - 3)\left(\lambda^2 - 8\lambda + 15\right) = 0$$

$$\boxed{\lambda_1 = 3} \qquad \lambda^2 - 8\lambda + 15 = 0$$

$$\lambda_{2,3} = \frac{-(-8) \pm \sqrt{(-8)^2 - 4 \cdot 1 \cdot 15}}{2 \cdot 1}$$

$$\lambda_{2,3} = \frac{8 \pm 2}{2} \qquad \lambda_2 = 5$$

$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$

$$\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$$

$$1, -1, 3, -3, 5, -5, 9, -9, 15, -15, 45, -45$$

| | 1 | $-11$ | 39 | $-45$ |
|---|---|---|---|---|
| 3 | 1 | $-8$ | 15 | 0 |

$$(\lambda - 3)\left(\lambda^2 - 8\lambda + 15\right) = 0$$

$$\boxed{\lambda_1 = 3} \qquad \lambda^2 - 8\lambda + 15 = 0$$

$$\lambda_{2,3} = \frac{-(-8) \pm \sqrt{(-8)^2 - 4 \cdot 1 \cdot 15}}{2 \cdot 1}$$

$$\lambda_{2,3} = \frac{8 \pm 2}{2} \qquad \lambda_2 = 5,\ \lambda_3 = 3$$

$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$

$$\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$$

$$1, -1, 3, -3, 5, -5, 9, -9, 15, -15, 45, -45$$

| | 1 | −11 | 39 | −45 |
|---|---|---|---|---|
| 3 | 1 | −8 | 15 | 0 |

$$(\lambda - 3)\left(\lambda^2 - 8\lambda + 15\right) = 0$$

$$\boxed{\lambda_1 = 3}$$

$$\lambda^2 - 8\lambda + 15 = 0$$

$$\lambda_{2,3} = \frac{-(-8) \pm \sqrt{(-8)^2 - 4 \cdot 1 \cdot 15}}{2 \cdot 1}$$

$$\lambda_{2,3} = \frac{8 \pm 2}{2} \qquad \boxed{\lambda_2 = 5,\ \lambda_3 = 3}$$

$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$

$$\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$$

$$1, -1, 3, -3, 5, -5, 9, -9, 15, -15, 45, -45$$

$\sigma(A) =$

$$\begin{array}{c|c|c|c|c} & 1 & -11 & 39 & -45 \\ \hline 3 & 1 & -8 & 15 & 0 \end{array}$$

$$(\lambda - 3)\left(\lambda^2 - 8\lambda + 15\right) = 0$$

$$\boxed{\lambda_1 = 3} \qquad \lambda^2 - 8\lambda + 15 = 0$$

$$\lambda_{2,3} = \frac{-(-8) \pm \sqrt{(-8)^2 - 4 \cdot 1 \cdot 15}}{2 \cdot 1}$$

$$\lambda_{2,3} = \frac{8 \pm 2}{2} \qquad \boxed{\lambda_2 = 5,\ \lambda_3 = 3}$$

$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$

$$\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$$

$$1, -1, 3, -3, 5, -5, 9, -9, 15, -15, 45, -45$$

$$\sigma(A) = \{3, 5\}$$

| | 1 | −11 | 39 | −45 |
|---|---|---|---|---|
| 3 | 1 | −8 | 15 | 0 |

$$(\lambda - 3)\left(\lambda^2 - 8\lambda + 15\right) = 0$$

$$\boxed{\lambda_1 = 3}$$

$$\lambda^2 - 8\lambda + 15 = 0$$

$$\lambda_{2,3} = \frac{-(-8) \pm \sqrt{(-8)^2 - 4 \cdot 1 \cdot 15}}{2 \cdot 1}$$

$$\lambda_{2,3} = \frac{8 \pm 2}{2} \qquad \boxed{\lambda_2 = 5,\ \lambda_3 = 3}$$

$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$

$$\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$$

$$1, -1, 3, -3, 5, -5, 9, -9, 15, -15, 45, -45$$

$$\sigma(A) = \{3, 5\}$$

algebarska kratnost jednaka je 2

|   | 1 | −11 | 39 | −45 |
|---|---|---|---|---|
| 3 | 1 | −8 | 15 | 0 |

$$(\lambda - 3)\left(\lambda^2 - 8\lambda + 15\right) = 0$$

$$\boxed{\lambda_1 = 3}$$

$$\lambda^2 - 8\lambda + 15 = 0$$

$$\lambda_{2,3} = \frac{-(-8) \pm \sqrt{(-8)^2 - 4 \cdot 1 \cdot 15}}{2 \cdot 1}$$

$$\lambda_{2,3} = \frac{8 \pm 2}{2} \qquad \boxed{\lambda_2 = 5,\ \lambda_3 = 3}$$

$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$

$$\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$$

$$1, -1, 3, -3, 5, -5, 9, -9, 15, -15, 45, -45$$

| | 1 | −11 | 39 | −45 |
|---|---|---|---|---|
| 3 | 1 | −8 | 15 | 0 |

$$(\lambda - 3)\left(\lambda^2 - 8\lambda + 15\right) = 0$$

$$\boxed{\lambda_1 = 3}$$

$$\lambda^2 - 8\lambda + 15 = 0$$

$$\lambda_{2,3} = \frac{-(-8) \pm \sqrt{(-8)^2 - 4 \cdot 1 \cdot 15}}{2 \cdot 1}$$

$$\lambda_{2,3} = \frac{8 \pm 2}{2} \qquad \boxed{\lambda_2 = 5,\ \lambda_3 = 3}$$

$$\sigma(A) = \{3, 5\}$$

algebarska kratnost jednaka je 1

algebarska kratnost jednaka je 2

$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$

algebarska kratnost jednaka je 1

$$\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$$

$$1, -1, 3, -3, 5, -5, 9, -9, 15, -15, 45, -45$$

| | 1 | −11 | 39 | −45 |
|---|---|---|---|---|
| 3 | 1 | −8 | 15 | 0 |

$$\sigma(A) = \{3, 5\}$$

algebarska kratnost jednaka je 2

$$(\lambda - 3)\left(\lambda^2 - 8\lambda + 15\right) = 0$$

$$\boxed{\lambda_1 = 3}$$

$$\lambda^2 - 8\lambda + 15 = 0$$

$$\lambda_{2,3} = \frac{-(-8) \pm \sqrt{(-8)^2 - 4 \cdot 1 \cdot 15}}{2 \cdot 1}$$

$$k_A(\lambda) =$$

$$\lambda_{2,3} = \frac{8 \pm 2}{2} \qquad \boxed{\lambda_2 = 5,\ \lambda_3 = 3}$$

$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$

algebarska kratnost jednaka je 1

$$\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$$

$$1, -1, 3, -3, 5, -5, 9, -9, 15, -15, 45, -45$$

| | 1 | −11 | 39 | −45 |
|---|---|---|---|---|
| 3 | 1 | −8 | 15 | 0 |

$$\sigma(A) = \{3, 5\}$$

algebarska kratnost jednaka je 2

$$(\lambda - 3)\left(\lambda^2 - 8\lambda + 15\right) = 0$$

$$\boxed{\lambda_1 = 3}$$

$$\lambda^2 - 8\lambda + 15 = 0$$

$$\lambda_{2,3} = \frac{-(-8) \pm \sqrt{(-8)^2 - 4 \cdot 1 \cdot 15}}{2 \cdot 1}$$

$$k_A(\lambda) = (\lambda - 3)^2$$

$$\lambda_{2,3} = \frac{8 \pm 2}{2}$$

$$\boxed{\lambda_2 = 5,\ \lambda_3 = 3}$$

$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$

algebarska kratnost jednaka je 1

$$\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$$

$$1, -1, 3, -3, 5, -5, 9, -9, 15, -15, 45, -45$$

| | 1 | $-11$ | 39 | $-45$ |
|---|---|---|---|---|
| 3 | 1 | $-8$ | 15 | 0 |

$$\sigma(A) = \{3, 5\}$$

algebarska kratnost jednaka je 2

$$(\lambda - 3)\left(\lambda^2 - 8\lambda + 15\right) = 0$$

$$\boxed{\lambda_1 = 3}$$

$$\lambda^2 - 8\lambda + 15 = 0$$

$$\lambda_{2,3} = \frac{-(-8) \pm \sqrt{(-8)^2 - 4 \cdot 1 \cdot 15}}{2 \cdot 1}$$

$$k_A(\lambda) = (\lambda - 3)^2 \cdot (\lambda - 5)$$

$$\lambda_{2,3} = \frac{8 \pm 2}{2}$$

$$\boxed{\lambda_2 = 5,\ \lambda_3 = 3}$$

b)

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\boxed{S(\lambda) = \{x \in V : f(x) = \lambda x\}}$$

$$\boxed{(F_{\mathcal{B}} - \lambda I) X_{\mathcal{B}} = O}$$

b)

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$(A - 3I)X = O$$

$$\boxed{S(\lambda) = \left\{x \in V : f(x) = \lambda x\right\}}$$

$$\boxed{\left(F_{\mathcal{B}} - \lambda I\right) X_{\mathcal{B}} = O}$$

b)

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$(A - 3I)X = O$$

$$\begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$$

$$\boxed{S(\lambda) = \left\{x \in V : f(x) = \lambda x\right\}} \qquad \boxed{\left(F_{\mathcal{B}} - \lambda I\right)X_{\mathcal{B}} = O}$$

b)

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$(A - 3I)X = O$$

$$\begin{bmatrix} 1 & & \\ & & \\ & & \end{bmatrix}$$

$$\boxed{S(\lambda) = \{x \in V : f(x) = \lambda x\}}$$

$$\boxed{\left(F_{\mathcal{B}} - \lambda I\right) X_{\mathcal{B}} = O}$$

b)

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$(A - 3I)X = O$$

$$\begin{bmatrix} 1 & & \\ & 2 & \\ & & \end{bmatrix}$$

$$\boxed{S(\lambda) = \{x \in V : f(x) = \lambda x\}}$$

$$\boxed{\left(F_{\mathcal{B}} - \lambda I\right) X_{\mathcal{B}} = O}$$

b)

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$(A - 3I)X = O$$

$$\begin{bmatrix} 1 & & \\ & 2 & \\ & & -1 \end{bmatrix}$$

$$\boxed{S(\lambda) = \left\{x \in V : f(x) = \lambda x\right\}}$$

$$\boxed{\left(F_{\mathcal{B}} - \lambda I\right) X_{\mathcal{B}} = O}$$

b)

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$(A - 3I)X = O$$

$$\begin{bmatrix} 1 & 1 & -1 \\ & 2 & -2 \\ & & -1 \end{bmatrix}$$

$$\boxed{S(\lambda) = \{x \in V : f(x) = \lambda x\}}$$

$$\boxed{(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O}$$

b)

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$(A - 3I)X = O$$

$$\begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & -2 \\ 1 & 1 & -1 \end{bmatrix}$$

$$\boxed{S(\lambda) = \left\{x \in V : f(x) = \lambda x\right\}}$$

$$\boxed{\left(F_{\mathcal{B}} - \lambda I\right) X_{\mathcal{B}} = O}$$

b)

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$(A - 3I)X = O$$

$$\begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & -2 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$\boxed{S(\lambda) = \{x \in V : f(x) = \lambda x\}}$$

$$\boxed{\left(F_{\mathcal{B}} - \lambda I\right) X_{\mathcal{B}} = O}$$

b)

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$(A - 3I)X = O$$

$$\begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & -2 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\boxed{S(\lambda) = \{x \in V : f(x) = \lambda x\}}$$

$$\boxed{\left(F_{\mathcal{B}} - \lambda I\right) X_{\mathcal{B}} = O}$$

b)

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

| $x_1$ | $x_2$ | $x_3$ | |
|---|---|---|---|

$$(A - 3I)X = O$$

$$\begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & -2 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\boxed{S(\lambda) = \{x \in V : f(x) = \lambda x\}}$$

$$\boxed{\left(F_{\mathcal{B}} - \lambda I\right) X_{\mathcal{B}} = O}$$

b)

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

| $x_1$ | $x_2$ | $x_3$ | |
|---|---|---|---|
| 1 | 1 | $-1$ | 0 |

$$(A - 3I)X = O$$

$$\begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & -2 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\boxed{S(\lambda) = \{x \in V : f(x) = \lambda x\}}$$

$$\boxed{\left(F_{\mathcal{B}} - \lambda I\right) X_{\mathcal{B}} = O}$$

b)

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

| $x_1$ | $x_2$ | $x_3$ | |
|---|---|---|---|
| 1 | 1 | $-1$ | 0 |
| 2 | 2 | $-2$ | 0 |

$$(A - 3I)X = O$$

$$\begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & -2 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\boxed{S(\lambda) = \left\{x \in V : f(x) = \lambda x\right\}}$$

$$\boxed{\left(F_{\mathcal{B}} - \lambda I\right) X_{\mathcal{B}} = O}$$

b)

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

| $x_1$ | $x_2$ | $x_3$ | |
|---|---|---|---|
| 1 | 1 | $-1$ | 0 |
| 2 | 2 | $-2$ | 0 |
| 1 | 1 | $-1$ | 0 |

$$(A - 3I)X = O$$

$$\begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & -2 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\boxed{S(\lambda) = \{x \in V : f(x) = \lambda x\}}$$

$$\boxed{\left(F_{\mathcal{B}} - \lambda I\right) X_{\mathcal{B}} = O}$$

b)

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

| $x_1$ | $x_2$ | $x_3$ | |
|---|---|---|---|
| 1 | 1 | $-1$ | 0 |
| 2 | 2 | $-2$ | 0 |
| 1 | 1 | $-1$ | 0 |

$$(A - 3I)X = O$$

$$\begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & -2 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\boxed{S(\lambda) = \{x \in V : f(x) = \lambda x\}} \qquad \boxed{\left(F_{\mathcal{B}} - \lambda I\right) X_{\mathcal{B}} = O}$$

b)

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

| $x_1$ | $x_2$ | $x_3$ | | |
|---|---|---|---|---|
| 1 | 1 | $-1$ | 0 | |
| 2 | 2 | $-2$ | 0 | $/:2$ |
| 1 | 1 | $-1$ | 0 | |

$$(A-3I)X = O$$

$$\begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & -2 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\boxed{S(\lambda) = \{x \in V : f(x) = \lambda x\}}$$

$$\boxed{\left(F_{\mathcal{B}} - \lambda I\right) X_{\mathcal{B}} = O}$$

b)

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$(A - 3I)X = O$$

$$\begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & -2 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

| $x_1$ | $x_2$ | $x_3$ | | |
|---|---|---|---|---|
| 1 | 1 | $-1$ | 0 | |
| 2 | 2 | $-2$ | 0 | $/:2$ |
| 1 | 1 | $-1$ | 0 | |
| 1 | 1 | $-1$ | 0 | |

$$\boxed{S(\lambda) = \{x \in V : f(x) = \lambda x\}}$$

$$\boxed{\left(F_{\mathcal{B}} - \lambda I\right) X_{\mathcal{B}} = O}$$

b)

