

# Seminari 10

## MATEMATIČKE METODE ZA INFORMATIČARE

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FOI, Varaždin

# Sadržaj

prvi zadatak

Problem svojstvenih vrijednosti

drugi zadatak

treći zadatak

# **prvi zadatak**

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## Zadatak 1

Odredite sliku, jezgru, rang i defekt linearog operatora  $B : \mathbb{R}^5 \rightarrow \mathbb{R}^3$  zadanog matricom

$$B = \begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{bmatrix}$$

u paru kanonskih baza. Je li  $B$  izomorfizam?

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## Rješenje

Kako je  $\dim \mathbb{R}^5 \neq \dim \mathbb{R}^3$ , zaključujemo da  $\mathbb{R}^5$  i  $\mathbb{R}^3$  nisu izomorfni vektorski prostori.

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Stoga ne postoji niti jedan linearni operator  $\mathbb{R}^5 \rightarrow \mathbb{R}^3$  koji je bijekcija.

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Dakle, linearni operator  $B$  nije izomorfizam.

Ker  $B$

$$B : \mathbb{R}^5 \rightarrow \mathbb{R}^3$$

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 $\cdot (-1)$

$$\begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

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+ ←

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$$1 \quad 2 \quad 5 \quad 2 \quad 1 \quad 0$$

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1	2	3	1	0	0
1	2	5	2	1	0 $\xleftarrow{+}$
1	2	5	2	1	0 $\xleftarrow{+}$
0					
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1	2	3	1	0	0
1	2	5	2	1	0 $\xleftarrow{+}$
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0	0				
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0	0	-2			
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$$\begin{array}{l} / \cdot (-1) \\ / \cdot (-1) \end{array}$$

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1	2	3	1	0	0
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0	0	-2	-1		
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$$\left| \begin{array}{ccccc|c} x_1 & x_2 & x_3 & x_4 & x_5 & 0 \\ 1 & 2 & 3 & 1 & 0 & 0 \\ \textcircled{1} & 2 & 5 & 2 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 & 0 \\ \hline 0 & 0 & -2 & -1 & -1 & 0 \\ 1 & 2 & 5 & 2 & 1 & 0 \end{array} \right.$$

Row operations shown:

- Row 2:  $\cdot(-1)$
- Row 3:  $\cdot(-1)$
- Row 4:  $+ R_2$

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$\xrightarrow{\cdot(-1)} \xrightarrow{\cdot(-1)}$

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$\xleftarrow{+}$        $\xleftarrow{/\cdot(-1)}$        $\xleftarrow{/\cdot(-1)}$   
 $\xleftarrow{+}$

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+      / \cdot (-1) / \cdot (-1)      +

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+ /·(-1) /·(-1) +

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$\nearrow +$        $\nearrow / \cdot (-1)$        $\nearrow / \cdot (-1)$   
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$$\begin{array}{cc|c} x_1 & x_2 & x_3 & x_4 & x_5 & \\ \hline 1 & 2 & 3 & 1 & 0 & 0 \\ \textcircled{1} & 2 & 5 & 2 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 & 0 \\ \hline 0 & 0 & -2 & -1 & -1 & 0 \\ 1 & 2 & 5 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}$$

+ /·(-1) /·(-1) +

$$Y_B = F_{(\mathcal{A}, \mathcal{B})} X_A$$

Ker  $B$

$B : \mathbb{R}^5 \rightarrow \mathbb{R}^3$

$$B(x) = \Theta_{\mathbb{R}^3} \xrightarrow{\text{wavy arrow}} BX = O$$

$$\begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{array}{cc|c} x_1 & x_2 & x_3 & x_4 & x_5 & \\ \hline 1 & 2 & 3 & 1 & 0 & 0 \\ \textcircled{1} & 2 & 5 & 2 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 & 0 \\ \hline 0 & 0 & -2 & -1 & -1 & 0 \\ 1 & 2 & 5 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & -2 & -1 & -1 & 0 \end{array}$$

$\xleftarrow{+}$        $\xleftarrow{/\cdot(-1)}$        $\xleftarrow{/\cdot(-1)}$   
 $\xleftarrow{+}$

$$Y_{\mathcal{B}} = F_{(\mathcal{A}, \mathcal{B})} X_{\mathcal{A}}$$

Ker  $B$

$B : \mathbb{R}^5 \rightarrow \mathbb{R}^3$

$$B(x) = \Theta_{\mathbb{R}^3} \xrightarrow{\text{~~~~~}} BX = O$$

$$\begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{array}{cc|c} x_1 & x_2 & x_3 & x_4 & x_5 & \\ \hline 1 & 2 & 3 & 1 & 0 & 0 \\ \textcircled{1} & 2 & 5 & 2 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 & 0 \\ \hline 0 & 0 & -2 & -1 & -1 & 0 \\ 1 & 2 & 5 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & -2 & -1 & -1 & 0 \\ 1 & 2 & 5 & 2 & 1 & 0 \end{array}$$

$\xleftarrow{+} / \cdot (-1) / \cdot (-1)$   
 $\xleftarrow{+}$

$$Y_B = F_{(\mathcal{A}, \mathcal{B})} X_A$$

Ker  $B$

$B : \mathbb{R}^5 \rightarrow \mathbb{R}^3$

$$B(x) = \Theta_{\mathbb{R}^3} \xrightarrow{\text{~~~~~}} BX = O$$

$$\begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{array}{cc|c} x_1 & x_2 & x_3 & x_4 & x_5 & \\ \hline 1 & 2 & 3 & 1 & 0 & 0 \\ \textcircled{1} & 2 & 5 & 2 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 & 0 \\ \hline 0 & 0 & -2 & -1 & -1 & 0 \\ 1 & 2 & 5 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & -2 & -1 & -1 & 0 \\ 1 & 2 & 5 & 2 & 1 & 0 \end{array}$$

+ /·(-1) /·(-1) +

$$Y_B = F_{(\mathcal{A}, \mathcal{B})} X_A$$

Ker  $B$

$B : \mathbb{R}^5 \rightarrow \mathbb{R}^3$

$$B(x) = \Theta_{\mathbb{R}^3} \xrightarrow{\text{~~~~~}} BX = O$$

$$\begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	
1	2	3	1	0	0
1	2	5	2	1	0
1	2	5	2	1	0
0	0	-2	-1	-1	0
1	2	5	2	1	0
0	0	0	0	0	0
0	0	-2	-1	-1	0
1	2	5	2	1	0

$$Y_B = F_{(\mathcal{A}, \mathcal{B})} X_A$$

Ker  $B$

$B : \mathbb{R}^5 \rightarrow \mathbb{R}^3$

$$B(x) = \Theta_{\mathbb{R}^3} \xrightarrow{\text{~~~~~}} BX = O$$

$$\begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x_1$	$x_2$	$x_3$	$x_4$	$x_5$		
1	2	3	1	0	0	$\leftarrow +$
1	2	5	2	1	0	$/ \cdot (-1) / \cdot (-1)$
1	2	5	2	1	0	$\leftarrow +$
0	0	-2	-1	-1	0	
1	2	5	2	1	0	
0	0	0	0	0	0	
0	0	-2	-1	-1	0	$/ \cdot 1$
1	2	5	2	1	0	

$$Y_B = F_{(\mathcal{A}, \mathcal{B})} X_A$$

Ker  $B$

$B : \mathbb{R}^5 \rightarrow \mathbb{R}^3$

$$B(x) = \Theta_{\mathbb{R}^3} \xrightarrow{\text{~~~~~}} BX = O$$

$$\begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{array}{cc|c} x_1 & x_2 & x_3 & x_4 & x_5 & \\ \hline 1 & 2 & 3 & 1 & 0 & 0 \\ \textcircled{1} & 2 & 5 & 2 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 & 0 \\ \hline 0 & 0 & -2 & -1 & -1 & 0 \\ 1 & 2 & 5 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & -2 & -1 & \textcircled{-1} & 0 \\ 1 & 2 & 5 & 2 & 1 & 0 \end{array}$$

/·(-1) /·(-1)