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

| $x_1$ | $x_2$ | $x_3$ | | |
|---|---|---|---|---|
| 1 | 1 | $-1$ | 0 | |
| 2 | 2 | $-2$ | 0 | $/:2$ |
| 1 | 1 | $-1$ | 0 | |
| 1 | 1 | $-1$ | 0 | |
| 1 | 1 | $-1$ | 0 | |

$$(A-3I)X = O$$

$$\begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & -2 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\boxed{S(\lambda) = \{x \in V : f(x) = \lambda x\}}$$

$$\boxed{\left(F_{\mathcal{B}} - \lambda I\right) X_{\mathcal{B}} = O}$$

b)

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$(A - 3I)X = O$$

$$\begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & -2 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

| $x_1$ | $x_2$ | $x_3$ | | |
|---|---|---|---|---|
| 1 | 1 | $-1$ | 0 | |
| 2 | 2 | $-2$ | 0 | $/:2$ |
| 1 | 1 | $-1$ | 0 | |
| 1 | 1 | $-1$ | 0 | |
| 1 | 1 | $-1$ | 0 | |
| 1 | 1 | $-1$ | 0 | |

$$\boxed{S(\lambda) = \{x \in V : f(x) = \lambda x\}}$$

$$\boxed{\left(F_{\mathcal{B}} - \lambda I\right) X_{\mathcal{B}} = O}$$

b)

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$(A - 3I)X = O$$

$$\begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & -2 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

| $x_1$ | $x_2$ | $x_3$ | | |
|---|---|---|---|---|
| 1 | 1 | $-1$ | 0 | |
| 2 | 2 | $-2$ | 0 | $/ : 2$ |
| 1 | 1 | $-1$ | 0 | |
| 1 | 1 | $-1$ | 0 | |
| 1 | 1 | $-1$ | 0 | |
| 1 | 1 | $-1$ | 0 | |

$$\boxed{S(\lambda) = \{x \in V : f(x) = \lambda x\}}$$

$$\boxed{\left(F_{\mathcal{B}} - \lambda I\right) X_{\mathcal{B}} = O}$$

b)

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

| $x_1$ | $x_2$ | $x_3$ | | |
|---|---|---|---|---|
| 1 | 1 | $-1$ | 0 | |
| 2 | 2 | $-2$ | 0 | $/ : 2$ |
| 1 | 1 | $-1$ | 0 | |
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$$x_1 + x_2 - x_3 = 0$$

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$$x_1 + x_2 - x_3 = 0 \rightsquigarrow x_3 = x_1 + x_2$$

$$\boxed{S(\lambda) = \{x \in V : f(x) = \lambda x\}} \qquad \boxed{\left(F_{\mathcal{B}} - \lambda I\right) X_{\mathcal{B}} = O}$$

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$$(x_1, x_2, x_1 + x_2) = x_1 \cdot (1,$$

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$$(x_1, x_2, x_1 + x_2) = x_1 \cdot (1, 0,$$

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$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

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$$\boxed{S(\lambda) = \left\{ x \in V : f(x) = \lambda x \right\}} \qquad \boxed{\left(F_{\mathcal{B}} - \lambda I\right) X_{\mathcal{B}} = O}$$

b) $\mathcal{B}_{S(3)} = \{(1,0,1),\ (0,1,1)\}$ $\quad \dim S(3) = 2$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

| $x_1$ | $x_2$ | $x_3$ | | |
|---|---|---|---|---|
| 1 | 1 | $-1$ | 0 | |
| 2 | 2 | $-2$ | 0 | $/:2$ |
| 1 | 1 | $-1$ | 0 | |
| 1 | 1 | $-1$ | 0 | |
| 1 | 1 | $-1$ | 0 | |
| 1 | 1 | $-1$ | 0 | |
| 1 | 1 | $-1$ | 0 | |

$$(A - 3I)X = O$$

$$\begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & -2 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$x_1 + x_2 - x_3 = 0 \rightsquigarrow x_3 = x_1 + x_2$$

$$S(3) = \{(x_1, x_2, x_1 + x_2) : x_1, x_2 \in \mathbb{R}\}$$

$$(x_1, x_2, x_1 + x_2) = x_1 \cdot (1,0,1) + x_2 \cdot (0,1,1)$$

$$\boxed{S(\lambda) = \{x \in V : f(x) = \lambda x\}} \qquad \boxed{(F_{\mathcal{B}} - \lambda I) X_{\mathcal{B}} = O}$$

b) $\mathcal{B}_{S(3)} = \{(1,0,1),\ (0,1,1)\}$ $\dim S(3) = 2$

geometrijska kratnost
svojstvene vrijednosti $\lambda = 3$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

| $x_1$ | $x_2$ | $x_3$ | | |
|---|---|---|---|---|
| 1 | 1 | −1 | 0 | |
| 2 | 2 | −2 | 0 | $/:2$ |
| 1 | 1 | −1 | 0 | |
| 1 | 1 | −1 | 0 | |
| 1 | 1 | −1 | 0 | |
| 1 | 1 | −1 | 0 | |
| 1 | 1 | −1 | 0 | |

$$(A - 3I)X = O$$

$$\begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & -2 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$x_1 + x_2 - x_3 = 0 \rightsquigarrow x_3 = x_1 + x_2$$

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$$\boxed{S(\lambda) = \{x \in V : f(x) = \lambda x\}} \qquad \boxed{(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O}$$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\boxed{S(\lambda) = \left\{x \in V : f(x) = \lambda x\right\}} \qquad \boxed{\left(F_{\mathcal{B}} - \lambda I\right) X_{\mathcal{B}} = O}$$

$$(A - 5I)X = O \qquad A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\boxed{S(\lambda) = \{x \in V : f(x) = \lambda x\}} \qquad \boxed{\left(F_{\mathcal{B}} - \lambda I\right) X_{\mathcal{B}} = O}$$

$$(A-5I)X=O \qquad A=\begin{bmatrix}4 & 1 & -1\\ 2 & 5 & -2\\ 1 & 1 & 2\end{bmatrix}$$

$$\begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$$

$$\boxed{S(\lambda)=\{x\in V : f(x)=\lambda x\}} \qquad \boxed{\left(F_{\mathcal{B}}-\lambda I\right)X_{\mathcal{B}}=O}$$

$$(A - 5I)X = O \qquad A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} -1 & & \\ & & \\ & & \end{bmatrix}$$

$$\boxed{S(\lambda) = \{x \in V : f(x) = \lambda x\}} \qquad \boxed{\left(F_{\mathcal{B}} - \lambda I\right) X_{\mathcal{B}} = O}$$

$$(A - 5I)X = O \qquad A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} -1 & & \\ & 0 & \\ & & \end{bmatrix}$$

$$\boxed{S(\lambda) = \{x \in V : f(x) = \lambda x\}} \qquad \boxed{(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O}$$

$$(A - 5I)X = O \qquad A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} -1 & & \\ & 0 & \\ & & -3 \end{bmatrix}$$

$$\boxed{S(\lambda) = \{x \in V : f(x) = \lambda x\}} \qquad \boxed{(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O}$$

$$(A - 5I)X = O \qquad A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 & -1 \\ & 0 & -2 \\ & & -3 \end{bmatrix}$$

$$\boxed{S(\lambda) = \{x \in V : f(x) = \lambda x\}} \qquad \boxed{(F_{\mathcal{B}} - \lambda I) X_{\mathcal{B}} = O}$$

$$(A - 5I)X = O \qquad A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix}$$

$$\boxed{S(\lambda) = \left\{x \in V : f(x) = \lambda x\right\}} \qquad \boxed{\left(F_{\mathcal{B}} - \lambda I\right)X_{\mathcal{B}} = O}$$

$$(A-5I)X=O \qquad A=\begin{bmatrix}4 & 1 & -1\\ 2 & 5 & -2\\ 1 & 1 & 2\end{bmatrix}$$

$$\begin{bmatrix}-1 & 1 & -1\\ 2 & 0 & -2\\ 1 & 1 & -3\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}$$

$$\boxed{S(\lambda)=\{x\in V : f(x)=\lambda x\}} \qquad \boxed{\left(F_{\mathcal{B}}-\lambda I\right)X_{\mathcal{B}}=O}$$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$(A - 5I)X = O$$

$$\begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\boxed{S(\lambda) = \left\{x \in V : f(x) = \lambda x\right\}}$$

$$\boxed{\left(F_{\mathcal{B}} - \lambda I\right) X_{\mathcal{B}} = O}$$

| $x_1$ | $x_2$ | $x_3$ | |
|---|---|---|---|

$$(A-5I)X=O \qquad A=\begin{bmatrix}4&1&-1\\2&5&-2\\1&1&2\end{bmatrix}$$

$$\begin{bmatrix}-1&1&-1\\2&0&-2\\1&1&-3\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$\boxed{S(\lambda)=\left\{x\in V : f(x)=\lambda x\right\}} \qquad \boxed{\left(F_{\mathcal{B}}-\lambda I\right)X_{\mathcal{B}}=O}$$

| $x_1$ | $x_2$ | $x_3$ | |
|---|---|---|---|
| $-1$ | $1$ | $-1$ | $0$ |

$$(A-5I)X = O \qquad A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\boxed{S(\lambda) = \{x \in V : f(x) = \lambda x\}} \qquad \boxed{(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O}$$

| $x_1$ | $x_2$ | $x_3$ | |
|---|---|---|---|
| $-1$ | $1$ | $-1$ | $0$ |
| $2$ | $0$ | $-2$ | $0$ |

$$(A - 5I)X = O \qquad A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\boxed{S(\lambda) = \{x \in V : f(x) = \lambda x\}} \qquad \boxed{\left(F_{\mathcal{B}} - \lambda I\right) X_{\mathcal{B}} = O}$$

| $x_1$ | $x_2$ | $x_3$ | |
|---|---|---|---|
| $-1$ | $1$ | $-1$ | $0$ |
| $2$ | $0$ | $-2$ | $0$ |
| $1$ | $1$ | $-3$ | $0$ |

$$(A-5I)X=O \qquad A=\begin{bmatrix}4 & 1 & -1\\ 2 & 5 & -2\\ 1 & 1 & 2\end{bmatrix}$$

$$\begin{bmatrix}-1 & 1 & -1\\ 2 & 0 & -2\\ 1 & 1 & -3\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}=\begin{bmatrix}0\\ 0\\ 0\end{bmatrix}$$

$$\boxed{S(\lambda)=\{x\in V : f(x)=\lambda x\}} \qquad \boxed{\left(F_{\mathcal{B}}-\lambda I\right)X_{\mathcal{B}}=O}$$

| $x_1$ | $x_2$ | $x_3$ | |
|---|---|---|---|
| $-1$ | $1$ | $-1$ | $0$ |
| $2$ | $0$ | $-2$ | $0$ |
| $1$ | $1$ | $-3$ | $0$ |

$$(A-5I)X = O \qquad A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\boxed{S(\lambda) = \{x \in V : f(x) = \lambda x\}} \qquad \boxed{\left(F_{\mathcal{B}} - \lambda I\right) X_{\mathcal{B}} = O}$$

| $x_1$ | $x_2$ | $x_3$ | | |
|---|---|---|---|---|
| $-1$ | $1$ | $-1$ | $0$ | |
| $2$ | $0$ | $-2$ | $0$ | $/ : 2$ |
| $1$ | $1$ | $-3$ | $0$ | |

$$(A - 5I)X = O \qquad A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\boxed{S(\lambda) = \{x \in V : f(x) = \lambda x\}} \qquad \boxed{\left(F_{\mathcal{B}} - \lambda I\right) X_{\mathcal{B}} = O}$$

| $x_1$ | $x_2$ | $x_3$ | |
|---|---|---|---|
| $-1$ | $1$ | $-1$ | $0$ |
| $2$ | $0$ | $-2$ | $0$ $/:2$ |
| $1$ | $1$ | $-3$ | $0$ |
| $-1$ | $1$ | $-1$ | $0$ |

$$(A-5I)X=O \qquad A=\begin{bmatrix}4 & 1 & -1\\ 2 & 5 & -2\\ 1 & 1 & 2\end{bmatrix}$$

$$\begin{bmatrix}-1 & 1 & -1\\ 2 & 0 & -2\\ 1 & 1 & -3\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}=\begin{bmatrix}0\\ 0\\ 0\end{bmatrix}$$

$$\boxed{S(\lambda)=\{x\in V : f(x)=\lambda x\}}$$

$$\boxed{\left(F_{\mathcal{B}}-\lambda I\right)X_{\mathcal{B}}=O}$$

| $x_1$ | $x_2$ | $x_3$ | | |
|---|---|---|---|---|
| $-1$ | $1$ | $-1$ | $0$ | |
| $2$ | $0$ | $-2$ | $0$ | $/:2$ |
| $1$ | $1$ | $-3$ | $0$ | |
| $-1$ | $1$ | $-1$ | $0$ | |
| $1$ | $0$ | $-1$ | $0$ | |

$$(A-5I)X=O \qquad A=\begin{bmatrix}4&1&-1\\2&5&-2\\1&1&2\end{bmatrix}$$

$$\begin{bmatrix}-1&1&-1\\2&0&-2\\1&1&-3\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$\boxed{S(\lambda)=\{x\in V: f(x)=\lambda x\}} \qquad \boxed{\left(F_{\mathcal{B}}-\lambda I\right)X_{\mathcal{B}}=O}$$

| $x_1$ | $x_2$ | $x_3$ | |
|---|---|---|---|
| $-1$ | $1$ | $-1$ | $0$ |
| $2$ | $0$ | $-2$ | $0$ $/:2$ |
| $1$ | $1$ | $-3$ | $0$ |
| $-1$ | $1$ | $-1$ | $0$ |
| $1$ | $0$ | $-1$ | $0$ |
| $1$ | $1$ | $-3$ | $0$ |

$$(A-5I)X=O \qquad A=\begin{bmatrix}4 & 1 & -1\\ 2 & 5 & -2\\ 1 & 1 & 2\end{bmatrix}$$

$$\begin{bmatrix}-1 & 1 & -1\\ 2 & 0 & -2\\ 1 & 1 & -3\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}=\begin{bmatrix}0\\ 0\\ 0\end{bmatrix}$$

$$\boxed{S(\lambda)=\{x\in V : f(x)=\lambda x\}} \qquad \boxed{\big(F_{\mathcal{B}}-\lambda I\big)X_{\mathcal{B}}=O}$$

| $x_1$ | $x_2$ | $x_3$ | |
|---|---|---|---|
| $-1$ | $1$ | $-1$ | $0$ |
| $2$ | $0$ | $-2$ | $0$ |
| $1$ | $1$ | $-3$ | $0$ |
| $-1$ | $1$ | $-1$ | $0$ |
| $1$ | $0$ | $-1$ | $0$ |
| $1$ | $1$ | $-3$ | $0$ |

$/ : 2$

$$(A - 5I)X = O \qquad A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\boxed{S(\lambda) = \{x \in V : f(x) = \lambda x\}} \qquad \boxed{\left(F_{\mathcal{B}} - \lambda I\right) X_{\mathcal{B}} = O}$$

| $x_1$ | $x_2$ | $x_3$ | |
|---|---|---|---|
| $-1$ | $1$ | $-1$ | $0$ |
| $2$ | $0$ | $-2$ | $0$ |
| $1$ | $1$ | $-3$ | $0$ |
| $-1$ | $1$ | $-1$ | $0$ |
| $1$ | $0$ | $-1$ | $0$ |
| $1$ | $1$ | $-3$ | $0$ |

$/:2$

$$(A-5I)X=O \qquad A=\begin{bmatrix}4&1&-1\\2&5&-2\\1&1&2\end{bmatrix}$$

$$\begin{bmatrix}-1&1&-1\\2&0&-2\\1&1&-3\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$\boxed{S(\lambda)=\{x\in V : f(x)=\lambda x\}} \qquad \boxed{\left(F_{\mathcal{B}}-\lambda I\right)X_{\mathcal{B}}=O}$$

$$
\begin{array}{ccc|c}
x_1 & x_2 & x_3 & \\
\hline
-1 & 1 & -1 & 0 \\
2 & 0 & -2 & 0 \\
1 & 1 & -3 & 0 \\
\hline
-1 & 1 & -1 & 0 \\
1 & 0 & -1 & 0 \\
1 & 1 & -3 & 0 \\
\hline
\end{array}
\qquad
\begin{array}{l}
\\
\\
/:2 \\
\\
/\cdot(-1) \\
\\
\\
\end{array}
$$

$$(A - 5I)X = O \qquad A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\boxed{S(\lambda) = \{x \in V : f(x) = \lambda x\}} \qquad \boxed{(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O}$$

| $x_1$ | $x_2$ | $x_3$ | | |
|---|---|---|---|---|
| $-1$ | $1$ | $-1$ | $0$ | |
| $2$ | $0$ | $-2$ | $0$ | $/:2$ |
| $1$ | $1$ | $-3$ | $0$ | |
| $-1$ | $1$ | $-1$ | $0$ | $/\cdot(-1)$ |
| $1$ | $0$ | $-1$ | $0$ | |
| $1$ | $1$ | $-3$ | $0$ | $+$ |

$$(A-5I)X = O \qquad A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\boxed{S(\lambda) = \{x \in V : f(x) = \lambda x\}} \qquad \boxed{(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O}$$

| $x_1$ | $x_2$ | $x_3$ | | |
|---|---|---|---|---|
| $-1$ | $1$ | $-1$ | $0$ | |
| $2$ | $0$ | $-2$ | $0$ | $/:2$ |
| $1$ | $1$ | $-3$ | $0$ | |
| $-1$ | $1$ | $-1$ | $0$ | $/\cdot(-1)$ |
| $1$ | $0$ | $-1$ | $0$ | |
| $1$ | $1$ | $-3$ | $0$ | $+$ |
| $-1$ | $1$ | $-1$ | $0$ | |

$$(A-5I)X=O \qquad A=\begin{bmatrix}4&1&-1\\2&5&-2\\1&1&2\end{bmatrix}$$

$$\begin{bmatrix}-1&1&-1\\2&0&-2\\1&1&-3\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$\boxed{S(\lambda)=\{x\in V: f(x)=\lambda x\}} \qquad \boxed{(F_{\mathcal{B}}-\lambda I)X_{\mathcal{B}}=O}$$

| $x_1$ | $x_2$ | $x_3$ | | |
|---|---|---|---|---|
| $-1$ | $1$ | $-1$ | $0$ | |
| $2$ | $0$ | $-2$ | $0$ | $/:2$ |
| $1$ | $1$ | $-3$ | $0$ | |
| $-1$ | $1$ | $-1$ | $0$ | $/\cdot(-1)$ |
| $1$ | $0$ | $-1$ | $0$ | |
| $1$ | $1$ | $-3$ | $0$ | $+$ |
| $-1$ | $1$ | $-1$ | $0$ | |
| $1$ | $0$ | $-1$ | $0$ | |

$$(A-5I)X=O \qquad A=\begin{bmatrix}4&1&-1\\2&5&-2\\1&1&2\end{bmatrix}$$

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$$\boxed{S(\lambda)=\{x\in V: f(x)=\lambda x\}} \qquad \boxed{(F_{\mathcal{B}}-\lambda I)X_{\mathcal{B}}=O}$$

| $x_1$ | $x_2$ | $x_3$ | | |
|---|---|---|---|---|
| $-1$ | $1$ | $-1$ | $0$ | |
| $2$ | $0$ | $-2$ | $0$ | $/:2$ |
| $1$ | $1$ | $-3$ | $0$ | |
| $-1$ | $1$ | $-1$ | $0$ | $/\cdot(-1)$ |
| $1$ | $0$ | $-1$ | $0$ | |
| $1$ | $1$ | $-3$ | $0$ | $+$ |
| $-1$ | $1$ | $-1$ | $0$ | |
| $1$ | $0$ | $-1$ | $0$ | |
| $2$ | | | | |

$$(A-5I)X=O \qquad A=\begin{bmatrix}4 & 1 & -1\\ 2 & 5 & -2\\ 1 & 1 & 2\end{bmatrix}$$

$$\begin{bmatrix}-1 & 1 & -1\\ 2 & 0 & -2\\ 1 & 1 & -3\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$\boxed{S(\lambda)=\{x\in V : f(x)=\lambda x\}} \qquad \boxed{(F_{\mathcal{B}}-\lambda I)X_{\mathcal{B}}=O}$$

| $x_1$ | $x_2$ | $x_3$ | | |
|---|---|---|---|---|
| $-1$ | $1$ | $-1$ | $0$ | |
| $2$ | $0$ | $-2$ | $0$ | $/:2$ |
| $1$ | $1$ | $-3$ | $0$ | |
| $-1$ | $1$ | $-1$ | $0$ | $/\cdot(-1)$ |
| $1$ | $0$ | $-1$ | $0$ | |
| $1$ | $1$ | $-3$ | $0$ | $+$ |
| $-1$ | $1$ | $-1$ | $0$ | |
| $1$ | $0$ | $-1$ | $0$ | |
| $2$ | $0$ | | | |

$$(A-5I)X=O \qquad A=\begin{bmatrix}4&1&-1\\2&5&-2\\1&1&2\end{bmatrix}$$

$$\begin{bmatrix}-1&1&-1\\2&0&-2\\1&1&-3\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$\boxed{S(\lambda)=\{x\in V : f(x)=\lambda x\}} \qquad \boxed{(F_{\mathcal{B}}-\lambda I)X_{\mathcal{B}}=O}$$

| $x_1$ | $x_2$ | $x_3$ | | |
|---|---|---|---|---|
| $-1$ | $1$ | $-1$ | $0$ | |
| $2$ | $0$ | $-2$ | $0$ | $/:2$ |
| $1$ | $1$ | $-3$ | $0$ | |
| $-1$ | $1$ | $-1$ | $0$ | $/\cdot(-1)$ |
| $1$ | $0$ | $-1$ | $0$ | |
| $1$ | $1$ | $-3$ | $0$ | $+$ |
| $-1$ | $1$ | $-1$ | $0$ | |
| $1$ | $0$ | $-1$ | $0$ | |
| $2$ | $0$ | $-2$ | | |

$$(A-5I)X=O \qquad A=\begin{bmatrix}4&1&-1\\2&5&-2\\1&1&2\end{bmatrix}$$

$$\begin{bmatrix}-1&1&-1\\2&0&-2\\1&1&-3\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$\boxed{S(\lambda)=\{x\in V: f(x)=\lambda x\}} \qquad \boxed{(F_{\mathcal{B}}-\lambda I)X_{\mathcal{B}}=O}$$

$$
\begin{array}{ccc|cl}
x_1 & x_2 & x_3 & & \\
\hline
-1 & 1 & -1 & 0 & \\
2 & 0 & -2 & 0 & /:2 \\
1 & 1 & -3 & 0 & \\
\hline
-1 & 1 & -1 & 0 & /\cdot(-1) \\
1 & 0 & -1 & 0 & \\
1 & 1 & -3 & 0 & \leftarrow + \\
\hline
-1 & 1 & -1 & 0 & \\
1 & 0 & -1 & 0 & \\
2 & 0 & -2 & 0 &
\end{array}
$$

$$(A-5I)X=O \qquad A=\begin{bmatrix}4 & 1 & -1\\ 2 & 5 & -2\\ 1 & 1 & 2\end{bmatrix}$$

$$\begin{bmatrix}-1 & 1 & -1\\ 2 & 0 & -2\\ 1 & 1 & -3\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}=\begin{bmatrix}0\\ 0\\ 0\end{bmatrix}$$

$$\boxed{S(\lambda)=\{x\in V : f(x)=\lambda x\}} \qquad \boxed{(F_{\mathcal{B}}-\lambda I)X_{\mathcal{B}}=O}$$

| $x_1$ | $x_2$ | $x_3$ | | |
|---|---|---|---|---|
| $-1$ | $1$ | $-1$ | $0$ | |
| $2$ | $0$ | $-2$ | $0$ | $/:2$ |
| $1$ | $1$ | $-3$ | $0$ | |
| $-1$ | $1$ | $-1$ | $0$ | $/\cdot(-1)$ |
| $1$ | $0$ | $-1$ | $0$ | |
| $1$ | $1$ | $-3$ | $0$ | $+$ |
| $-1$ | $1$ | $-1$ | $0$ | |
| $1$ | $0$ | $-1$ | $0$ | |
| $2$ | $0$ | $-2$ | $0$ | |

$$(A-5I)X=O \qquad A=\begin{bmatrix}4 & 1 & -1\\ 2 & 5 & -2\\ 1 & 1 & 2\end{bmatrix}$$

$$\begin{bmatrix}-1 & 1 & -1\\ 2 & 0 & -2\\ 1 & 1 & -3\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}=\begin{bmatrix}0\\ 0\\ 0\end{bmatrix}$$

$$\boxed{S(\lambda)=\{x\in V: f(x)=\lambda x\}} \qquad \boxed{\left(F_{\mathcal{B}}-\lambda I\right)X_{\mathcal{B}}=O}$$

| $x_1$ | $x_2$ | $x_3$ | | |
|---|---|---|---|---|
| $-1$ | $1$ | $-1$ | $0$ | |
| $2$ | $0$ | $-2$ | $0$ | $/:2$ |
| $1$ | $1$ | $-3$ | $0$ | |
| $-1$ | $1$ | $-1$ | $0$ | $/\cdot(-1)$ |
| $1$ | $0$ | $-1$ | $0$ | |
| $1$ | $1$ | $-3$ | $0$ | $+$ |
| $-1$ | $1$ | $-1$ | $0$ | |
| $1$ | $0$ | $-1$ | $0$ | |
| $2$ | $0$ | $-2$ | $0$ | |
| $-1$ | $1$ | $-1$ | $0$ | |

$$(A-5I)X=O \qquad A=\begin{bmatrix}4 & 1 & -1\\ 2 & 5 & -2\\ 1 & 1 & 2\end{bmatrix}$$

$$\begin{bmatrix}-1 & 1 & -1\\ 2 & 0 & -2\\ 1 & 1 & -3\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}=\begin{bmatrix}0\\ 0\\ 0\end{bmatrix}$$

$$\boxed{S(\lambda)=\{x\in V : f(x)=\lambda x\}} \qquad \boxed{(F_{\mathcal{B}}-\lambda I)X_{\mathcal{B}}=O}$$

| $x_1$ | $x_2$ | $x_3$ | | |
|---|---|---|---|---|
| $-1$ | $1$ | $-1$ | $0$ | |
| $2$ | $0$ | $-2$ | $0$ | $/:2$ |
| $1$ | $1$ | $-3$ | $0$ | |
| $-1$ | $1$ | $-1$ | $0$ | $/\cdot(-1)$ |
| $1$ | $0$ | $-1$ | $0$ | |
| $1$ | $1$ | $-3$ | $0$ | $+$ |
| $-1$ | $1$ | $-1$ | $0$ | |
| $1$ | $0$ | $-1$ | $0$ | |
| $2$ | $0$ | $-2$ | $0$ | |
| $-1$ | $1$ | $-1$ | $0$ | |
| $1$ | $0$ | $-1$ | $0$ | |

$$(A-5I)X=O \qquad A=\begin{bmatrix}4 & 1 & -1\\ 2 & 5 & -2\\ 1 & 1 & 2\end{bmatrix}$$

$$\begin{bmatrix}-1 & 1 & -1\\ 2 & 0 & -2\\ 1 & 1 & -3\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}=\begin{bmatrix}0\\ 0\\ 0\end{bmatrix}$$

$$\boxed{S(\lambda)=\{x\in V : f(x)=\lambda x\}} \qquad \boxed{(F_{\mathcal{B}}-\lambda I)X_{\mathcal{B}}=O}$$

| $x_1$ | $x_2$ | $x_3$ | | |
|---|---|---|---|---|
| $-1$ | $1$ | $-1$ | $0$ | |
| $2$ | $0$ | $-2$ | $0$ | $/:2$ |
| $1$ | $1$ | $-3$ | $0$ | |
| $-1$ | $1$ | $-1$ | $0$ | $/\cdot(-1)$ |
| $1$ | $0$ | $-1$ | $0$ | |
| $1$ | $1$ | $-3$ | $0$ | $+$ |
| $-1$ | $1$ | $-1$ | $0$ | |
| $1$ | $0$ | $-1$ | $0$ | |
| $2$ | $0$ | $-2$ | $0$ | |
| $-1$ | $1$ | $-1$ | $0$ | |
| $1$ | $0$ | $-1$ | $0$ | |

$$(A-5I)X=O \qquad A=\begin{bmatrix}4 & 1 & -1\\ 2 & 5 & -2\\ 1 & 1 & 2\end{bmatrix}$$

$$\begin{bmatrix}-1 & 1 & -1\\ 2 & 0 & -2\\ 1 & 1 & -3\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$\boxed{S(\lambda)=\{x\in V : f(x)=\lambda x\}} \qquad \boxed{(F_{\mathcal{B}}-\lambda I)X_{\mathcal{B}}=O}$$

| $x_1$ | $x_2$ | $x_3$ | |
|---|---|---|---|
| $-1$ | $1$ | $-1$ | $0$ |
| $2$ | $0$ | $-2$ | $0$ $/:2$ |
| $1$ | $1$ | $-3$ | $0$ |
| $-1$ | $1$ | $-1$ | $0$ $/\cdot(-1)$ |
| $1$ | $0$ | $-1$ | $0$ |
| $1$ | $1$ | $-3$ | $0$ $+$ |
| $-1$ | $1$ | $-1$ | $0$ |
| $1$ | $0$ | $-1$ | $0$ |
| $2$ | $0$ | $-2$ | $0$ |
| $-1$ | $1$ | $-1$ | $0$ |
| $1$ | $0$ | $-1$ | $0$ |