+ +

/·1

+

$$Y_B = F_{(\mathcal{A}, \mathcal{B})} X_A$$

Ker  $B$

$B : \mathbb{R}^5 \rightarrow \mathbb{R}^3$

$$B(x) = \Theta_{\mathbb{R}^3} \xrightarrow{\text{~~~~~}} BX = O$$

$$\begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	
1	2	3	1	0	0
1	2	5	2	1	0
1	2	5	2	1	0
0	0	-2	-1	-1	0
1	2	5	2	1	0
0	0	0	0	0	0
0	0	-2	-1	-1	0
1	2	5	2	1	0
0	0	-2	-1	-1	0

$$\begin{array}{l} / \cdot (-1) \\ / \cdot (-1) \end{array}$$

$$+$$

$$Y_B = F_{(\mathcal{A}, \mathcal{B})} X_A$$

Ker  $B$

$B : \mathbb{R}^5 \rightarrow \mathbb{R}^3$

$$B(x) = \Theta_{\mathbb{R}^3} \xrightarrow{\text{~~~~~}} BX = O$$

$$\begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{array}{cc|c} x_1 & x_2 & x_3 & x_4 & x_5 & \\ \hline 1 & 2 & 3 & 1 & 0 & 0 \\ \textcircled{1} & 2 & 5 & 2 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 & 0 \\ \hline 0 & 0 & -2 & -1 & -1 & 0 \\ 1 & 2 & 5 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & -2 & -1 & \textcircled{-1} & 0 \\ 1 & 2 & 5 & 2 & 1 & 0 \\ \hline 0 & 0 & -2 & -1 & -1 & 0 \\ 1 & & & & & \end{array}$$

$$Y_B = F_{(\mathcal{A}, \mathcal{B})} X_A$$

Ker  $B$

$B : \mathbb{R}^5 \rightarrow \mathbb{R}^3$

$$B(x) = \Theta_{\mathbb{R}^3} \xrightarrow{\text{~~~~~}} BX = O$$

$$\begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{array}{ccccc|c} x_1 & x_2 & x_3 & x_4 & x_5 & \\ \hline 1 & 2 & 3 & 1 & 0 & 0 \\ \textcircled{1} & 2 & 5 & 2 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 & 0 \\ \hline 0 & 0 & -2 & -1 & -1 & 0 \\ 1 & 2 & 5 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & -2 & -1 & \textcircled{-1} & 0 \\ 1 & 2 & 5 & 2 & 1 & 0 \\ \hline 0 & 0 & -2 & -1 & -1 & 0 \\ 1 & 2 & & & & \end{array}$$

Handwritten annotations:

- A blue arrow points from the first row to the second row with the label  $/ \cdot (-1)$ .
- A blue arrow points from the second row to the third row with the label  $/ \cdot (-1)$ .
- A blue arrow points from the fourth row to the fifth row with the label  $+ \cdot 1$ .
- A blue arrow points from the fifth row to the sixth row with the label  $+ \cdot 1$ .

$$Y_B = F_{(\mathcal{A}, \mathcal{B})} X_A$$

Ker  $B$

$B : \mathbb{R}^5 \rightarrow \mathbb{R}^3$

$$B(x) = \Theta_{\mathbb{R}^3} \xrightarrow{\text{~~~~~}} BX = O$$

$$\begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{array}{cc|ccccc} x_1 & x_2 & x_3 & x_4 & x_5 & & \\ \hline 1 & 2 & 3 & 1 & 0 & 0 & + \\ \textcircled{1} & 2 & 5 & 2 & 1 & 0 & / \cdot (-1) / \cdot (-1) \\ 1 & 2 & 5 & 2 & 1 & 0 & + \\ \hline 0 & 0 & -2 & -1 & -1 & 0 & \\ 1 & 2 & 5 & 2 & 1 & 0 & \\ 0 & 0 & 0 & 0 & 0 & 0 & \\ \hline 0 & 0 & -2 & -1 & \textcircled{-1} & 0 & / \cdot 1 \\ 1 & 2 & 5 & 2 & 1 & 0 & + \\ \hline 0 & 0 & -2 & -1 & -1 & 0 & \\ 1 & 2 & 3 & & & & \end{array}$$

$$Y_B = F_{(\mathcal{A}, \mathcal{B})} X_A$$

Ker  $B$

$B : \mathbb{R}^5 \rightarrow \mathbb{R}^3$

$$B(x) = \Theta_{\mathbb{R}^3} \xrightarrow{\text{~~~~~}} BX = O$$

$$\begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{array}{cc|ccccc} x_1 & x_2 & x_3 & x_4 & x_5 & & \\ \hline 1 & 2 & 3 & 1 & 0 & 0 & + \\ \textcircled{1} & 2 & 5 & 2 & 1 & 0 & / \cdot (-1) / \cdot (-1) \\ 1 & 2 & 5 & 2 & 1 & 0 & + \\ \hline 0 & 0 & -2 & -1 & -1 & 0 & \\ 1 & 2 & 5 & 2 & 1 & 0 & \\ 0 & 0 & 0 & 0 & 0 & 0 & \\ \hline 0 & 0 & -2 & -1 & \textcircled{-1} & 0 & / \cdot 1 \\ 1 & 2 & 5 & 2 & 1 & 0 & + \\ \hline 0 & 0 & -2 & -1 & -1 & 0 & \\ 1 & 2 & 3 & 1 & & & \end{array}$$

$$Y_B = F_{(\mathcal{A}, \mathcal{B})} X_A$$

Ker  $B$

$B : \mathbb{R}^5 \rightarrow \mathbb{R}^3$

$$B(x) = \Theta_{\mathbb{R}^3} \xrightarrow{\text{~~~~~}} BX = O$$

$$\begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{array}{cc|c} x_1 & x_2 & x_3 & x_4 & x_5 & \\ \hline 1 & 2 & 3 & 1 & 0 & 0 \\ \textcircled{1} & 2 & 5 & 2 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 & 0 \\ \hline 0 & 0 & -2 & -1 & -1 & 0 \\ 1 & 2 & 5 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & -2 & -1 & \textcircled{-1} & 0 \\ 1 & 2 & 5 & 2 & 1 & 0 \\ \hline 0 & 0 & -2 & -1 & -1 & 0 \\ 1 & 2 & 3 & 1 & 0 & \end{array}$$

/·(-1) /·(-1)

+ +

/·1

+

$$Y_B = F_{(\mathcal{A}, \mathcal{B})} X_A$$

Ker  $B$

$B : \mathbb{R}^5 \rightarrow \mathbb{R}^3$

$$B(x) = \Theta_{\mathbb{R}^3} \xrightarrow{\text{~~~~~}} BX = O$$

$$\begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{array}{cc|c} x_1 & x_2 & x_3 & x_4 & x_5 & \\ \hline 1 & 2 & 3 & 1 & 0 & 0 \\ \textcircled{1} & 2 & 5 & 2 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 & 0 \\ \hline 0 & 0 & -2 & -1 & -1 & 0 \\ 1 & 2 & 5 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & -2 & -1 & \textcircled{-1} & 0 \\ 1 & 2 & 5 & 2 & 1 & 0 \\ \hline 0 & 0 & -2 & -1 & -1 & 0 \\ 1 & 2 & 3 & 1 & 0 & 0 \end{array}$$

/·(-1) /·(-1)

+ +

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+

$$Y_B = F_{(\mathcal{A}, \mathcal{B})} X_A$$

Ker  $B$  $B : \mathbb{R}^5 \rightarrow \mathbb{R}^3$ 

$$B(x) = \Theta_{\mathbb{R}^3} \xrightarrow{\text{~~~~~}} BX = O$$

$$\begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{array}{cc|c}
 x_1 & x_2 & x_3 & x_4 & x_5 & \\
 \hline
 1 & 2 & 3 & 1 & 0 & 0 \\
 \textcircled{1} & 2 & 5 & 2 & 1 & 0 \\
 1 & 2 & 5 & 2 & 1 & 0 \\
 \hline
 0 & 0 & -2 & -1 & -1 & 0 \\
 1 & 2 & 5 & 2 & 1 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 \\
 \hline
 0 & 0 & -2 & -1 & \textcircled{-1} & 0 \\
 1 & 2 & 5 & 2 & 1 & 0 \\
 \hline
 0 & 0 & -2 & -1 & -1 & 0 \\
 1 & 2 & 3 & 1 & 0 & 0
 \end{array}$$