$$(A-5I)X=O \qquad A=\begin{bmatrix}4&1&-1\\2&5&-2\\1&1&2\end{bmatrix}$$

$$\begin{bmatrix}-1&1&-1\\2&0&-2\\1&1&-3\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$\boxed{S(\lambda)=\{x\in V: f(x)=\lambda x\}} \qquad \boxed{\left(F_{\mathcal{B}}-\lambda I\right)X_{\mathcal{B}}=O}$$

| $x_1$ | $x_2$ | $x_3$ | |
|---|---|---|---|
| $-1$ | $1$ | $-1$ | $0$ |
| $2$ | $0$ | $-2$ | $0$ $/:2$ |
| $1$ | $1$ | $-3$ | $0$ |
| $-1$ | $1$ | $-1$ | $0$ $/\cdot(-1)$ |
| $1$ | $0$ | $-1$ | $0$ |
| $1$ | $1$ | $-3$ | $0$ $+$ |
| $-1$ | $1$ | $-1$ | $0$ |
| $1$ | $0$ | $-1$ | $0$ |
| $2$ | $0$ | $-2$ | $0$ |
| $-1$ | $1$ | $-1$ | $0$ |
| $1$ | $0$ | $-1$ | $0$ $/\cdot 1$ |

$$(A-5I)X=O \qquad A=\begin{bmatrix}4 & 1 & -1\\ 2 & 5 & -2\\ 1 & 1 & 2\end{bmatrix}$$

$$\begin{bmatrix}-1 & 1 & -1\\ 2 & 0 & -2\\ 1 & 1 & -3\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}=\begin{bmatrix}0\\ 0\\ 0\end{bmatrix}$$

$$\boxed{S(\lambda)=\{x\in V : f(x)=\lambda x\}} \qquad \boxed{\left(F_{\mathcal{B}}-\lambda I\right)X_{\mathcal{B}}=O}$$

| $x_1$ | $x_2$ | $x_3$ | | |
|---|---|---|---|---|
| $-1$ | $1$ | $-1$ | $0$ | |
| $2$ | $0$ | $-2$ | $0$ | $/:2$ |
| $1$ | $1$ | $-3$ | $0$ | |
| $-1$ | $1$ | $-1$ | $0$ | $/\cdot(-1)$ |
| $1$ | $0$ | $-1$ | $0$ | |
| $1$ | $1$ | $-3$ | $0$ | $+$ |
| $-1$ | $1$ | $-1$ | $0$ | |
| $1$ | $0$ | $-1$ | $0$ | |
| $2$ | $0$ | $-2$ | $0$ | |
| $-1$ | $1$ | $-1$ | $0$ | $+$ |
| $1$ | $0$ | $-1$ | $0$ | $/\cdot 1$ |

$$(A-5I)X = O \qquad A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\boxed{S(\lambda) = \{x \in V : f(x) = \lambda x\}} \qquad \boxed{\left(F_{\mathcal{B}} - \lambda I\right) X_{\mathcal{B}} = O}$$

| $x_1$ | $x_2$ | $x_3$ | | |
|---|---|---|---|---|
| $-1$ | $1$ | $-1$ | $0$ | |
| $2$ | $0$ | $-2$ | $0$ | $/:2$ |
| $1$ | $1$ | $-3$ | $0$ | |
| $-1$ | $1$ | $-1$ | $0$ | $/\cdot(-1)$ |
| $1$ | $0$ | $-1$ | $0$ | |
| $1$ | $1$ | $-3$ | $0$ | $+$ |
| $-1$ | $1$ | $-1$ | $0$ | |
| $1$ | $0$ | $-1$ | $0$ | |
| $2$ | $0$ | $-2$ | $0$ | |
| $-1$ | $1$ | $-1$ | $0$ | $+$ |
| $1$ | $0$ | $-1$ | $0$ | $/\cdot 1$ |
| | | | | |
| $1$ | $0$ | $-1$ | $0$ | |

$$(A-5I)X=O \qquad A=\begin{bmatrix}4&1&-1\\2&5&-2\\1&1&2\end{bmatrix}$$

$$\begin{bmatrix}-1&1&-1\\2&0&-2\\1&1&-3\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$\boxed{S(\lambda)=\left\{x\in V : f(x)=\lambda x\right\}} \qquad \boxed{\left(F_{\mathcal{B}}-\lambda I\right)X_{\mathcal{B}}=O}$$

$$
\begin{array}{ccc|cl}
x_1 & x_2 & x_3 & & \\
\hline
-1 & 1 & -1 & 0 & \\
2 & 0 & -2 & 0 & /:2 \\
1 & 1 & -3 & 0 & \\
\hline
-1 & 1 & -1 & 0 & /\cdot(-1) \\
1 & 0 & -1 & 0 & \\
1 & 1 & -3 & 0 & + \\
\hline
-1 & 1 & -1 & 0 & + \\
1 & 0 & -1 & 0 & \\
2 & 0 & -2 & 0 & \\
\hline
-1 & 1 & -1 & 0 & + \\
1 & 0 & -1 & 0 & /\cdot 1 \\
\hline
0 & & & & \\
1 & 0 & -1 & 0 &
\end{array}
$$

$$(A-5I)X = O \qquad A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\boxed{S(\lambda) = \{x \in V : f(x) = \lambda x\}} \qquad \boxed{(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O}$$

| $x_1$ | $x_2$ | $x_3$ | | |
|---|---|---|---|---|
| $-1$ | $1$ | $-1$ | $0$ | |
| $2$ | $0$ | $-2$ | $0$ | $/:2$ |
| $1$ | $1$ | $-3$ | $0$ | |
| $-1$ | $1$ | $-1$ | $0$ | $/\cdot(-1)$ |
| $1$ | $0$ | $-1$ | $0$ | |
| $1$ | $1$ | $-3$ | $0$ | $+$ |
| $-1$ | $1$ | $-1$ | $0$ | |
| $1$ | $0$ | $-1$ | $0$ | |
| $2$ | $0$ | $-2$ | $0$ | |
| $-1$ | $1$ | $-1$ | $0$ | $+$ |
| $1$ | $0$ | $-1$ | $0$ | $/\cdot 1$ |
| $0$ | $1$ | | | |
| $1$ | $0$ | $-1$ | $0$ | |

$$(A-5I)X=O \qquad A=\begin{bmatrix}4&1&-1\\2&5&-2\\1&1&2\end{bmatrix}$$

$$\begin{bmatrix}-1&1&-1\\2&0&-2\\1&1&-3\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$\boxed{S(\lambda)=\{x\in V : f(x)=\lambda x\}} \qquad \boxed{\left(F_{\mathcal{B}}-\lambda I\right)X_{\mathcal{B}}=O}$$

| $x_1$ | $x_2$ | $x_3$ | | |
|---|---|---|---|---|
| $-1$ | $1$ | $-1$ | $0$ | |
| $2$ | $0$ | $-2$ | $0$ | $/:2$ |
| $1$ | $1$ | $-3$ | $0$ | |
| $-1$ | $1$ | $-1$ | $0$ | $/\cdot(-1)$ |
| $1$ | $0$ | $-1$ | $0$ | |
| $1$ | $1$ | $-3$ | $0$ | $+$ |
| $-1$ | $1$ | $-1$ | $0$ | |
| $1$ | $0$ | $-1$ | $0$ | |
| $2$ | $0$ | $-2$ | $0$ | |
| $-1$ | $1$ | $-1$ | $0$ | $+$ |
| $1$ | $0$ | $-1$ | $0$ | $/\cdot 1$ |
| $0$ | $1$ | $-2$ | | |
| $1$ | $0$ | $-1$ | $0$ | |

$$(A-5I)X = O \qquad A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\boxed{S(\lambda) = \{x \in V : f(x) = \lambda x\}} \qquad \boxed{(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O}$$

| $x_1$ | $x_2$ | $x_3$ | | |
|---|---|---|---|---|
| $-1$ | $1$ | $-1$ | $0$ | |
| $2$ | $0$ | $-2$ | $0$ | $/:2$ |
| $1$ | $1$ | $-3$ | $0$ | |
| $-1$ | $1$ | $-1$ | $0$ | $/\cdot(-1)$ |
| $1$ | $0$ | $-1$ | $0$ | |
| $1$ | $1$ | $-3$ | $0$ | $+$ |
| $-1$ | $1$ | $-1$ | $0$ | |
| $1$ | $0$ | $-1$ | $0$ | |
| $2$ | $0$ | $-2$ | $0$ | |
| $-1$ | $1$ | $-1$ | $0$ | $+$ |
| $1$ | $0$ | $-1$ | $0$ | $/\cdot 1$ |
| $0$ | $1$ | $-2$ | $0$ | |
| $1$ | $0$ | $-1$ | $0$ | |

$$(A-5I)X=O \qquad A=\begin{bmatrix}4 & 1 & -1\\ 2 & 5 & -2\\ 1 & 1 & 2\end{bmatrix}$$

$$\begin{bmatrix}-1 & 1 & -1\\ 2 & 0 & -2\\ 1 & 1 & -3\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}=\begin{bmatrix}0\\ 0\\ 0\end{bmatrix}$$

$$\boxed{S(\lambda)=\left\{x\in V : f(x)=\lambda x\right\}} \qquad \boxed{\left(F_{\mathcal{B}}-\lambda I\right)X_{\mathcal{B}}=O}$$

| $x_1$ | $x_2$ | $x_3$ | |
|---|---|---|---|
| $-1$ | $1$ | $-1$ | $0$ |
| $2$ | $0$ | $-2$ | $0$ | $/:2$
| $1$ | $1$ | $-3$ | $0$ |
| $-1$ | $1$ | $-1$ | $0$ | $/\cdot(-1)$
| $1$ | $0$ | $-1$ | $0$ |
| $1$ | $1$ | $-3$ | $0$ | $+$
| $-1$ | $1$ | $-1$ | $0$ |
| $1$ | $0$ | $-1$ | $0$ |
| $2$ | $0$ | $-2$ | $0$ |
| $-1$ | $1$ | $-1$ | $0$ | $+$
| $1$ | $0$ | $-1$ | $0$ | $/\cdot 1$
| $0$ | $1$ | $-2$ | $0$ |
| $1$ | $0$ | $-1$ | $0$ |

$$(A-5I)X = O \qquad A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$x_2 - 2x_3 = 0$$

$$\boxed{S(\lambda) = \{x \in V : f(x) = \lambda x\}} \qquad \boxed{(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O}$$

| $x_1$ | $x_2$ | $x_3$ | | |
|---|---|---|---|---|
| $-1$ | $1$ | $-1$ | $0$ | |
| $2$ | $0$ | $-2$ | $0$ | $/:2$ |
| $1$ | $1$ | $-3$ | $0$ | |
| $-1$ | $1$ | $-1$ | $0$ | $/\cdot(-1)$ |
| $1$ | $0$ | $-1$ | $0$ | |
| $1$ | $1$ | $-3$ | $0$ | $+$ |
| $-1$ | $1$ | $-1$ | $0$ | |
| $1$ | $0$ | $-1$ | $0$ | |
| $2$ | $0$ | $-2$ | $0$ | |
| $-1$ | $1$ | $-1$ | $0$ | $+$ |
| $1$ | $0$ | $-1$ | $0$ | $/\cdot 1$ |
| $0$ | $1$ | $-2$ | $0$ | |
| $1$ | $0$ | $-1$ | $0$ | |

$$(A-5I)X=O \qquad A=\begin{bmatrix}4&1&-1\\2&5&-2\\1&1&2\end{bmatrix}$$

$$\begin{bmatrix}-1&1&-1\\2&0&-2\\1&1&-3\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$x_2-2x_3=0$$
$$x_1-x_3=0$$

$$\boxed{S(\lambda)=\{x\in V: f(x)=\lambda x\}}$$

$$\boxed{(F_{\mathcal{B}}-\lambda I)X_{\mathcal{B}}=O}$$

| $x_1$ | $x_2$ | $x_3$ | | |
|---|---|---|---|---|
| $-1$ | $1$ | $-1$ | $0$ | |
| $2$ | $0$ | $-2$ | $0$ | $/:2$ |
| $1$ | $1$ | $-3$ | $0$ | |
| $-1$ | $1$ | $-1$ | $0$ | $/\cdot(-1)$ |
| $1$ | $0$ | $-1$ | $0$ | |
| $1$ | $1$ | $-3$ | $0$ | $+$ |
| $-1$ | $1$ | $-1$ | $0$ | |
| $1$ | $0$ | $-1$ | $0$ | |
| $2$ | $0$ | $-2$ | $0$ | |
| $-1$ | $1$ | $-1$ | $0$ | $+$ |
| $1$ | $0$ | $-1$ | $0$ | $/\cdot 1$ |
| $0$ | $1$ | $-2$ | $0$ | |
| $1$ | $0$ | $-1$ | $0$ | |

$$(A-5I)X=O \qquad A=\begin{bmatrix}4 & 1 & -1\\ 2 & 5 & -2\\ 1 & 1 & 2\end{bmatrix}$$

$$\begin{bmatrix}-1 & 1 & -1\\ 2 & 0 & -2\\ 1 & 1 & -3\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}=\begin{bmatrix}0\\ 0\\ 0\end{bmatrix}$$

$$\left.\begin{aligned} x_2-2x_3&=0\\ x_1-x_3&=0\end{aligned}\right\}$$

$$\boxed{S(\lambda)=\{x\in V : f(x)=\lambda x\}} \qquad \boxed{(F_{\mathcal{B}}-\lambda I)X_{\mathcal{B}}=O}$$

| $x_1$ | $x_2$ | $x_3$ | | |
|---|---|---|---|---|
| $-1$ | $1$ | $-1$ | $0$ | |
| $2$ | $0$ | $-2$ | $0$ | $/:2$ |
| $1$ | $1$ | $-3$ | $0$ | |
| $-1$ | $1$ | $-1$ | $0$ | $/\cdot(-1)$ |
| $1$ | $0$ | $-1$ | $0$ | |
| $1$ | $1$ | $-3$ | $0$ | $+$ |
| $-1$ | $1$ | $-1$ | $0$ | |
| $1$ | $0$ | $-1$ | $0$ | |
| $2$ | $0$ | $-2$ | $0$ | |
| $-1$ | $1$ | $-1$ | $0$ | $+$ |
| $1$ | $0$ | $-1$ | $0$ | $/\cdot 1$ |
| $0$ | $1$ | $-2$ | $0$ | |
| $1$ | $0$ | $-1$ | $0$ | |

$$(A-5I)X = O \qquad A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\left.\begin{aligned} x_2 - 2x_3 &= 0 \\ x_1 - x_3 &= 0 \end{aligned}\right\} \rightsquigarrow x_2 = 2x_3$$

$$\boxed{S(\lambda) = \{x \in V : f(x) = \lambda x\}} \qquad \boxed{\left(F_{\mathcal{B}} - \lambda I\right) X_{\mathcal{B}} = O}$$