+ ←  
 $/ \cdot (-1)$   $/ \cdot (-1)$   
 + ←  
 $-2x_3 - x_4 - x_5 = 0$

$$Y_B = F_{(\mathcal{A}, \mathcal{B})} X_A$$

Ker  $B$  $B : \mathbb{R}^5 \rightarrow \mathbb{R}^3$ 

$$B(x) = \Theta_{\mathbb{R}^3} \xrightarrow{\text{~~~~~}} BX = O$$

$$\begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	
1	2	3	1	0	0
1	2	5	2	1	0 / \cdot (-1) / \cdot (-1)
1	2	5	2	1	0 +
0	0	-2	-1	-1	0
1	2	5	2	1	0
0	0	0	0	0	0
0	0	-2	-1	-1	0 / \cdot 1
1	2	5	2	1	0 +
0	0	-2	-1	-1	0
1	2	3	1	0	0

$$Y_B = F_{(\mathcal{A}, \mathcal{B})} X_A$$

Ker  $B$  $B : \mathbb{R}^5 \rightarrow \mathbb{R}^3$ 

$$B(x) = \Theta_{\mathbb{R}^3} \xrightarrow{\text{~~~~~}} BX = O$$

$$\begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	
1	2	3	1	0	0
1	2	5	2	1	0 / \cdot (-1) / \cdot (-1)
1	2	5	2	1	0 +
0	0	-2	-1	-1	0
1	2	5	2	1	0
0	0	0	0	0	0
0	0	-2	-1	-1	0 / \cdot 1
1	2	5	2	1	0 +
0	0	-2	-1	-1	0
1	2	3	1	0	0

$$\left. \begin{array}{l} -2x_3 - x_4 - x_5 = 0 \\ x_1 + 2x_2 + 3x_3 + x_4 = 0 \end{array} \right\}$$

$$Y_B = F_{(\mathcal{A}, \mathcal{B})} X_A$$

Ker  $B$  $B : \mathbb{R}^5 \rightarrow \mathbb{R}^3$ 

$$B(x) = \Theta_{\mathbb{R}^3} \xrightarrow{\text{~~~~~}} BX = O$$

$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	
1	2	3	1	0	0
1	2	5	2	1	0
1	2	5	2	1	0
0	0	-2	-1	-1	0
1	2	5	2	1	0
0	0	0	0	0	0
0	0	-2	-1	-1	0
1	2	5	2	1	0
0	0	-2	-1	-1	0
1	2	3	1	0	0

$$\begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\left. \begin{array}{l} -2x_3 - x_4 - x_5 = 0 \\ x_1 + 2x_2 + 3x_3 + x_4 = 0 \end{array} \right\}$$

$$Y_B = F_{(\mathcal{A}, \mathcal{B})} X_A$$

Ker  $B$  $B : \mathbb{R}^5 \rightarrow \mathbb{R}^3$ 

$$B(x) = \Theta_{\mathbb{R}^3} \xrightarrow{\text{~~~~~}} BX = O$$

$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	
1	2	3	1	0	0
1	2	5	2	1	0
1	2	5	2	1	0
0	0	-2	-1	-1	0
1	2	5	2	1	0
0	0	0	0	0	0
0	0	-2	-1	-1	0
1	2	5	2	1	0
0	0	-2	-1	-1	0
1	2	3	1	0	0

$$\begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\left. \begin{array}{l} -2x_3 - x_4 - x_5 = 0 \\ x_1 + 2x_2 + 3x_3 + x_4 = 0 \end{array} \right\}$$

$$x_5 = -2x_3 - x_4$$

$$Y_B = F_{(\mathcal{A}, \mathcal{B})} X_A$$

Ker  $B$  $B : \mathbb{R}^5 \rightarrow \mathbb{R}^3$ 

$$B(x) = \Theta_{\mathbb{R}^3} \xrightarrow{\text{~~~~~}} BX = O$$

$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	
1	2	3	1	0	0
1	2	5	2	1	0
1	2	5	2	1	0
0	0	-2	-1	-1	0
1	2	5	2	1	0
0	0	0	0	0	0
0	0	-2	-1	-1	0
1	2	5	2	1	0
0	0	-2	-1	-1	0
1	2	3	1	0	0

$$\begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

+  
/·(-1) /·(-1)

$$\left. \begin{array}{l} -2x_3 - x_4 - x_5 = 0 \\ x_1 + 2x_2 + 3x_3 + x_4 = 0 \end{array} \right\}$$

$x_5 = -2x_3 - x_4$

$x_1 = -2x_2 - 3x_3 - x_4$

$$Y_B = F_{(\mathcal{A}, \mathcal{B})} X_A$$

Ker  $B$

$$B : \mathbb{R}^5 \rightarrow \mathbb{R}^3$$

$$x_5 = -2x_3 - x_4$$

$$x_1 = -2x_2 - 3x_3 - x_4$$

Ker  $B$

$$B : \mathbb{R}^5 \rightarrow \mathbb{R}^3$$

$$x_5 = -2x_3 - x_4$$

$$x_1 = -2x_2 - 3x_3 - x_4$$

$$\text{Ker } B =$$

Ker  $B$

$$B : \mathbb{R}^5 \rightarrow \mathbb{R}^3$$

$$x_5 = -2x_3 - x_4$$

$$x_1 = -2x_2 - 3x_3 - x_4$$

$$\text{Ker } B = \{ ($$

Ker  $B$

$$B : \mathbb{R}^5 \rightarrow \mathbb{R}^3$$

$$x_5 = -2x_3 - x_4$$

$$x_1 = -2x_2 - 3x_3 - x_4$$

$$\text{Ker } B = \{(-2x_2 - 3x_3 - x_4,$$

Ker  $B$

$$B : \mathbb{R}^5 \rightarrow \mathbb{R}^3$$

$$x_5 = -2x_3 - x_4$$

$$x_1 = -2x_2 - 3x_3 - x_4$$

$$\text{Ker } B = \{(-2x_2 - 3x_3 - x_4, x_2,$$

Ker  $B$

$$B : \mathbb{R}^5 \rightarrow \mathbb{R}^3$$

$$x_5 = -2x_3 - x_4$$

$$x_1 = -2x_2 - 3x_3 - x_4$$

$$\text{Ker } B = \{(-2x_2 - 3x_3 - x_4, x_2, x_3,$$

Ker  $B$

$$B : \mathbb{R}^5 \rightarrow \mathbb{R}^3$$

$$x_5 = -2x_3 - x_4$$

$$x_1 = -2x_2 - 3x_3 - x_4$$

$$\text{Ker } B = \{(-2x_2 - 3x_3 - x_4, x_2, x_3, x_4,$$

Ker  $B$

$$B : \mathbb{R}^5 \rightarrow \mathbb{R}^3$$

$$x_5 = -2x_3 - x_4$$

$$x_1 = -2x_2 - 3x_3 - x_4$$

$$\text{Ker } B = \{(-2x_2 - 3x_3 - x_4, x_2, x_3, x_4, -2x_3 - x_4)$$

Ker  $B$

$$B : \mathbb{R}^5 \rightarrow \mathbb{R}^3$$

$$x_5 = -2x_3 - x_4$$

$$x_1 = -2x_2 - 3x_3 - x_4$$

$$\text{Ker } B = \{(-2x_2 - 3x_3 - x_4, x_2, x_3, x_4, -2x_3 - x_4) : x_2, x_3, x_4 \in \mathbb{R}\}$$

Ker  $B$

$$B : \mathbb{R}^5 \rightarrow \mathbb{R}^3$$

$$x_5 = -2x_3 - x_4$$

$$x_1 = -2x_2 - 3x_3 - x_4$$

$$\text{Ker } B = \{(-2x_2 - 3x_3 - x_4, x_2, x_3, x_4, -2x_3 - x_4) : x_2, x_3, x_4 \in \mathbb{R}\}$$

$$(-2x_2 - 3x_3 - x_4, x_2, x_3, x_4, -2x_3 - x_4) =$$

Ker  $B$

$$B : \mathbb{R}^5 \rightarrow \mathbb{R}^3$$

$$x_5 = -2x_3 - x_4$$

$$x_1 = -2x_2 - 3x_3 - x_4$$

$$\text{Ker } B = \{(-2x_2 - 3x_3 - x_4, x_2, x_3, x_4, -2x_3 - x_4) : x_2, x_3, x_4 \in \mathbb{R}\}$$

$$(-2x_2 - 3x_3 - x_4, x_2, x_3, x_4, -2x_3 - x_4) =$$

$$= x_2 \cdot ($$

Ker  $B$

$$B : \mathbb{R}^5 \rightarrow \mathbb{R}^3$$

$$x_5 = -2x_3 - x_4$$

$$x_1 = -2x_2 - 3x_3 - x_4$$

$$\text{Ker } B = \{(-2x_2 - 3x_3 - x_4, x_2, x_3, x_4, -2x_3 - x_4) : x_2, x_3, x_4 \in \mathbb{R}\}$$

$$(-2x_2 - 3x_3 - x_4, x_2, x_3, x_4, -2x_3 - x_4) =$$

$$= x_2 \cdot (-2,$$

Ker  $B$

$$B : \mathbb{R}^5 \rightarrow \mathbb{R}^3$$

$$x_5 = -2x_3 - x_4$$

$$x_1 = -2x_2 - 3x_3 - x_4$$

$$\text{Ker } B = \{(-2x_2 - 3x_3 - x_4, x_2, x_3, x_4, -2x_3 - x_4) : x_2, x_3, x_4 \in \mathbb{R}\}$$

$$(-2x_2 - 3x_3 - x_4, x_2, x_3, x_4, -2x_3 - x_4) =$$

$$= x_2 \cdot (-2, 1,$$

Ker  $B$

$$B : \mathbb{R}^5 \rightarrow \mathbb{R}^3$$

$$x_5 = -2x_3 - x_4$$

$$x_1 = -2x_2 - 3x_3 - x_4$$

$$\text{Ker } B = \{(-2x_2 - 3x_3 - x_4, x_2, x_3, x_4, -2x_3 - x_4) : x_2, x_3, x_4 \in \mathbb{R}\}$$

$$(-2x_2 - 3x_3 - x_4, x_2, x_3, x_4, -2x_3 - x_4) =$$

$$= x_2 \cdot (-2, 1, 0,$$

Ker  $B$

$$B : \mathbb{R}^5 \rightarrow \mathbb{R}^3$$

$$x_5 = -2x_3 - x_4$$

$$x_1 = -2x_2 - 3x_3 - x_4$$

$$\text{Ker } B = \{(-2x_2 - 3x_3 - x_4, x_2, x_3, x_4, -2x_3 - x_4) : x_2, x_3, x_4 \in \mathbb{R}\}$$

$$(-2x_2 - 3x_3 - x_4, x_2, x_3, x_4, -2x_3 - x_4) =$$

$$= x_2 \cdot (-2, 1, 0, 0,$$

Ker  $B$

$$B : \mathbb{R}^5 \rightarrow \mathbb{R}^3$$

$$x_5 = -2x_3 - x_4$$

$$x_1 = -2x_2 - 3x_3 - x_4$$

$$\text{Ker } B = \{(-2x_2 - 3x_3 - x_4, x_2, x_3, x_4, -2x_3 - x_4) : x_2, x_3, x_4 \in \mathbb{R}\}$$

$$(-2x_2 - 3x_3 - x_4, x_2, x_3, x_4, -2x_3 - x_4) =$$

$$= x_2 \cdot (-2, 1, 0, 0, 0)$$

Ker  $B$

$$B : \mathbb{R}^5 \rightarrow \mathbb{R}^3$$

$$x_5 = -2x_3 - x_4$$

$$x_1 = -2x_2 - 3x_3 - x_4$$

$$\text{Ker } B = \{(-2x_2 - 3x_3 - x_4, x_2, x_3, x_4, -2x_3 - x_4) : x_2, x_3, x_4 \in \mathbb{R}\}$$

$$(-2x_2 - 3x_3 - x_4, x_2, x_3, x_4, -2x_3 - x_4) =$$

$$= x_2 \cdot (-2, 1, 0, 0, 0) +$$

Ker  $B$

$$B : \mathbb{R}^5 \rightarrow \mathbb{R}^3$$

$$x_5 = -2x_3 - x_4$$

$$x_1 = -2x_2 - 3x_3 - x_4$$

$$\text{Ker } B = \{(-2x_2 - 3x_3 - x_4, x_2, x_3, x_4, -2x_3 - x_4) : x_2, x_3, x_4 \in \mathbb{R}\}$$

$$(-2x_2 - 3x_3 - x_4, x_2, x_3, x_4, -2x_3 - x_4) =$$

$$= x_2 \cdot (-2, 1, 0, 0, 0) + x_3 \cdot ($$

Ker  $B$

$$B : \mathbb{R}^5 \rightarrow \mathbb{R}^3$$

$$x_5 = -2x_3 - x_4$$

$$x_1 = -2x_2 - 3x_3 - x_4$$

$$\text{Ker } B = \{(-2x_2 - 3x_3 - x_4, x_2, x_3, x_4, -2x_3 - x_4) : x_2, x_3, x_4 \in \mathbb{R}\}$$

$$(-2x_2 - 3x_3 - x_4, x_2, x_3, x_4, -2x_3 - x_4) =$$

$$= x_2 \cdot (-2, 1, 0, 0, 0) + x_3 \cdot (-3,$$

Ker  $B$

$$B : \mathbb{R}^5 \rightarrow \mathbb{R}^3$$

$$x_5 = -2x_3 - x_4$$

$$x_1 = -2x_2 - 3x_3 - x_4$$

$$\text{Ker } B = \{(-2x_2 - 3x_3 - x_4, x_2, x_3, x_4, -2x_3 - x_4) : x_2, x_3, x_4 \in \mathbb{R}\}$$

$$(-2x_2 - 3x_3 - x_4, x_2, x_3, x_4, -2x_3 - x_4) =$$

$$= x_2 \cdot (-2, 1, 0, 0, 0) + x_3 \cdot (-3, 0,$$

Ker  $B$

$$B : \mathbb{R}^5 \rightarrow \mathbb{R}^3$$

$$x_5 = -2x_3 - x_4$$

$$x_1 = -2x_2 - 3x_3 - x_4$$

$$\text{Ker } B = \{(-2x_2 - 3x_3 - x_4, x_2, x_3, x_4, -2x_3 - x_4) : x_2, x_3, x_4 \in \mathbb{R}\}$$

$$(-2x_2 - 3x_3 - x_4, x_2, x_3, x_4, -2x_3 - x_4) =$$

$$= x_2 \cdot (-2, 1, 0, 0, 0) + x_3 \cdot (-3, 0, 1,$$

Ker  $B$

$$B : \mathbb{R}^5 \rightarrow \mathbb{R}^3$$

$$x_5 = -2x_3 - x_4$$

$$x_1 = -2x_2 - 3x_3 - x_4$$

$$\text{Ker } B = \{(-2x_2 - 3x_3 - x_4, x_2, x_3, x_4, -2x_3 - x_4) : x_2, x_3, x_4 \in \mathbb{R}\}$$

$$(-2x_2 - 3x_3 - x_4, x_2, x_3, x_4, -2x_3 - x_4) =$$

$$= x_2 \cdot (-2, 1, 0, 0, 0) + x_3 \cdot (-3, 0, 1, 0,$$

Ker  $B$

$$B : \mathbb{R}^5 \rightarrow \mathbb{R}^3$$

$$x_5 = -2x_3 - x_4$$

$$x_1 = -2x_2 - 3x_3 - x_4$$

$$\text{Ker } B = \{(-2x_2 - 3x_3 - x_4, x_2, x_3, x_4, -2x_3 - x_4) : x_2, x_3, x_4 \in \mathbb{R}\}$$