$$
\begin{array}{ccc|c}
x_1 & x_2 & x_3 & \\
\hline
-1 & 1 & -1 & 0 \\
2 & 0 & -2 & 0 \\
1 & 1 & -3 & 0 \\
\hline
-1 & 1 & -1 & 0 \\
1 & 0 & -1 & 0 \\
1 & 1 & -3 & 0 \\
\hline
-1 & 1 & -1 & 0 \\
1 & 0 & -1 & 0 \\
2 & 0 & -2 & 0 \\
\hline
-1 & 1 & -1 & 0 \\
1 & 0 & -1 & 0 \\
\hline
0 & 1 & -2 & 0 \\
1 & 0 & -1 & 0
\end{array}
\begin{array}{l}
\\
\\
/:2 \\
\\
/\cdot(-1) \\
\\
+ \\
\\
\\
\\
+ \\
/\cdot 1 \\
\\
\\
\end{array}
$$

$$(A-5I)X=O \qquad A=\begin{bmatrix}4 & 1 & -1\\ 2 & 5 & -2\\ 1 & 1 & 2\end{bmatrix}$$

$$\begin{bmatrix}-1 & 1 & -1\\ 2 & 0 & -2\\ 1 & 1 & -3\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}=\begin{bmatrix}0\\ 0\\ 0\end{bmatrix}$$

$$\left.\begin{aligned} x_2-2x_3&=0\\ x_1-x_3&=0\end{aligned}\right\} \begin{aligned} &\rightsquigarrow x_2=2x_3\\ &\rightsquigarrow x_1=x_3\end{aligned}$$

$$\boxed{S(\lambda)=\{x\in V : f(x)=\lambda x\}} \qquad \boxed{(F_{\mathcal{B}}-\lambda I)X_{\mathcal{B}}=O}$$

| $x_1$ | $x_2$ | $x_3$ | | |
|---|---|---|---|---|
| $-1$ | $1$ | $-1$ | $0$ | |
| $2$ | $0$ | $-2$ | $0$ | $/:2$ |
| $1$ | $1$ | $-3$ | $0$ | |
| $-1$ | $1$ | $-1$ | $0$ | $/\cdot(-1)$ |
| $1$ | $0$ | $-1$ | $0$ | |
| $1$ | $1$ | $-3$ | $0$ | $+$ |
| $-1$ | $1$ | $-1$ | $0$ | |
| $1$ | $0$ | $-1$ | $0$ | |
| $2$ | $0$ | $-2$ | $0$ | |
| $-1$ | $1$ | $-1$ | $0$ | $+$ |
| $1$ | $0$ | $-1$ | $0$ | $/\cdot 1$ |
| $0$ | $1$ | $-2$ | $0$ | |
| $1$ | $0$ | $-1$ | $0$ | |

$$(A-5I)X = O \qquad A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\left.\begin{aligned} x_2 - 2x_3 &= 0 \\ x_1 - x_3 &= 0 \end{aligned}\right\} \begin{aligned} &\rightsquigarrow x_2 = 2x_3 \\ &\rightsquigarrow x_1 = x_3 \end{aligned}$$

$$S(5) =$$

$$\boxed{S(\lambda) = \{x \in V : f(x) = \lambda x\}} \qquad \boxed{(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O}$$

| $x_1$ | $x_2$ | $x_3$ | | |
|---|---|---|---|---|
| $-1$ | $1$ | $-1$ | $0$ | |
| $2$ | $0$ | $-2$ | $0$ | $/ : 2$ |
| $1$ | $1$ | $-3$ | $0$ | |
| $-1$ | $1$ | $-1$ | $0$ | $/\cdot(-1)$ |
| $1$ | $0$ | $-1$ | $0$ | |
| $1$ | $1$ | $-3$ | $0$ | $+$ |
| $-1$ | $1$ | $-1$ | $0$ | |
| $1$ | $0$ | $-1$ | $0$ | |
| $2$ | $0$ | $-2$ | $0$ | |
| $-1$ | $1$ | $-1$ | $0$ | $+$ |
| $1$ | $0$ | $-1$ | $0$ | $/\cdot 1$ |
| $0$ | $1$ | $-2$ | $0$ | |
| $1$ | $0$ | $-1$ | $0$ | |

$$(A-5I)X = O \qquad A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\left.\begin{aligned} x_2 - 2x_3 &= 0 \\ x_1 - x_3 &= 0 \end{aligned}\right\} \begin{aligned} &\rightsquigarrow x_2 = 2x_3 \\ &\rightsquigarrow x_1 = x_3 \end{aligned}$$

$$S(5) = \{($$

$$\boxed{S(\lambda) = \{x \in V : f(x) = \lambda x\}}$$

$$\boxed{(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O}$$

| $x_1$ | $x_2$ | $x_3$ | | |
|---|---|---|---|---|
| $-1$ | $1$ | $-1$ | $0$ | |
| $2$ | $0$ | $-2$ | $0$ | $/:2$ |
| $1$ | $1$ | $-3$ | $0$ | |
| $-1$ | $1$ | $-1$ | $0$ | $/\cdot(-1)$ |
| $1$ | $0$ | $-1$ | $0$ | |
| $1$ | $1$ | $-3$ | $0$ | $+$ |
| $-1$ | $1$ | $-1$ | $0$ | |
| $1$ | $0$ | $-1$ | $0$ | |
| $2$ | $0$ | $-2$ | $0$ | |
| $-1$ | $1$ | $-1$ | $0$ | $+$ |
| $1$ | $0$ | $-1$ | $0$ | $/\cdot 1$ |
| $0$ | $1$ | $-2$ | $0$ | |
| $1$ | $0$ | $-1$ | $0$ | |

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$(A - 5I)X = O$$

$$\begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\left.\begin{aligned} x_2 - 2x_3 &= 0 \\ x_1 - x_3 &= 0 \end{aligned}\right\} \begin{aligned} &\rightsquigarrow x_2 = 2x_3 \\ &\rightsquigarrow x_1 = x_3 \end{aligned}$$

$$S(5) = \{(x_3,$$

$$\boxed{S(\lambda) = \{x \in V : f(x) = \lambda x\}}$$

$$\boxed{(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O}$$

$$\begin{array}{ccc|cl} x_1 & x_2 & x_3 & & \\ \hline -1 & 1 & -1 & 0 & \\ 2 & 0 & -2 & 0 & /:2 \\ 1 & 1 & -3 & 0 & \\ \hline -1 & 1 & -1 & 0 & /\cdot(-1) \\ 1 & 0 & -1 & 0 & \\ 1 & 1 & -3 & 0 & + \\ \hline -1 & 1 & -1 & 0 & \\ 1 & 0 & -1 & 0 & \\ 2 & 0 & -2 & 0 & \\ \hline -1 & 1 & -1 & 0 & + \\ 1 & 0 & -1 & 0 & /\cdot 1 \\ \hline 0 & 1 & -2 & 0 & \\ 1 & 0 & -1 & 0 & \end{array}$$

$$(A-5I)X = O \qquad A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\left.\begin{array}{r} x_2 - 2x_3 = 0 \\ x_1 - x_3 = 0 \end{array}\right\} \begin{array}{l} \rightsquigarrow x_2 = 2x_3 \\ \rightsquigarrow x_1 = x_3 \end{array}$$

$$S(5) = \{(x_3, 2x_3,$$

$$\boxed{S(\lambda) = \{x \in V : f(x) = \lambda x\}} \qquad \boxed{(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O}$$

| $x_1$ | $x_2$ | $x_3$ | | |
|---|---|---|---|---|
| $-1$ | $1$ | $-1$ | $0$ | |
| $2$ | $0$ | $-2$ | $0$ | $/:2$ |
| $1$ | $1$ | $-3$ | $0$ | |
| $-1$ | $1$ | $-1$ | $0$ | $/\cdot(-1)$ |
| $1$ | $0$ | $-1$ | $0$ | |
| $1$ | $1$ | $-3$ | $0$ | $+$ |
| $-1$ | $1$ | $-1$ | $0$ | |
| $1$ | $0$ | $-1$ | $0$ | |
| $2$ | $0$ | $-2$ | $0$ | |
| $-1$ | $1$ | $-1$ | $0$ | $+$ |
| $1$ | $0$ | $-1$ | $0$ | $/\cdot 1$ |
| $0$ | $1$ | $-2$ | $0$ | |
| $1$ | $0$ | $-1$ | $0$ | |

$$(A-5I)X=O \qquad A=\begin{bmatrix}4&1&-1\\2&5&-2\\1&1&2\end{bmatrix}$$

$$\begin{bmatrix}-1&1&-1\\2&0&-2\\1&1&-3\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$\left.\begin{aligned}x_2-2x_3&=0\\x_1-x_3&=0\end{aligned}\right\}\begin{aligned}&\rightsquigarrow x_2=2x_3\\&\rightsquigarrow x_1=x_3\end{aligned}$$

$$S(5)=\{(x_3,2x_3,x_3)$$

$$\boxed{S(\lambda)=\{x\in V : f(x)=\lambda x\}} \qquad \boxed{(F_{\mathcal{B}}-\lambda I)X_{\mathcal{B}}=O}$$

| $x_1$ | $x_2$ | $x_3$ | | |
|---|---|---|---|---|
| $-1$ | $1$ | $-1$ | $0$ | |
| $2$ | $0$ | $-2$ | $0$ | $/:2$ |
| $1$ | $1$ | $-3$ | $0$ | |
| $-1$ | $1$ | $-1$ | $0$ | $/\cdot(-1)$ |
| $1$ | $0$ | $-1$ | $0$ | |
| $1$ | $1$ | $-3$ | $0$ | $+$ |
| $-1$ | $1$ | $-1$ | $0$ | |
| $1$ | $0$ | $-1$ | $0$ | |
| $2$ | $0$ | $-2$ | $0$ | |
| $-1$ | $1$ | $-1$ | $0$ | $+$ |
| $1$ | $0$ | $-1$ | $0$ | $/\cdot 1$ |
| $0$ | $1$ | $-2$ | $0$ | |
| $1$ | $0$ | $-1$ | $0$ | |

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$$\begin{bmatrix}-1&1&-1\\2&0&-2\\1&1&-3\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$\left.\begin{aligned}x_2-2x_3&=0\\x_1-x_3&=0\end{aligned}\right\}\begin{aligned}&\rightsquigarrow x_2=2x_3\\&\rightsquigarrow x_1=x_3\end{aligned}$$

$$S(5)=\{(x_3,2x_3,x_3):x_3\in\mathbb{R}\}$$

$$\boxed{S(\lambda)=\{x\in V:f(x)=\lambda x\}}$$

$$\boxed{(F_{\mathcal{B}}-\lambda I)X_{\mathcal{B}}=O}$$

| $x_1$ | $x_2$ | $x_3$ | | |
|---|---|---|---|---|
| $-1$ | $1$ | $-1$ | $0$ | |
| $2$ | $0$ | $-2$ | $0$ | $/:2$ |
| $1$ | $1$ | $-3$ | $0$ | |
| $-1$ | $1$ | $-1$ | $0$ | $/\cdot(-1)$ |
| $1$ | $0$ | $-1$ | $0$ | |
| $1$ | $1$ | $-3$ | $0$ | $+$ |
| $-1$ | $1$ | $-1$ | $0$ | |
| $1$ | $0$ | $-1$ | $0$ | |
| $2$ | $0$ | $-2$ | $0$ | |
| $-1$ | $1$ | $-1$ | $0$ | $+$ |
| $1$ | $0$ | $-1$ | $0$ | $/\cdot 1$ |
| $0$ | $1$ | $-2$ | $0$ | |
| $1$ | $0$ | $-1$ | $0$ | |

$$(A-5I)X = O \qquad A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\left.\begin{aligned} x_2 - 2x_3 &= 0 \\ x_1 - x_3 &= 0 \end{aligned}\right\} \begin{aligned} &\rightsquigarrow x_2 = 2x_3 \\ &\rightsquigarrow x_1 = x_3 \end{aligned}$$

$$S(5) = \{(x_3, 2x_3, x_3) : x_3 \in \mathbb{R}\}$$

$$(x_3, 2x_3, x_3) =$$

$$\boxed{S(\lambda) = \{x \in V : f(x) = \lambda x\}} \qquad \boxed{(F_{\mathcal{B}} - \lambda I) X_{\mathcal{B}} = O}$$

| $x_1$ | $x_2$ | $x_3$ | |
|---|---|---|---|
| $-1$ | $1$ | $-1$ | $0$ |
| $2$ | $0$ | $-2$ | $0$ $/:2$ |
| $1$ | $1$ | $-3$ | $0$ |
| $-1$ | $1$ | $-1$ | $0$ $/\cdot(-1)$ |
| $1$ | $0$ | $-1$ | $0$ |
| $1$ | $1$ | $-3$ | $0$ $+$ |
| $-1$ | $1$ | $-1$ | $0$ |
| $1$ | $0$ | $-1$ | $0$ |
| $2$ | $0$ | $-2$ | $0$ |
| $-1$ | $1$ | $-1$ | $0$ $+$ |
| $1$ | $0$ | $-1$ | $0$ $/\cdot 1$ |
| $0$ | $1$ | $-2$ | $0$ |
| $1$ | $0$ | $-1$ | $0$ |

$$(A-5I)X=O \qquad A=\begin{bmatrix}4&1&-1\\2&5&-2\\1&1&2\end{bmatrix}$$

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$$\left.\begin{aligned}x_2-2x_3&=0\\x_1-x_3&=0\end{aligned}\right\}\begin{aligned}&\rightsquigarrow x_2=2x_3\\&\rightsquigarrow x_1=x_3\end{aligned}$$