$$(-2x_2 - 3x_3 - x_4, x_2, x_3, x_4, -2x_3 - x_4) =$$

$$= x_2 \cdot (-2, 1, 0, 0, 0) + x_3 \cdot (-3, 0, 1, 0, -2)$$

Ker  $B$

$$B : \mathbb{R}^5 \rightarrow \mathbb{R}^3$$

$$x_5 = -2x_3 - x_4$$

$$x_1 = -2x_2 - 3x_3 - x_4$$

$$\text{Ker } B = \{(-2x_2 - 3x_3 - x_4, x_2, x_3, x_4, -2x_3 - x_4) : x_2, x_3, x_4 \in \mathbb{R}\}$$

$$(-2x_2 - 3x_3 - x_4, x_2, x_3, x_4, -2x_3 - x_4) =$$

$$= x_2 \cdot (-2, 1, 0, 0, 0) + x_3 \cdot (-3, 0, 1, 0, -2) +$$

Ker  $B$

$$B : \mathbb{R}^5 \rightarrow \mathbb{R}^3$$

$$x_5 = -2x_3 - x_4$$

$$x_1 = -2x_2 - 3x_3 - x_4$$

$$\text{Ker } B = \{(-2x_2 - 3x_3 - x_4, x_2, x_3, x_4, -2x_3 - x_4) : x_2, x_3, x_4 \in \mathbb{R}\}$$

$$(-2x_2 - 3x_3 - x_4, x_2, x_3, x_4, -2x_3 - x_4) =$$

$$= x_2 \cdot (-2, 1, 0, 0, 0) + x_3 \cdot (-3, 0, 1, 0, -2) + x_4 \cdot ($$

Ker  $B$

$$B : \mathbb{R}^5 \rightarrow \mathbb{R}^3$$

$$x_5 = -2x_3 - x_4$$

$$x_1 = -2x_2 - 3x_3 - x_4$$

$$\text{Ker } B = \{(-2x_2 - 3x_3 - x_4, x_2, x_3, x_4, -2x_3 - x_4) : x_2, x_3, x_4 \in \mathbb{R}\}$$

$$(-2x_2 - 3x_3 - x_4, x_2, x_3, x_4, -2x_3 - x_4) =$$

$$= x_2 \cdot (-2, 1, 0, 0, 0) + x_3 \cdot (-3, 0, 1, 0, -2) + x_4 \cdot (-1,$$

Ker  $B$

$$B : \mathbb{R}^5 \rightarrow \mathbb{R}^3$$

$$x_5 = -2x_3 - x_4$$

$$x_1 = -2x_2 - 3x_3 - x_4$$

$$\text{Ker } B = \{(-2x_2 - 3x_3 - x_4, x_2, x_3, x_4, -2x_3 - x_4) : x_2, x_3, x_4 \in \mathbb{R}\}$$

$$(-2x_2 - 3x_3 - x_4, x_2, x_3, x_4, -2x_3 - x_4) =$$

$$= x_2 \cdot (-2, 1, 0, 0, 0) + x_3 \cdot (-3, 0, 1, 0, -2) + x_4 \cdot (-1, 0,$$

Ker  $B$

$$B : \mathbb{R}^5 \rightarrow \mathbb{R}^3$$

$$x_5 = -2x_3 - x_4$$

$$x_1 = -2x_2 - 3x_3 - x_4$$

$$\text{Ker } B = \{(-2x_2 - 3x_3 - x_4, x_2, x_3, x_4, -2x_3 - x_4) : x_2, x_3, x_4 \in \mathbb{R}\}$$

$$(-2x_2 - 3x_3 - x_4, x_2, x_3, x_4, -2x_3 - x_4) =$$

$$= x_2 \cdot (-2, 1, 0, 0, 0) + x_3 \cdot (-3, 0, 1, 0, -2) + x_4 \cdot (-1, 0, 0,$$

Ker  $B$

$$B : \mathbb{R}^5 \rightarrow \mathbb{R}^3$$

$$x_5 = -2x_3 - x_4$$

$$x_1 = -2x_2 - 3x_3 - x_4$$

$$\text{Ker } B = \{(-2x_2 - 3x_3 - x_4, x_2, x_3, x_4, -2x_3 - x_4) : x_2, x_3, x_4 \in \mathbb{R}\}$$

$$(-2x_2 - 3x_3 - x_4, x_2, x_3, x_4, -2x_3 - x_4) =$$

$$= x_2 \cdot (-2, 1, 0, 0, 0) + x_3 \cdot (-3, 0, 1, 0, -2) + x_4 \cdot (-1, 0, 0, 1,$$

Ker  $B$

$$B : \mathbb{R}^5 \rightarrow \mathbb{R}^3$$

$$x_5 = -2x_3 - x_4$$

$$x_1 = -2x_2 - 3x_3 - x_4$$

$$\text{Ker } B = \{(-2x_2 - 3x_3 - x_4, x_2, x_3, x_4, -2x_3 - x_4) : x_2, x_3, x_4 \in \mathbb{R}\}$$

$$(-2x_2 - 3x_3 - x_4, x_2, x_3, x_4, -2x_3 - x_4) =$$

$$= x_2 \cdot (-2, 1, 0, 0, 0) + x_3 \cdot (-3, 0, 1, 0, -2) + x_4 \cdot (-1, 0, 0, 1, -1)$$

Ker  $B$

$$B : \mathbb{R}^5 \rightarrow \mathbb{R}^3$$

$$x_5 = -2x_3 - x_4$$

$$x_1 = -2x_2 - 3x_3 - x_4$$

$$\text{Ker } B = \{(-2x_2 - 3x_3 - x_4, x_2, x_3, x_4, -2x_3 - x_4) : x_2, x_3, x_4 \in \mathbb{R}\}$$

$$(-2x_2 - 3x_3 - x_4, x_2, x_3, x_4, -2x_3 - x_4) =$$

$$= x_2 \cdot (-2, 1, 0, 0, 0) + x_3 \cdot (-3, 0, 1, 0, -2) + x_4 \cdot (-1, 0, 0, 1, -1)$$

$$\mathcal{B}_{\text{Ker } B} = \{(-2, 1, 0, 0, 0), (-3, 0, 1, 0, -2), (-1, 0, 0, 1, -1)\}$$

Ker  $B$

$$B : \mathbb{R}^5 \rightarrow \mathbb{R}^3$$

$$x_5 = -2x_3 - x_4$$

$$x_1 = -2x_2 - 3x_3 - x_4$$

$$\text{Ker } B = \{(-2x_2 - 3x_3 - x_4, x_2, x_3, x_4, -2x_3 - x_4) : x_2, x_3, x_4 \in \mathbb{R}\}$$

$$(-2x_2 - 3x_3 - x_4, x_2, x_3, x_4, -2x_3 - x_4) =$$

$$= x_2 \cdot (-2, 1, 0, 0, 0) + x_3 \cdot (-3, 0, 1, 0, -2) + x_4 \cdot (-1, 0, 0, 1, -1)$$

$$\mathcal{B}_{\text{Ker } B} = \{(-2, 1, 0, 0, 0), (-3, 0, 1, 0, -2), (-1, 0, 0, 1, -1)\}$$

$$d(B) = 3$$

Ker  $B$

$$B : \mathbb{R}^5 \rightarrow \mathbb{R}^3$$

$$x_5 = -2x_3 - x_4$$

$$x_1 = -2x_2 - 3x_3 - x_4$$

$$\text{Ker } B = \{(-2x_2 - 3x_3 - x_4, x_2, x_3, x_4, -2x_3 - x_4) : x_2, x_3, x_4 \in \mathbb{R}\}$$

$$(-2x_2 - 3x_3 - x_4, x_2, x_3, x_4, -2x_3 - x_4) =$$

$$= x_2 \cdot (-2, 1, 0, 0, 0) + x_3 \cdot (-3, 0, 1, 0, -2) + x_4 \cdot (-1, 0, 0, 1, -1)$$

$$\mathcal{B}_{\text{Ker } B} = \{(-2, 1, 0, 0, 0), (-3, 0, 1, 0, -2), (-1, 0, 0, 1, -1)\}$$

$$d(B) = 3 \xrightarrow{\quad} B \text{ nije injekcija}$$

Im  $B$

$$B : \mathbb{R}^5 \rightarrow \mathbb{R}^3$$

$\text{Im } B$

$$B : \mathbb{R}^5 \rightarrow \mathbb{R}^3$$

$$B = \begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{bmatrix}$$

Im  $B$

$$B : \mathbb{R}^5 \rightarrow \mathbb{R}^3$$

$$r(B) + d(B) = \dim \mathbb{R}^5$$

$$B = \begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{bmatrix}$$

Im  $B$

$$B : \mathbb{R}^5 \rightarrow \mathbb{R}^3$$

$$r(B) + d(B) = \dim \mathbb{R}^5$$

$$r(B) + 3 = 5$$

$$B = \begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{bmatrix}$$

**Im  $B$**

$$B : \mathbb{R}^5 \rightarrow \mathbb{R}^3$$

$$r(B) + d(B) = \dim \mathbb{R}^5$$

$$r(B) + 3 = 5$$

$$r(B) = 2$$

$$B = \begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{bmatrix}$$

Im  $B$

$$B : \mathbb{R}^5 \rightarrow \mathbb{R}^3$$

$$r(B) + d(B) = \dim \mathbb{R}^5$$

$$r(B) + 3 = 5$$

$$r(B) = 2$$

$$r(B) \neq \dim \mathbb{R}^3$$

$$B = \begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{bmatrix}$$

Im  $B$

$$B : \mathbb{R}^5 \rightarrow \mathbb{R}^3$$

$$r(B) + d(B) = \dim \mathbb{R}^5$$

$$r(B) + 3 = 5$$

$$r(B) = 2$$

$$B = \begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{bmatrix}$$

$$r(B) \neq \dim \mathbb{R}^3 \xrightarrow{\text{blue arrow}} B \text{ nije surjekcija}$$

Im  $B$

$$B : \mathbb{R}^5 \rightarrow \mathbb{R}^3$$

$$r(B) + d(B) = \dim \mathbb{R}^5$$

$$r(B) + 3 = 5$$

$$r(B) = 2$$

$$B = \begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{bmatrix}$$

$$r(B) \neq \dim \mathbb{R}^3 \quad \textcolor{blue}{\rightarrow} B \text{ nije surjekcija}$$

$$\begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{bmatrix}$$

Im  $B$

$$B : \mathbb{R}^5 \rightarrow \mathbb{R}^3$$

$$r(B) + d(B) = \dim \mathbb{R}^5$$

$$r(B) + 3 = 5$$

$$r(B) = 2$$

$$B = \begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{bmatrix}$$

$$r(B) \neq \dim \mathbb{R}^3 \longrightarrow B \text{ nije surjekcija}$$

$$\begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{bmatrix}$$

Im  $B$

$$B : \mathbb{R}^5 \rightarrow \mathbb{R}^3$$

$$r(B) + d(B) = \dim \mathbb{R}^5$$

$$r(B) + 3 = 5$$

$$r(B) = 2$$

$r(B) \neq \dim \mathbb{R}^3 \longrightarrow B$  nije surjekcija

$$B = \begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{bmatrix} / \cdot (-1)$$

Im  $B$

$$B : \mathbb{R}^5 \rightarrow \mathbb{R}^3$$

$$r(B) + d(B) = \dim \mathbb{R}^5$$

$$r(B) + 3 = 5$$

$$r(B) = 2$$

$r(B) \neq \dim \mathbb{R}^3 \rightarrow B$  nije surjekcija

$$B = \begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{bmatrix} / \cdot (-1)$$


Im  $B$

$$B : \mathbb{R}^5 \rightarrow \mathbb{R}^3$$

$$r(B) + d(B) = \dim \mathbb{R}^5$$

$$r(B) + 3 = 5$$

$$r(B) = 2$$

$r(B) \neq \dim \mathbb{R}^3 \rightarrow B$  nije surjekcija

$$B = \begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{bmatrix} / \cdot (-1) / \cdot (-1)$$


Im  $B$

$$B : \mathbb{R}^5 \rightarrow \mathbb{R}^3$$

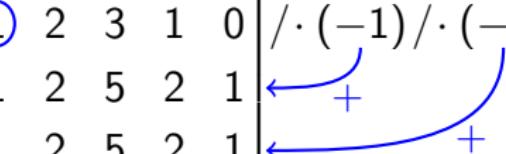
$$r(B) + d(B) = \dim \mathbb{R}^5$$

$$r(B) + 3 = 5$$

$$r(B) = 2$$

$r(B) \neq \dim \mathbb{R}^3 \rightarrow B$  nije surjekcija

$$B = \begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{bmatrix} / \cdot (-1) / \cdot (-1)$$


Im  $B$

$$B : \mathbb{R}^5 \rightarrow \mathbb{R}^3$$

$$r(B) + d(B) = \dim \mathbb{R}^5$$

$$r(B) + 3 = 5$$

$$r(B) = 2$$

$r(B) \neq \dim \mathbb{R}^3 \rightarrow B$  nije surjekcija

$$B = \begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{bmatrix}$$

$$\left[ \begin{array}{ccccc} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{array} \right] / \cdot (-1) / \cdot (-1) \sim \left[ \begin{array}{ccccc} & & & & \\ & & & & \\ & & & & \end{array} \right]$$

Diagram illustrating row reduction steps:

- Step 1: Row 1 is multiplied by -1.
- Step 2: Row 1 is multiplied by -1 again.
- Step 3: Row 2 is added to Row 1.
- Step 4: Row 3 is added to Row 1.

Im  $B$

$$B : \mathbb{R}^5 \rightarrow \mathbb{R}^3$$

$$r(B) + d(B) = \dim \mathbb{R}^5$$

$$r(B) + 3 = 5$$

$$r(B) = 2$$

$r(B) \neq \dim \mathbb{R}^3 \rightarrow B$  nije surjekcija

$$B = \begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{bmatrix}$$

$$\left[ \begin{array}{ccccc} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{array} \right] / \cdot (-1) / \cdot (-1) \sim \left[ \begin{array}{ccccc} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{array} \right]$$

Diagram illustrating row operations on the matrix:

- The first row has a circled '1' at the beginning.
- A blue arrow labeled '+' points from the second row to the third row, indicating a swap operation.
- A blue arrow labeled '+' points from the second row to the third row, indicating a swap operation.
- A blue arrow labeled '+' points from the second row to the third row, indicating a swap operation.

Im  $B$

$$B : \mathbb{R}^5 \rightarrow \mathbb{R}^3$$

$$r(B) + d(B) = \dim \mathbb{R}^5$$

$$r(B) + 3 = 5$$

$$r(B) = 2$$

$r(B) \neq \dim \mathbb{R}^3 \rightarrow B$  nije surjekcija

$$B = \begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{bmatrix}$$

$$\left[ \begin{array}{ccccc} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{array} \right] / \cdot (-1) / \cdot (-1) \sim \left[ \begin{array}{ccccc} 1 & 2 & 3 & 1 & 0 \\ 0 & & & & \\ & & & & \end{array} \right]$$

Diagram illustrating row reduction steps:

- Step 1:  $\textcircled{1}$  (circled 1) indicates the first row.
- Step 2: A blue arrow labeled  $+$  points from the first row to the second row, indicating the operation  $R_2 \leftarrow R_2 - R_1$ .
- Step 3: A blue arrow labeled  $+$  points from the first row to the third row, indicating the operation  $R_3 \leftarrow R_3 - R_1$ .

Im  $B$

$$B : \mathbb{R}^5 \rightarrow \mathbb{R}^3$$

$$r(B) + d(B) = \dim \mathbb{R}^5$$

$$r(B) + 3 = 5$$

$$r(B) = 2$$

$r(B) \neq \dim \mathbb{R}^3 \rightarrow B$  nije surjekcija

$$B = \begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{bmatrix}$$

$$\left[ \begin{array}{ccccc} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 1 & 2 & 5 & 2 & 1 \end{array} \right] / \cdot (-1) / \cdot (-1) \sim \left[ \begin{array}{ccccc} 1 & 2 & 3 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

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Diagram illustrating row reduction steps:

- Step 1: Row 1 is multiplied by -1.
- Step 2: Row 1 is multiplied by -1 again.
- Step 3: Row 2 is multiplied by -1.
- Step 4: Row 2 is multiplied by -1 again.
- Step 5: Row 2 is added to Row 1.
- Step 6: Row 2 is added to Row 3.