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$$(x_3,2x_3,x_3)=x_3\cdot($$

$$\boxed{S(\lambda)=\{x\in V: f(x)=\lambda x\}} \qquad \boxed{(F_{\mathcal{B}}-\lambda I)X_{\mathcal{B}}=O}$$

| $x_1$ | $x_2$ | $x_3$ | | |
|---|---|---|---|---|
| $-1$ | $1$ | $-1$ | $0$ | |
| $2$ | $0$ | $-2$ | $0$ | $/:2$ |
| $1$ | $1$ | $-3$ | $0$ | |
| $-1$ | $1$ | $-1$ | $0$ | $/\cdot(-1)$ |
| $1$ | $0$ | $-1$ | $0$ | |
| $1$ | $1$ | $-3$ | $0$ | $+$ |
| $-1$ | $1$ | $-1$ | $0$ | |
| $1$ | $0$ | $-1$ | $0$ | |
| $2$ | $0$ | $-2$ | $0$ | |
| $-1$ | $1$ | $-1$ | $0$ | $+$ |
| $1$ | $0$ | $-1$ | $0$ | $/\cdot 1$ |
| $0$ | $1$ | $-2$ | $0$ | |
| $1$ | $0$ | $-1$ | $0$ | |

$$(A-5I)X = O \qquad A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\left.\begin{aligned} x_2 - 2x_3 &= 0 \\ x_1 - x_3 &= 0 \end{aligned}\right\} \begin{aligned} &\rightsquigarrow x_2 = 2x_3 \\ &\rightsquigarrow x_1 = x_3 \end{aligned}$$

$$S(5) = \{(x_3, 2x_3, x_3) : x_3 \in \mathbb{R}\}$$

$$(x_3, 2x_3, x_3) = x_3 \cdot (1,$$

$$\boxed{S(\lambda) = \{x \in V : f(x) = \lambda x\}} \qquad \boxed{(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O}$$

| $x_1$ | $x_2$ | $x_3$ | | |
|---|---|---|---|---|
| $-1$ | $1$ | $-1$ | $0$ | |
| $2$ | $0$ | $-2$ | $0$ | $/:2$ |
| $1$ | $1$ | $-3$ | $0$ | |
| $-1$ | $1$ | $-1$ | $0$ | $/\cdot(-1)$ |
| $1$ | $0$ | $-1$ | $0$ | |
| $1$ | $1$ | $-3$ | $0$ | $+$ |
| $-1$ | $1$ | $-1$ | $0$ | |
| $1$ | $0$ | $-1$ | $0$ | |
| $2$ | $0$ | $-2$ | $0$ | |
| $-1$ | $1$ | $-1$ | $0$ | $+$ |
| $1$ | $0$ | $-1$ | $0$ | $/\cdot 1$ |
| $0$ | $1$ | $-2$ | $0$ | |
| $1$ | $0$ | $-1$ | $0$ | |

$$(A-5I)X = O \qquad A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\left.\begin{aligned} x_2 - 2x_3 &= 0 \\ x_1 - x_3 &= 0 \end{aligned}\right\} \begin{aligned} &\rightsquigarrow x_2 = 2x_3 \\ &\rightsquigarrow x_1 = x_3 \end{aligned}$$

$$S(5) = \big\{(x_3, 2x_3, x_3) : x_3 \in \mathbb{R}\big\}$$

$$(x_3, 2x_3, x_3) = x_3 \cdot (1, 2,$$

$$\boxed{S(\lambda) = \big\{x \in V : f(x) = \lambda x\big\}} \qquad \boxed{\big(F_{\mathcal{B}} - \lambda I\big)X_{\mathcal{B}} = O}$$

| $x_1$ | $x_2$ | $x_3$ | | |
|---|---|---|---|---|
| $-1$ | $1$ | $-1$ | $0$ | |
| $2$ | $0$ | $-2$ | $0$ | $/:2$ |
| $1$ | $1$ | $-3$ | $0$ | |
| $-1$ | $1$ | $-1$ | $0$ | $/\cdot(-1)$ |
| $1$ | $0$ | $-1$ | $0$ | |
| $1$ | $1$ | $-3$ | $0$ | $+$ |
| $-1$ | $1$ | $-1$ | $0$ | |
| $1$ | $0$ | $-1$ | $0$ | |
| $2$ | $0$ | $-2$ | $0$ | |
| $-1$ | $1$ | $-1$ | $0$ | $+$ |
| $1$ | $0$ | $-1$ | $0$ | $/\cdot 1$ |
| $0$ | $1$ | $-2$ | $0$ | |
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$$S(5) = \{(x_3, 2x_3, x_3) : x_3 \in \mathbb{R}\}$$

$$(x_3, 2x_3, x_3) = x_3 \cdot (1, 2, 1)$$

$$\boxed{S(\lambda) = \{x \in V : f(x) = \lambda x\}} \qquad \boxed{(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O}$$

| $x_1$ | $x_2$ | $x_3$ | | |
|---|---|---|---|---|
| $-1$ | $1$ | $-1$ | $0$ | |
| $2$ | $0$ | $-2$ | $0$ | $/:2$ |
| $1$ | $1$ | $-3$ | $0$ | |
| $-1$ | $1$ | $-1$ | $0$ | $/\cdot(-1)$ |
| $1$ | $0$ | $-1$ | $0$ | |
| $1$ | $1$ | $-3$ | $0$ | $+$ |
| $-1$ | $1$ | $-1$ | $0$ | |
| $1$ | $0$ | $-1$ | $0$ | |
| $2$ | $0$ | $-2$ | $0$ | |
| $-1$ | $1$ | $-1$ | $0$ | $+$ |
| $1$ | $0$ | $-1$ | $0$ | $/\cdot 1$ |
| $0$ | $1$ | $-2$ | $0$ | |
| $1$ | $0$ | $-1$ | $0$ | |

$$(A-5I)X = O \qquad A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\left.\begin{aligned} x_2 - 2x_3 &= 0 \\ x_1 - x_3 &= 0 \end{aligned}\right\} \begin{aligned} &\rightsquigarrow x_2 = 2x_3 \\ &\rightsquigarrow x_1 = x_3 \end{aligned}$$

$$S(5) = \{(x_3, 2x_3, x_3) : x_3 \in \mathbb{R}\}$$

$$(x_3, 2x_3, x_3) = x_3 \cdot (1, 2, 1)$$

$$\mathcal{B}_{S(5)} = \{(1, 2, 1)\}$$

$$\boxed{S(\lambda) = \{x \in V : f(x) = \lambda x\}} \qquad \boxed{(F_{\mathcal{B}} - \lambda I) X_{\mathcal{B}} = O}$$

| $x_1$ | $x_2$ | $x_3$ | | |
|---|---|---|---|---|
| $-1$ | $1$ | $-1$ | $0$ | |
| $2$ | $0$ | $-2$ | $0$ | $/:2$ |
| $1$ | $1$ | $-3$ | $0$ | |
| $-1$ | $1$ | $-1$ | $0$ | $/\cdot(-1)$ |
| $1$ | $0$ | $-1$ | $0$ | |
| $1$ | $1$ | $-3$ | $0$ | $+$ |
| $-1$ | $1$ | $-1$ | $0$ | |
| $1$ | $0$ | $-1$ | $0$ | |
| $2$ | $0$ | $-2$ | $0$ | |
| $-1$ | $1$ | $-1$ | $0$ | $+$ |
| $1$ | $0$ | $-1$ | $0$ | $/\cdot 1$ |
| $0$ | $1$ | $-2$ | $0$ | |
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$$(A-5I)X = O \qquad A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

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$$\left.\begin{aligned} x_2 - 2x_3 &= 0 \\ x_1 - x_3 &= 0 \end{aligned}\right\} \begin{aligned} &\rightsquigarrow x_2 = 2x_3 \\ &\rightsquigarrow x_1 = x_3 \end{aligned}$$

$$S(5) = \{(x_3, 2x_3, x_3) : x_3 \in \mathbb{R}\}$$

$$(x_3, 2x_3, x_3) = x_3 \cdot (1, 2, 1) \qquad \dim S(5) = 1$$

$$\mathcal{B}_{S(5)} = \{(1, 2, 1)\}$$

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| $x_1$ | $x_2$ | $x_3$ | | |
|---|---|---|---|---|
| $-1$ | $1$ | $-1$ | $0$ | |
| $2$ | $0$ | $-2$ | $0$ | $/:2$ |
| $1$ | $1$ | $-3$ | $0$ | |
| $-1$ | $1$ | $-1$ | $0$ | $/\cdot(-1)$ |
| $1$ | $0$ | $-1$ | $0$ | |
| $1$ | $1$ | $-3$ | $0$ | $+$ |
| $-1$ | $1$ | $-1$ | $0$ | |
| $1$ | $0$ | $-1$ | $0$ | |
| $2$ | $0$ | $-2$ | $0$ | |
| $-1$ | $1$ | $-1$ | $0$ | $+$ |
| $1$ | $0$ | $-1$ | $0$ | $/\cdot 1$ |
| $0$ | $1$ | $-2$ | $0$ | |
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$$(A-5I)X = O \qquad A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\left.\begin{aligned} x_2 - 2x_3 &= 0 \\ x_1 - x_3 &= 0 \end{aligned}\right\} \begin{aligned} &\rightsquigarrow x_2 = 2x_3 \\ &\rightsquigarrow x_1 = x_3 \end{aligned}$$

$$S(5) = \{(x_3, 2x_3, x_3) : x_3 \in \mathbb{R}\}$$

$$(x_3, 2x_3, x_3) = x_3 \cdot (1,2,1) \qquad \dim S(5) = 1$$

$$\mathcal{B}_{S(5)} = \{(1,2,1)\}$$

geometrijska kratnost svojstvene vrijednosti $\lambda = 5$

$$\boxed{S(\lambda) = \{x \in V : f(x) = \lambda x\}} \qquad \boxed{(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O}$$

c)

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

c) $k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

c) $k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$

$k_A(\lambda) = (\lambda - 3)^2 \cdot (\lambda - 5)$

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c) $k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$

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$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$(A - 3I) \cdot (A - 5I) =$

c) $k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$

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$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$(A - 3I) \cdot (A - 5I) = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$$

c) $k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$

$k_A(\lambda) = (\lambda - 3)^2 \cdot (\lambda - 5)$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$(A - 3I) \cdot (A - 5I) = \begin{bmatrix} 1 & & \\ & & \\ & & \end{bmatrix}$$

c) $k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$

$k_A(\lambda) = (\lambda - 3)^2 \cdot (\lambda - 5)$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

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$$(A - 3I) \cdot (A - 5I) = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & -2 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

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d)

c) $k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

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---

d) $k_A(A) = O$

c) $k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$k_A(\lambda) = (\lambda - 3)^2 \cdot (\lambda - 5)$

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---

d)

$$k_A(A) = O$$

$$A^3 - 11A^2 + 39A - 45I = O$$

c) $k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$k_A(\lambda) = (\lambda - 3)^2 \cdot (\lambda - 5)$

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---

d)

$$k_A(A) = O$$

$$A^3 - 11A^2 + 39A - 45I = O$$

$$45I =$$

c) $k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

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---

d)

$$k_A(A) = O$$

$$A^3 - 11A^2 + 39A - 45I = O$$

$$45I = A^3 - 11A^2 + 39A$$

c) $k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$k_A(\lambda) = (\lambda - 3)^2 \cdot (\lambda - 5)$

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---

d)

$$k_A(A) = O$$

$$A^3 - 11A^2 + 39A - 45I = O$$

$$45I = A^3 - 11A^2 + 39A \;/ \cdot A^{-1}$$

c) $k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$

$k_A(\lambda) = (\lambda - 3)^2 \cdot (\lambda - 5)$

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---

d)

$$k_A(A) = O$$

$$A^3 - 11A^2 + 39A - 45I = O$$

$$45I = A^3 - 11A^2 + 39A \;/ \cdot A^{-1}$$

$$45A^{-1} =$$

c) $k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$k_A(\lambda) = (\lambda - 3)^2 \cdot (\lambda - 5)$

$m_A(\lambda) = (\lambda - 3) \cdot (\lambda - 5)$ $\quad \boxed{m_A(\lambda) = \lambda^2 - 8\lambda + 15}$

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---

d)

$$k_A(A) = O$$

$$A^3 - 11A^2 + 39A - 45I = O$$

$$45I = A^3 - 11A^2 + 39A \;/ \cdot A^{-1}$$

$$45A^{-1} = A^2$$

c) $k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$k_A(\lambda) = (\lambda - 3)^2 \cdot (\lambda - 5)$

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---

d)

$$k_A(A) = O$$

$$A^3 - 11A^2 + 39A - 45I = O$$

$$45I = A^3 - 11A^2 + 39A \; / \cdot A^{-1}$$

$$45A^{-1} = A^2 - 11A$$

c) $k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$

$k_A(\lambda) = (\lambda - 3)^2 \cdot (\lambda - 5)$

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$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

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d)

$$k_A(A) = O$$

$$A^3 - 11A^2 + 39A - 45I = O$$

$$45I = A^3 - 11A^2 + 39A \,/\cdot A^{-1}$$

$$45A^{-1} = A^2 - 11A + 39I$$

c) $k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$

$k_A(\lambda) = (\lambda - 3)^2 \cdot (\lambda - 5)$

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d)

$$k_A(A) = O$$

$$A^3 - 11A^2 + 39A - 45I = O$$

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c) $k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$

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$k_A(\lambda) = (\lambda - 3)^2 \cdot (\lambda - 5)$

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---

d)

$$k_A(A) = O$$

$$A^3 - 11A^2 + 39A - 45I = O$$

$$45I = A^3 - 11A^2 + 39A \,/\cdot A^{-1}$$

$$45A^{-1} = A^2 - 11A + 39I \,/ : 45$$

$$A^{-1} =$$

c) $k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$k_A(\lambda) = (\lambda - 3)^2 \cdot (\lambda - 5)$

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---

d)

$$k_A(A) = O$$

$$A^3 - 11A^2 + 39A - 45I = O$$

$$45I = A^3 - 11A^2 + 39A \; / \cdot A^{-1}$$

$$45A^{-1} = A^2 - 11A + 39I \; / : 45$$

$$A^{-1} = \frac{1}{45}A^2$$

c) $k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$k_A(\lambda) = (\lambda - 3)^2 \cdot (\lambda - 5)$$

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d)

$$k_A(A) = O$$

$$A^3 - 11A^2 + 39A - 45I = O$$

$$45I = A^3 - 11A^2 + 39A \;/ \cdot A^{-1}$$

$$45A^{-1} = A^2 - 11A + 39I \;/ : 45$$

$$A^{-1} = \frac{1}{45}A^2 - \frac{11}{45}A$$

c) $k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

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d)

$$k_A(A) = O$$

$$A^3 - 11A^2 + 39A - 45I = O$$

$$45I = A^3 - 11A^2 + 39A \ / \cdot A^{-1}$$

$$45A^{-1} = A^2 - 11A + 39I \ / : 45$$

$$A^{-1} = \frac{1}{45}A^2 - \frac{11}{45}A + \frac{39}{45}I$$

c) $k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$k_A(\lambda) = (\lambda - 3)^2 \cdot (\lambda - 5)$$

$$m_A(\lambda) = (\lambda - 3) \cdot (\lambda - 5) \qquad \boxed{m_A(\lambda) = \lambda^2 - 8\lambda + 15}$$

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---

d)

$$k_A(A) = O$$

$$A^3 - 11A^2 + 39A - 45I = O$$

$$45I = A^3 - 11A^2 + 39A \; / \cdot A^{-1}$$

$$45A^{-1} = A^2 - 11A + 39I \; / : 45$$

$$\boxed{A^{-1} = \frac{1}{45}A^2 - \frac{11}{45}A + \frac{39}{45}I}$$

c) $k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

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d) $k_A(A) = O$ $\qquad m_A(A) = O$

$$A^3 - 11A^2 + 39A - 45I = O$$

$$45I = A^3 - 11A^2 + 39A \;/ \cdot A^{-1}$$

$$45A^{-1} = A^2 - 11A + 39I \;/ :45$$

$$\boxed{A^{-1} = \frac{1}{45}A^2 - \frac{11}{45}A + \frac{39}{45}I}$$

c) $k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$

$k_A(\lambda) = (\lambda - 3)^2 \cdot (\lambda - 5)$

$m_A(\lambda) = (\lambda - 3) \cdot (\lambda - 5)$ $\boxed{m_A(\lambda) = \lambda^2 - 8\lambda + 15}$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$(A - 3I) \cdot (A - 5I) = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & -2 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

---

d)

$$k_A(A) = O \qquad\qquad m_A(A) = O$$

$$A^3 - 11A^2 + 39A - 45I = O \qquad\qquad A^2 - 8A + 15I = O$$

$$45I = A^3 - 11A^2 + 39A \;/ \cdot A^{-1}$$

$$45A^{-1} = A^2 - 11A + 39I \;/ : 45$$

$$\boxed{A^{-1} = \frac{1}{45}A^2 - \frac{11}{45}A + \frac{39}{45}I}$$

c) $k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$

$k_A(\lambda) = (\lambda - 3)^2 \cdot (\lambda - 5)$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$m_A(\lambda) = (\lambda - 3) \cdot (\lambda - 5)$ $\boxed{m_A(\lambda) = \lambda^2 - 8\lambda + 15}$

$$(A - 3I) \cdot (A - 5I) = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & -2 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

---

d)

$k_A(A) = O$

$A^3 - 11A^2 + 39A - 45I = O$

$45I = A^3 - 11A^2 + 39A \;/ \cdot A^{-1}$

$45A^{-1} = A^2 - 11A + 39I \;/ : 45$

$$\boxed{A^{-1} = \frac{1}{45}A^2 - \frac{11}{45}A + \frac{39}{45}I}$$

$m_A(A) = O$

$A^2 - 8A + 15I = O$

$15I =$

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# treći zadatak

## Zadatak 3

*Postoji li linearni operator* $f : \mathbb{R}^3 \to \mathbb{R}^2$ *za kojeg vrijedi*

$$f(1,0,0) = (1,0), \quad f(0,1,0) = (1,3), \quad f(1,1,1) = (2,4)?$$

*Ako postoji, odredite u tom slučaju* $f(0,0,1)$ *i njegovu matricu u paru kanonskih baza.*

## Rješenje

$$f : \mathbb{R}^3 \to \mathbb{R}^2$$

$$f(1,0,0) = (1,0), \quad f(0,1,0) = (1,3), \quad f(1,1,1) = (2,4)$$

## Rješenje

$$f : \mathbb{R}^3 \to \mathbb{R}^2$$

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$$\mathcal{B} = \big\{(1,0,0),\ (0,1,0),\ (1,1,1)\big\}$$

## Rješenje

$$f : \mathbb{R}^3 \to \mathbb{R}^2$$

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$$\begin{bmatrix} & \\ & \\ & \end{bmatrix}$$

## Rješenje

$$f : \mathbb{R}^3 \to \mathbb{R}^2$$

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$$\mathcal{B} = \{(1,0,0),\ (0,1,0),\ (1,1,1)\}$$

$$\begin{bmatrix} 1 & & \\ 0 & & \\ 0 & & \end{bmatrix}$$

## Rješenje

$$f : \mathbb{R}^3 \to \mathbb{R}^2$$

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$$\begin{bmatrix} 1 & 0 & \\ 0 & 1 & \\ 0 & 0 & \end{bmatrix}$$

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Svaki linearni operator zadan je svojim djelovanjem na nekoj bazi.

## Rješenje

$$f : \mathbb{R}^3 \to \mathbb{R}^2$$

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Svaki linearni operator zadan je svojim djelovanjem na nekoj bazi. Postoji jedinstveni linearni operator $f$ koji zadovoljava zadane uvjete.

## Rješenje

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$$f(0,0,1) = ?$$

## Rješenje

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$$(0,0,1) = \alpha_1 \cdot (1,0,0)$$

## Rješenje

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$$f(0,0,1) = ?$$

$$(0,0,1) = \alpha_1 \cdot (1,0,0) + \alpha_2 \cdot (0,1,0)$$

## Rješenje

$$f : \mathbb{R}^3 \to \mathbb{R}^2$$

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## Rješenje

$$f : \mathbb{R}^3 \to \mathbb{R}^2$$

$$f(1,0,0) = (1,0), \quad f(0,1,0) = (1,3), \quad f(1,1,1) = (2,4)$$

$\mathcal{B} = \{(1,0,0), (0,1,0), (1,1,1)\}$ je baza za $\mathbb{R}^3$.

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

Svaki linearni operator zadan je svojim djelovanjem na nekoj bazi. Postoji jedinstveni linearni operator $f$ koji zadovoljava zadane uvjete.

$$f(0,0,1) = ?$$

$$(0,0,1) = \alpha_1 \cdot (1,0,0) + \alpha_2 \cdot (0,1,0) + \alpha_3 \cdot (1,1,1)$$

$$\left.\begin{aligned} \alpha_1 + \alpha_3 &= 0 \\ \alpha_2 + \alpha_3 &= 0 \\ \alpha_3 &= 1 \end{aligned}\right\} \qquad \alpha_3 = 1, \quad \alpha_2 = -1, \quad \alpha_1 = -1$$

$f$ je linearni operator

$$f(0,0,1) = f\big(-1 \cdot (1,0,0) + (-1) \cdot (0,1,0) + 1 \cdot (1,1,1)\big) =$$

$$= -1 \cdot f(1,0,0) + (-1) \cdot f(0,1,0) + 1 \cdot f(1,1,1) =$$

$$= -(1,0)$$

## Rješenje

$$f : \mathbb{R}^3 \to \mathbb{R}^2$$

$$f(1,0,0) = (1,0), \quad f(0,1,0) = (1,3), \quad f(1,1,1) = (2,4)$$

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$$f(0,0,1) = f\big(-1 \cdot (1,0,0) + (-1) \cdot (0,1,0) + 1 \cdot (1,1,1)\big) =$$

$$= -1 \cdot f(1,0,0) + (-1) \cdot f(0,1,0) + 1 \cdot f(1,1,1) =$$

$$= -(1,0) - (1,3)$$

## Rješenje

$$f : \mathbb{R}^3 \to \mathbb{R}^2$$

$$f(1,0,0) = (1,0), \quad f(0,1,0) = (1,3), \quad f(1,1,1) = (2,4)$$

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$f$ je linearni operator

$$f(0,0,1) = f\big(-1 \cdot (1,0,0) + (-1) \cdot (0,1,0) + 1 \cdot (1,1,1)\big) =$$

$$= -1 \cdot f(1,0,0) + (-1) \cdot f(0,1,0) + 1 \cdot f(1,1,1) =$$

$$= -(1,0) - (1,3) + (2,4)$$

## Rješenje

$$f : \mathbb{R}^3 \to \mathbb{R}^2$$

$$f(1,0,0) = (1,0), \quad f(0,1,0) = (1,3), \quad f(1,1,1) = (2,4)$$

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$f$ je linearni operator

$$f(0,0,1) = f\big(-1 \cdot (1,0,0) + (-1) \cdot (0,1,0) + 1 \cdot (1,1,1)\big) =$$

$$= -1 \cdot f(1,0,0) + (-1) \cdot f(0,1,0) + 1 \cdot f(1,1,1) =$$

$$= -(1,0) - (1,3) + (2,4) = (0,1)$$

$$f : \mathbb{R}^3 \to \mathbb{R}^2$$

$$f(1,0,0) = (1,0), \quad f(0,1,0) = (1,3), \quad f(1,1,1) = (2,4)$$

$\mathcal{B} = \big\{(1,0,0),\, (0,1,0),\, (1,1,1)\big\}$

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$$F_{(\mathcal{B},\,\mathcal{A}_{\mathsf{kan}})} = \left[ \quad \quad \right]$$

$$f : \mathbb{R}^3 \to \mathbb{R}^2$$

$$f(1,0,0) = (1,0), \quad f(0,1,0) = (1,3), \quad f(1,1,1) = (2,4)$$

$$\mathcal{B} = \{(1,0,0),\, (0,1,0),\, (1,1,1)\} \qquad \mathcal{A}_{\text{kan}} = \{(1,0),\, (0,1)\}$$

$$F_{(\mathcal{B},\, \mathcal{A}_{\text{kan}})} = \begin{bmatrix} 1 & & \\ 0 & & \end{bmatrix}$$

$$f : \mathbb{R}^3 \to \mathbb{R}^2$$

$$f(1,0,0) = (1,0), \quad f(0,1,0) = (1,3), \quad f(1,1,1) = (2,4)$$

$$\mathcal{B} = \big\{(1,0,0),\ (0,1,0),\ (1,1,1)\big\} \qquad \mathcal{A}_{\mathsf{kan}} = \big\{(1,0),\ (0,1)\big\}$$

$$F_{(\mathcal{B},\,\mathcal{A}_{\mathsf{kan}})} = \begin{bmatrix} 1 & 1 & \phantom{0} \\ 0 & 3 & \phantom{0} \end{bmatrix}$$

$$f : \mathbb{R}^3 \to \mathbb{R}^2$$

$$f(1,0,0) = (1,0), \quad f(0,1,0) = (1,3), \quad f(1,1,1) = (2,4)$$

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$$F_{(\mathcal{B},\, \mathcal{A}_{\mathsf{kan}})} = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 3 & 4 \end{bmatrix}$$

$$f : \mathbb{R}^3 \to \mathbb{R}^2$$

$$f(1,0,0) = (1,0), \quad f(0,1,0) = (1,3), \quad f(1,1,1) = (2,4)$$

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$$F_{(\mathcal{B},\,\mathcal{A}_{\text{kan}})} = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 3 & 4 \end{bmatrix}$$

$$\boxed{Y_{\mathcal{A}_{\text{kan}}} = F_{(\mathcal{B},\,\mathcal{A}_{\text{kan}})} X_{\mathcal{B}}}$$

$$f(0,0,1) = ?$$

$$f : \mathbb{R}^3 \to \mathbb{R}^2$$

$$f(1,0,0) = (1,0), \quad f(0,1,0) = (1,3), \quad f(1,1,1) = (2,4)$$

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$$\boxed{Y_{\mathcal{A}_{\text{kan}}} = F_{(\mathcal{B},\,\mathcal{A}_{\text{kan}})} X_{\mathcal{B}}}$$

$$f(0,0,1) = ?$$

$$X_{\mathcal{B}} = \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$$

$$f : \mathbb{R}^3 \to \mathbb{R}^2$$

$$f(1,0,0) = (1,0), \quad f(0,1,0) = (1,3), \quad f(1,1,1) = (2,4)$$

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$$f(0,0,1) = ?$$

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$$Y_{\mathcal{A}_{\text{kan}}} =$$

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$$Y_{\mathcal{A}_{\text{kan}}} = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 3 & 4 \end{bmatrix} \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$$

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$$\begin{aligned} f(0,0,1) &= f\big(-1 \cdot (1,0,0) + (-1) \cdot (0,1,0) + 1 \cdot (1,1,1)\big) = \\ &= -1 \cdot f(1,0,0) + (-1) \cdot f(0,1,0) + 1 \cdot f(1,1,1) = \\ &= -(1,0) - (1,3) + (2,4) = (0,1) \end{aligned}$$

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$$\boxed{F_{(\mathcal{B}_{\mathsf{kan}},\,\mathcal{A}_{\mathsf{kan}})} = T^{-1} F_{(\mathcal{B},\,\mathcal{A}_{\mathsf{kan}})} S}$$

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$$F_{(\mathcal{B},\, \mathcal{A}_{\text{kan}})} = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 3 & 4 \end{bmatrix}$$

$$\boxed{F_{(\mathcal{B}_{\text{kan}},\, \mathcal{A}_{\text{kan}})} = T^{-1} F_{(\mathcal{B},\, \mathcal{A}_{\text{kan}})} S}$$

$$\mathcal{B} \xrightarrow{\quad S \quad} \mathcal{B}_{\text{kan}}$$

$$f : \mathbb{R}^3 \to \mathbb{R}^2$$

$$f(1,0,0) = (1,0), \quad f(0,1,0) = (1,3), \quad f(1,1,1) = (2,4)$$

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$$F_{(\mathcal{B},\,\mathcal{A}_{\text{kan}})} = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 3 & 4 \end{bmatrix}$$

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$$\mathcal{B} \xrightarrow{\quad S \quad} \mathcal{B}_{\text{kan}} \qquad \mathcal{A}_{\text{kan}} \xrightarrow{\quad T \quad} \mathcal{A}_{\text{kan}}$$

$$f : \mathbb{R}^3 \to \mathbb{R}^2$$

$$f(1,0,0) = (1,0), \quad f(0,1,0) = (1,3), \quad f(1,1,1) = (2,4)$$

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$$F_{(\mathcal{B},\,\mathcal{A}_{\text{kan}})} = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 3 & 4 \end{bmatrix}$$

$$\boxed{F_{(\mathcal{B}_{\text{kan}},\,\mathcal{A}_{\text{kan}})} = T^{-1} F_{(\mathcal{B},\,\mathcal{A}_{\text{kan}})} S}$$

$$\begin{tikzcd} \mathcal{B} \arrow[r, "S"] & \mathcal{B}_{\text{kan}} \arrow[l, bend left, dashed, "S^{-1}"] \end{tikzcd} \qquad \mathcal{A}_{\text{kan}} \xrightarrow{T} \mathcal{A}_{\text{kan}}$$

$$f : \mathbb{R}^3 \to \mathbb{R}^2$$

$$f(1,0,0) = (1,0), \quad f(0,1,0) = (1,3), \quad f(1,1,1) = (2,4)$$

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$$\mathcal{B}_{\text{kan}} = \{(1,0,0),\ (0,1,0),\ (0,0,1)\}$$

$$F_{(\mathcal{B},\, \mathcal{A}_{\text{kan}})} = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 3 & 4 \end{bmatrix}$$

$$S^{-1} = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$$

$$\boxed{F_{(\mathcal{B}_{\text{kan}},\, \mathcal{A}_{\text{kan}})} = T^{-1} F_{(\mathcal{B},\, \mathcal{A}_{\text{kan}})} S}$$

$$\begin{tikzcd} \mathcal{B} \arrow[r, "S"] & \mathcal{B}_{\text{kan}} \arrow[l, bend left, dashed, "S^{-1}"] \end{tikzcd} \qquad \mathcal{A}_{\text{kan}} \xrightarrow{T} \mathcal{A}_{\text{kan}}$$

$$f : \mathbb{R}^3 \to \mathbb{R}^2$$

$$f(1,0,0) = (1,0), \quad f(0,1,0) = (1,3), \quad f(1,1,1) = (2,4)$$

$$\mathcal{B} = \{(1,0,0),\ (0,1,0),\ (1,1,1)\} \qquad \mathcal{A}_{\text{kan}} = \{(1,0),\ (0,1)\}$$

$$\mathcal{B}_{\text{kan}} = \{(1,0,0),\ (0,1,0),\ (0,0,1)\}$$

$$F_{(\mathcal{B},\, \mathcal{A}_{\text{kan}})} = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 3 & 4 \end{bmatrix}$$

$$\boxed{F_{(\mathcal{B}_{\text{kan}},\, \mathcal{A}_{\text{kan}})} = T^{-1} F_{(\mathcal{B},\, \mathcal{A}_{\text{kan}})} S}$$