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- Step 7: Row 2 is added to Row 1.
- Step 8: Row 3 is added to Row 1.

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Handwritten annotations show blue arrows indicating row operations: one arrow labeled '+' points from the first row to the second, and another arrow labeled '+' points from the first row to the third.

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Handwritten annotations show blue arrows indicating row operations: a vertical arrow labeled with a circled '1' points to the first row; a horizontal arrow labeled with a '+' points from the second row to the third row; another horizontal arrow labeled with a '+' points from the second row to the third row.

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Diagram illustrating row operations on the matrix:

- The first row has a circled '1' at the beginning.
- A blue arrow labeled '+' points from the second row to the third row, indicating a swap operation.
- A blue arrow labeled '+' points from the first row to the second row, indicating a swap operation.
- A blue arrow labeled '+' points from the first row to the third row, indicating a swap operation.
- A blue arrow labeled '+' points from the first column to the second column, indicating a scalar multiplication by -1.
- A blue arrow labeled '+' points from the first column to the third column, indicating a scalar multiplication by -1.

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Diagram illustrating row operations on the matrix:

- The first row is multiplied by  $-1$  (indicated by a blue circle).
- The first row is multiplied by  $-1$  again (indicated by a blue circle).
- The second row is added to the third row (indicated by a blue plus sign).
- The third row is added to the second row (indicated by a blue plus sign).

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- A blue arrow labeled '+' points from the first row to the third row, indicating a swap operation.
- A blue arrow labeled '+' points from the first column to the second column, indicating a multiplication by -1.
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Im  $B$

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Diagram illustrating row operations on the matrix:

- The first row is multiplied by  $-1$  (indicated by a blue circle "1" and a blue arrow).
- The second row is multiplied by  $-1$  (indicated by a blue circle "2" and a blue arrow).
- The third row is multiplied by  $-1$  (indicated by a blue circle "2" and a blue arrow).

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Diagram illustrating row operations on the matrix:

- The first row is circled in blue.
- A blue arrow labeled with a plus sign (+) points from the first row to the second row.
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- The second row is circled in blue.

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Ako je  $\dim U = \dim V$ , je li linearни operator  $f : U \rightarrow V$  izomorfizam?

# **Problem svojstvenih vrijednosti**

---

# Glavne minore

Neka je  $A \in M_n(F)$  pri čemu je  $F$  polje.

- **Glavna podmatrica** reda  $r$  matrice  $A$  je svaka podmatrica  $A_{i_1, i_2, \dots, i_r}$  koja se sastoji od onih elemenata matrice  $A$  koji se nalaze na presjeku  $r$  redaka i  $r$  stupaca s istim indeksima  $i_1, i_2, \dots, i_r$ .
- Glavnih podmatrica reda  $r$  matrice  $A$  ima ukupno  $\binom{n}{r}$ .
- **Glavna minora**  $\Delta_{i_1, i_2, \dots, i_r}$  reda  $r$  matrice  $A$  je determinanta pripadne glavne podmatrice, tj.  $\Delta_{i_1, i_2, \dots, i_r} = \det A_{i_1, i_2, \dots, i_r}$ .

# Karakteristični polinom

- $k_A^{(1)}(\lambda) = \det(A - \lambda I)$

$$k_A^{(1)}(\lambda) = (-1)^n \lambda^n + a_1 \lambda^{n-1} + a_2 \lambda^{n-2} + \dots + a_{n-1} \lambda + a_n$$

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- $k_A^{(2)}(\lambda) = \det(\lambda I - A)$

$$k_A^{(2)}(\lambda) = \lambda^n + c_1 \lambda^{n-1} + c_2 \lambda^{n-2} + \dots + c_{n-1} \lambda + c_n$$

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- $c_r = (-1)^r \sum_{i_1 < i_2 < \dots < i_r} \Delta_{i_1, i_2, \dots, i_r}, \quad \{i_1, i_2, \dots, i_r\} \subseteq \{1, 2, \dots, n\}$

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- $c_1 = -\operatorname{tr} A, \quad c_n = (-1)^n \det A$

# Problem svojstvenih vrijednosti

- $V$  konačnodimenzionalni vektorski prostor nad poljem  $F$
- $\mathcal{B}$  neka baza za vektorski prostor  $V$

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$$f : V \rightarrow V$$

$$f(x) = \lambda x$$

# Problem svojstvenih vrijednosti

- $V$  konačnodimenzionalni vektorski prostor nad poljem  $F$
- $\mathcal{B}$  neka baza za vektorski prostor  $V$

$$f : V \rightarrow V$$

$$f(x) = \lambda x \quad F_{\mathcal{B}} X_{\mathcal{B}} = \lambda X_{\mathcal{B}}$$

# Problem svojstvenih vrijednosti

- $V$  konačnodimenzionalni vektorski prostor nad poljem  $F$
- $\mathcal{B}$  neka baza za vektorski prostor  $V$

$$f : V \rightarrow V$$

$$f(x) = \lambda x \quad F_{\mathcal{B}} X_{\mathcal{B}} = \lambda X_{\mathcal{B}}$$

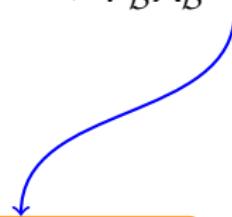

$$(F_{\mathcal{B}} - \lambda I) X_{\mathcal{B}} = O$$

# Problem svojstvenih vrijednosti

- $V$  konačnodimenzionalni vektorski prostor nad poljem  $F$
- $\mathcal{B}$  neka baza za vektorski prostor  $V$

$$f : V \rightarrow V$$

$$f(x) = \lambda x \quad F_{\mathcal{B}} X_{\mathcal{B}} = \lambda X_{\mathcal{B}}$$



$$(F_{\mathcal{B}} - \lambda I) X_{\mathcal{B}} = O$$

$$S(\lambda) = \{x \in V : f(x) = \lambda x\}$$

## **drugi zadatak**

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## Zadatak 2

Zadana je matrica  $A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$ .

- Odredite svojstvene vrijednosti matrice  $A$ .
- Odredite svojstvene potprostore matrice  $A$ .
- Odredite minimalni polinom matrice  $A$ .
- Izrazite  $A^{-1}$  pomoću potencija matrice  $A$ .

## Rješenje

a)  $k_A(\lambda) = \lambda^3 + c_1\lambda^2 + c_2\lambda + c_3$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

## Rješenje

a)  $k_A(\lambda) = \lambda^3 + c_1\lambda^2 + c_2\lambda + c_3$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$c_1 =$$

## Rješenje

a)  $k_A(\lambda) = \lambda^3 + c_1\lambda^2 + c_2\lambda + c_3$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$c_1 = (-1)^1 \cdot (\Delta_1 + \Delta_2 + \Delta_3)$$

## Rješenje

a)  $k_A(\lambda) = \lambda^3 + c_1\lambda^2 + c_2\lambda + c_3$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$c_1 = (-1)^1 \cdot (\Delta_1 + \Delta_2 + \Delta_3) = -($$

## Rješenje

a)  $k_A(\lambda) = \lambda^3 + c_1\lambda^2 + c_2\lambda + c_3$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

1.