$$S^{-1} = \begin{bmatrix} 1 & & \\ 0 & & \\ 0 & & \end{bmatrix}$$

$$\begin{tikzcd} \mathcal{B} \arrow[r, "S"] & \mathcal{B}_{\text{kan}} \arrow[l, bend left, dashed, "S^{-1}"] \end{tikzcd} \qquad \mathcal{A}_{\text{kan}} \xrightarrow{T} \mathcal{A}_{\text{kan}}$$

$$f : \mathbb{R}^3 \to \mathbb{R}^2$$

$$f(1,0,0) = (1,0), \quad f(0,1,0) = (1,3), \quad f(1,1,1) = (2,4)$$

$$\mathcal{B} = \{(1,0,0),\ (0,1,0),\ (1,1,1)\} \qquad \mathcal{A}_{\text{kan}} = \{(1,0),\ (0,1)\}$$

$$\mathcal{B}_{\text{kan}} = \{(1,0,0),\ (0,1,0),\ (0,0,1)\}$$

$$F_{(\mathcal{B},\,\mathcal{A}_{\text{kan}})} = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 3 & 4 \end{bmatrix}$$

$$S^{-1} = \begin{bmatrix} 1 & 0 & \\ 0 & 1 & \\ 0 & 0 & \end{bmatrix}$$

$$\boxed{F_{(\mathcal{B}_{\text{kan}},\,\mathcal{A}_{\text{kan}})} = T^{-1} F_{(\mathcal{B},\,\mathcal{A}_{\text{kan}})} S}$$

$$\begin{tikzcd} \mathcal{B} \arrow[r, "S"] & \mathcal{B}_{\text{kan}} \arrow[l, bend left, dashed, "S^{-1}"] \end{tikzcd} \qquad \mathcal{A}_{\text{kan}} \xrightarrow{T} \mathcal{A}_{\text{kan}}$$

$$f : \mathbb{R}^3 \to \mathbb{R}^2$$

$$f(1,0,0) = (1,0), \quad f(0,1,0) = (1,3), \quad f(1,1,1) = (2,4)$$

$$\mathcal{B} = \{(1,0,0),\ (0,1,0),\ (1,1,1)\} \qquad \mathcal{A}_{\text{kan}} = \{(1,0),\ (0,1)\}$$

$$\mathcal{B}_{\text{kan}} = \{(1,0,0),\ (0,1,0),\ (0,0,1)\}$$

$$F_{(\mathcal{B},\, \mathcal{A}_{\text{kan}})} = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 3 & 4 \end{bmatrix}$$

$$S^{-1} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\boxed{F_{(\mathcal{B}_{\text{kan}},\, \mathcal{A}_{\text{kan}})} = T^{-1} F_{(\mathcal{B},\, \mathcal{A}_{\text{kan}})} S}$$

$$\begin{tikzcd} \mathcal{B} \arrow[r, "S"] & \mathcal{B}_{\text{kan}} \arrow[l, bend left, dashed, "S^{-1}"] \end{tikzcd} \qquad \mathcal{A}_{\text{kan}} \xrightarrow{T} \mathcal{A}_{\text{kan}}$$

$$f : \mathbb{R}^3 \to \mathbb{R}^2$$

$$f(1,0,0) = (1,0), \quad f(0,1,0) = (1,3), \quad f(1,1,1) = (2,4)$$

$$\mathcal{B} = \{(1,0,0),\ (0,1,0),\ (1,1,1)\} \qquad \mathcal{A}_{\text{kan}} = \{(1,0),\ (0,1)\}$$

$$\mathcal{B}_{\text{kan}} = \{(1,0,0),\ (0,1,0),\ (0,0,1)\}$$

$$F_{(\mathcal{B},\,\mathcal{A}_{\text{kan}})} = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 3 & 4 \end{bmatrix}$$

$$S^{-1} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \qquad S = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\boxed{F_{(\mathcal{B}_{\text{kan}},\,\mathcal{A}_{\text{kan}})} = T^{-1} F_{(\mathcal{B},\,\mathcal{A}_{\text{kan}})} S}$$

$$\begin{tikzcd} \mathcal{B} \arrow[r, "S"] & \mathcal{B}_{\text{kan}} \arrow[l, bend left, dashed, "S^{-1}"] \end{tikzcd} \qquad \mathcal{A}_{\text{kan}} \xrightarrow{T} \mathcal{A}_{\text{kan}}$$

$$f : \mathbb{R}^3 \to \mathbb{R}^2$$

$$f(1,0,0) = (1,0), \quad f(0,1,0) = (1,3), \quad f(1,1,1) = (2,4)$$

$$\mathcal{B} = \{(1,0,0),\ (0,1,0),\ (1,1,1)\} \qquad \mathcal{A}_{\text{kan}} = \{(1,0),\ (0,1)\}$$

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$$F_{(\mathcal{B},\, \mathcal{A}_{\text{kan}})} = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 3 & 4 \end{bmatrix}$$

$$S^{-1} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \qquad \text{DZ} \quad S = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\boxed{F_{(\mathcal{B}_{\text{kan}},\, \mathcal{A}_{\text{kan}})} = T^{-1} F_{(\mathcal{B},\, \mathcal{A}_{\text{kan}})} S}$$

$$\begin{tikzcd} \mathcal{B} \arrow[r, "S"] & \mathcal{B}_{\text{kan}} \arrow[l, bend left, dashed, "S^{-1}"] \end{tikzcd} \qquad \mathcal{A}_{\text{kan}} \xrightarrow{T} \mathcal{A}_{\text{kan}}$$

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$$F_{(\mathcal{B},\, \mathcal{A}_{\text{kan}})} = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 3 & 4 \end{bmatrix}$$

$$S^{-1} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \qquad \boxed{\text{DZ}} \quad S = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\boxed{F_{(\mathcal{B}_{\text{kan}},\, \mathcal{A}_{\text{kan}})} = T^{-1} F_{(\mathcal{B},\, \mathcal{A}_{\text{kan}})} S}$$

$$\begin{tikzcd} \mathcal{B} \arrow[r, "S"] & \mathcal{B}_{\text{kan}} \arrow[l, bend left, dashed, "S^{-1}"] \end{tikzcd} \qquad \mathcal{A}_{\text{kan}} \xrightarrow{T} \mathcal{A}_{\text{kan}}$$

$$T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$f : \mathbb{R}^3 \to \mathbb{R}^2$$

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$$\mathcal{B}_{\text{kan}} = \{(1,0,0),\ (0,1,0),\ (0,0,1)\}$$

$$F_{(\mathcal{B},\,\mathcal{A}_{\text{kan}})} = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 3 & 4 \end{bmatrix}$$

$$S^{-1} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \qquad \boxed{\text{DZ}} \quad S = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\boxed{F_{(\mathcal{B}_{\text{kan}},\,\mathcal{A}_{\text{kan}})} = T^{-1} F_{(\mathcal{B},\,\mathcal{A}_{\text{kan}})} S}$$

$$\begin{tikzcd} \mathcal{B} \arrow[r, "S"] & \mathcal{B}_{\text{kan}} \arrow[l, bend left, dashed, "S^{-1}"] \end{tikzcd} \qquad \mathcal{A}_{\text{kan}} \xrightarrow{T} \mathcal{A}_{\text{kan}}$$

$$T^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

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$$F_{(\mathcal{B},\,\mathcal{A}_{\mathsf{kan}})} = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 3 & 4 \end{bmatrix}$$

$$\boxed{F_{(\mathcal{B}_{\mathsf{kan}},\,\mathcal{A}_{\mathsf{kan}})} = T^{-1} F_{(\mathcal{B},\,\mathcal{A}_{\mathsf{kan}})} S}$$

$$S^{-1} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \qquad \boxed{\text{DZ}} \quad S = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{tikzcd} \mathcal{B} \arrow[r, "S"] & \mathcal{B}_{\mathsf{kan}} \arrow[l, bend left, dashed, "S^{-1}"] \end{tikzcd} \qquad \mathcal{A}_{\mathsf{kan}} \xrightarrow{T} \mathcal{A}_{\mathsf{kan}}$$

$$T^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$F_{(\mathcal{B}_{\mathsf{kan}},\,\mathcal{A}_{\mathsf{kan}})} =$$

$$f : \mathbb{R}^3 \to \mathbb{R}^2$$

$$f(1,0,0) = (1,0), \quad f(0,1,0) = (1,3), \quad f(1,1,1) = (2,4)$$

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$$F_{(\mathcal{B},\,\mathcal{A}_{\text{kan}})} = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 3 & 4 \end{bmatrix}$$

$$S^{-1} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \qquad \boxed{\text{DZ}} \quad S = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\boxed{F_{(\mathcal{B}_{\text{kan}},\,\mathcal{A}_{\text{kan}})} = T^{-1} F_{(\mathcal{B},\,\mathcal{A}_{\text{kan}})} S}$$

$$\begin{tikzcd} \mathcal{B} \arrow[r, "S"] & \mathcal{B}_{\text{kan}} \arrow[l, bend left, dashed, "S^{-1}"] \end{tikzcd} \qquad \mathcal{A}_{\text{kan}} \xrightarrow{T} \mathcal{A}_{\text{kan}}$$

$$T^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

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$$F_{(\mathcal{B},\, \mathcal{A}_{\text{kan}})} = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 3 & 4 \end{bmatrix}$$

$$S^{-1} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \qquad \boxed{\text{DZ}} \quad S = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\boxed{F_{(\mathcal{B}_{\text{kan}},\, \mathcal{A}_{\text{kan}})} = T^{-1} F_{(\mathcal{B},\, \mathcal{A}_{\text{kan}})} S}$$

$$\begin{tikzcd} \mathcal{B} \arrow[r, "S"] & \mathcal{B}_{\text{kan}} \arrow[l, bend left, dashed, "S^{-1}"] \end{tikzcd} \qquad \mathcal{A}_{\text{kan}} \xrightarrow{T} \mathcal{A}_{\text{kan}}$$

$$T^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

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$$F_{(\mathcal{B},\, \mathcal{A}_{\text{kan}})} = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 3 & 4 \end{bmatrix}$$

$$S^{-1} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \qquad \boxed{\text{DZ}} \quad S = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\boxed{F_{(\mathcal{B}_{\text{kan}},\, \mathcal{A}_{\text{kan}})} = T^{-1} F_{(\mathcal{B},\, \mathcal{A}_{\text{kan}})} S}$$

$$\begin{tikzcd} \mathcal{B} \arrow[r, "S"] & \mathcal{B}_{\text{kan}} \arrow[l, bend left, dashed, "S^{-1}"] \end{tikzcd} \qquad \mathcal{A}_{\text{kan}} \xrightarrow{T} \mathcal{A}_{\text{kan}}$$

$$T^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

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$$F_{(\mathcal{B},\, \mathcal{A}_{\text{kan}})} = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 3 & 4 \end{bmatrix}$$

$$S^{-1} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

DZ

$$S = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\boxed{F_{(\mathcal{B}_{\text{kan}},\, \mathcal{A}_{\text{kan}})} = T^{-1} F_{(\mathcal{B},\, \mathcal{A}_{\text{kan}})} S}$$

$$\begin{tikzcd} \mathcal{B} \arrow[r, "S"] & \mathcal{B}_{\text{kan}} \arrow[l, bend left, dashed, "S^{-1}"] \end{tikzcd} \qquad \mathcal{A}_{\text{kan}} \xrightarrow{T} \mathcal{A}_{\text{kan}}$$

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$$F_{(\mathcal{B}_{\text{kan}},\, \mathcal{A}_{\text{kan}})} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 2 \\ 0 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 3 & 1 \end{bmatrix}$$

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$$f(0,0,1) = ?$$

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$$f(0,0,1) = ?$$

$$\boxed{Y_{\mathcal{A}_{\text{kan}}} = F_{(\mathcal{B}_{\text{kan}},\, \mathcal{A}_{\text{kan}})} X_{\mathcal{B}_{\text{kan}}}}$$

$$f : \mathbb{R}^3 \to \mathbb{R}^2$$

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$$F_{(\mathcal{B},\mathcal{A}_{\text{kan}})} = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 3 & 4 \end{bmatrix}$$

$$F_{(\mathcal{B}_{\text{kan}},\mathcal{A}_{\text{kan}})} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 3 & 1 \end{bmatrix}$$

$$f(0,0,1) = ?$$

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$$X_{\mathcal{B}_{\text{kan}}} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

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$$F_{(\mathcal{B},\, \mathcal{A}_{\text{kan}})} = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 3 & 4 \end{bmatrix}$$

$$F_{(\mathcal{B}_{\text{kan}},\, \mathcal{A}_{\text{kan}})} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 3 & 1 \end{bmatrix}$$

$$f(0,0,1) = ?$$

$$\boxed{Y_{\mathcal{A}_{\text{kan}}} = F_{(\mathcal{B}_{\text{kan}},\, \mathcal{A}_{\text{kan}})} X_{\mathcal{B}_{\text{kan}}}}$$

$$X_{\mathcal{B}_{\text{kan}}} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$Y_{\mathcal{A}_{\text{kan}}} =$$

$$f : \mathbb{R}^3 \to \mathbb{R}^2$$

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$$F_{(\mathcal{B},\mathcal{A}_{\text{kan}})} = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 3 & 4 \end{bmatrix}$$

$$F_{(\mathcal{B}_{\text{kan}},\mathcal{A}_{\text{kan}})} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 3 & 1 \end{bmatrix}$$

$$f(0,0,1) = ?$$

$$\boxed{Y_{\mathcal{A}_{\text{kan}}} = F_{(\mathcal{B}_{\text{kan}},\mathcal{A}_{\text{kan}})} X_{\mathcal{B}_{\text{kan}}}}$$

$$X_{\mathcal{B}_{\text{kan}}} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$Y_{\mathcal{A}_{\text{kan}}} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 3 & 1 \end{bmatrix}$$

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$$F_{(\mathcal{B},\,\mathcal{A}_{\text{kan}})} = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 3 & 4 \end{bmatrix}$$

$$F_{(\mathcal{B}_{\text{kan}},\,\mathcal{A}_{\text{kan}})} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 3 & 1 \end{bmatrix}$$

$$f(0,0,1) = ?$$

$$\boxed{Y_{\mathcal{A}_{\text{kan}}} = F_{(\mathcal{B}_{\text{kan}},\,\mathcal{A}_{\text{kan}})} X_{\mathcal{B}_{\text{kan}}}}$$

$$X_{\mathcal{B}_{\text{kan}}} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$Y_{\mathcal{A}_{\text{kan}}} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 3 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$f : \mathbb{R}^3 \to \mathbb{R}^2$$

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$$F_{(\mathcal{B}_{\text{kan}},\, \mathcal{A}_{\text{kan}})} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 3 & 1 \end{bmatrix}$$

$$f(0,0,1) = ?$$

$$\boxed{Y_{\mathcal{A}_{\text{kan}}} = F_{(\mathcal{B}_{\text{kan}},\, \mathcal{A}_{\text{kan}})} X_{\mathcal{B}_{\text{kan}}}}$$

$$X_{\mathcal{B}_{\text{kan}}} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$Y_{\mathcal{A}_{\text{kan}}} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 3 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$