1.

$$c_1 = (-1)^1 \cdot (\Delta_1 + \Delta_2 + \Delta_3) = -(4$$

## Rješenje

a)  $k_A(\lambda) = \lambda^3 + c_1\lambda^2 + c_2\lambda + c_3$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$c_1 = (-1)^1 \cdot (\Delta_1 + \Delta_2 + \Delta_3) = -(4 +$$

## Rješenje

a)  $k_A(\lambda) = \lambda^3 + c_1\lambda^2 + c_2\lambda + c_3$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

Diagram illustrating the cofactor expansion of the matrix along the second column (indicated by a red circle labeled "2." and a red arrow). The matrix is:

$$\begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

The second column is highlighted with a blue circle containing the number 5. A red arrow points from the label "2." at the bottom right to the second column.

$$c_1 = (-1)^1 \cdot (\Delta_1 + \Delta_2 + \Delta_3) = -(4 + 5)$$

## Rješenje

a)  $k_A(\lambda) = \lambda^3 + c_1\lambda^2 + c_2\lambda + c_3$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$c_1 = (-1)^1 \cdot (\Delta_1 + \Delta_2 + \Delta_3) = -(4 + 5 +$$

## Rješenje

a)  $k_A(\lambda) = \lambda^3 + c_1\lambda^2 + c_2\lambda + c_3$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

Diagram illustrating the cofactor expansion along the third column (3.). The entries 2 and 1 are circled in blue and red respectively. A red arrow points from the circled 2 to the circled 3., and another red arrow points from the circled 1 to the circled 3..

$$c_1 = (-1)^1 \cdot (\Delta_1 + \Delta_2 + \Delta_3) = -(4 + 5 + 2)$$

## Rješenje

a)  $k_A(\lambda) = \lambda^3 + c_1\lambda^2 + c_2\lambda + c_3$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$c_1 = (-1)^1 \cdot (\Delta_1 + \Delta_2 + \Delta_3) = -(4 + 5 + 2)$$

## Rješenje

a)  $k_A(\lambda) = \lambda^3 + c_1\lambda^2 + c_2\lambda + c_3$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$c_1 = (-1)^1 \cdot (\Delta_1 + \Delta_2 + \Delta_3) = -(4 + 5 + 2) = -11$$

## Rješenje

a)  $k_A(\lambda) = \lambda^3 + c_1\lambda^2 + c_2\lambda + c_3$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$c_1 = (-1)^1 \cdot (\Delta_1 + \Delta_2 + \Delta_3) = -(4 + 5 + 2) = -11$$

$$c_2 =$$

## Rješenje

a)  $k_A(\lambda) = \lambda^3 + c_1\lambda^2 + c_2\lambda + c_3$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$c_1 = (-1)^1 \cdot (\Delta_1 + \Delta_2 + \Delta_3) = -(4 + 5 + 2) = -11$$

$$c_2 = (-1)^2 \cdot (\Delta_{1,2} + \Delta_{1,3} + \Delta_{2,3})$$

## Rješenje

a)  $k_A(\lambda) = \lambda^3 + c_1\lambda^2 + c_2\lambda + c_3$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$c_1 = (-1)^1 \cdot (\Delta_1 + \Delta_2 + \Delta_3) = -(4 + 5 + 2) = -11$$

$$c_2 = (-1)^2 \cdot (\Delta_{1,2} + \Delta_{1,3} + \Delta_{2,3}) =$$

## Rješenje

a)  $k_A(\lambda) = \lambda^3 + c_1\lambda^2 + c_2\lambda + c_3$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

Diagram illustrating the cofactor expansion along the first row:

- Red arrows point from the numbers 1 and 2 in red circles at the bottom left to the first two columns of the matrix.
- Red wavy arrows point from the numbers 1 and 2 in red circles at the top right to the first two entries of the third column (-1 and -2).
- Red circles with numbers 1. and 2. are placed next to the wavy arrows.

$$c_1 = (-1)^1 \cdot (\Delta_1 + \Delta_2 + \Delta_3) = -(4 + 5 + 2) = -11$$

$$c_2 = (-1)^2 \cdot (\Delta_{1,2} + \Delta_{1,3} + \Delta_{2,3}) = \begin{vmatrix} 4 & 1 \\ 2 & 5 \end{vmatrix}$$

## Rješenje

a)  $k_A(\lambda) = \lambda^3 + c_1\lambda^2 + c_2\lambda + c_3$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$c_1 = (-1)^1 \cdot (\Delta_1 + \Delta_2 + \Delta_3) = -(4 + 5 + 2) = -11$$

$$c_2 = (-1)^2 \cdot (\Delta_{1,2} + \Delta_{1,3} + \Delta_{2,3}) = \begin{vmatrix} 4 & 1 \\ 2 & 5 \end{vmatrix} +$$

## Rješenje

a)  $k_A(\lambda) = \lambda^3 + c_1\lambda^2 + c_2\lambda + c_3$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

Diagram illustrating cofactor expansion:

- Row 1 circled in yellow, labeled "1." with a red arrow pointing to the circled -1.
- Row 3 circled in yellow, labeled "3." with a red arrow pointing to the circled 2.
- Column 1 circled in blue, labeled "1." with a red arrow pointing to the circled 1.
- Column 3 circled in blue, labeled "3." with a red arrow pointing to the circled -1.

$$c_1 = (-1)^1 \cdot (\Delta_1 + \Delta_2 + \Delta_3) = -(4 + 5 + 2) = -11$$

$$c_2 = (-1)^2 \cdot (\Delta_{1,2} + \Delta_{1,3} + \Delta_{2,3}) = \begin{vmatrix} 4 & 1 \\ 2 & 5 \end{vmatrix} + \begin{vmatrix} 4 & -1 \\ 1 & 2 \end{vmatrix}$$

## Rješenje

a)  $k_A(\lambda) = \lambda^3 + c_1\lambda^2 + c_2\lambda + c_3$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$c_1 = (-1)^1 \cdot (\Delta_1 + \Delta_2 + \Delta_3) = -(4 + 5 + 2) = -11$$

$$c_2 = (-1)^2 \cdot (\Delta_{1,2} + \Delta_{1,3} + \Delta_{2,3}) = \begin{vmatrix} 4 & 1 \\ 2 & 5 \end{vmatrix} + \begin{vmatrix} 4 & -1 \\ 1 & 2 \end{vmatrix} +$$

## Rješenje

a)  $k_A(\lambda) = \lambda^3 + c_1\lambda^2 + c_2\lambda + c_3$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

Diagram illustrating the cofactor expansion of the matrix A:

- The matrix A is shown with circled entries: 5 (top-right), 1 (middle-right), and 2 (bottom-right).
- Red arrows point from the circled entries to the labels "2." and "3.".
- Two red curly arrows point from the bottom row of the matrix to the labels "2." and "3.", indicating the rows used for the cofactor expansion.

$$c_1 = (-1)^1 \cdot (\Delta_1 + \Delta_2 + \Delta_3) = -(4 + 5 + 2) = -11$$

$$c_2 = (-1)^2 \cdot (\Delta_{1,2} + \Delta_{1,3} + \Delta_{2,3}) = \begin{vmatrix} 4 & 1 \\ 2 & 5 \end{vmatrix} + \begin{vmatrix} 4 & -1 \\ 1 & 2 \end{vmatrix} + \begin{vmatrix} 5 & -2 \\ 1 & 2 \end{vmatrix}$$

## Rješenje

a)  $k_A(\lambda) = \lambda^3 + c_1\lambda^2 + c_2\lambda + c_3$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$c_1 = (-1)^1 \cdot (\Delta_1 + \Delta_2 + \Delta_3) = -(4 + 5 + 2) = -11$$

$$c_2 = (-1)^2 \cdot (\Delta_{1,2} + \Delta_{1,3} + \Delta_{2,3}) = \begin{vmatrix} 4 & 1 \\ 2 & 5 \end{vmatrix} + \begin{vmatrix} 4 & -1 \\ 1 & 2 \end{vmatrix} + \begin{vmatrix} 5 & -2 \\ 1 & 2 \end{vmatrix} =$$

## Rješenje

a)  $k_A(\lambda) = \lambda^3 + c_1\lambda^2 + c_2\lambda + c_3$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$c_1 = (-1)^1 \cdot (\Delta_1 + \Delta_2 + \Delta_3) = -(4 + 5 + 2) = -11$$

$$c_2 = (-1)^2 \cdot (\Delta_{1,2} + \Delta_{1,3} + \Delta_{2,3}) = \begin{vmatrix} 4 & 1 \\ 2 & 5 \end{vmatrix} + \begin{vmatrix} 4 & -1 \\ 1 & 2 \end{vmatrix} + \begin{vmatrix} 5 & -2 \\ 1 & 2 \end{vmatrix} = \\ = 18$$

## Rješenje

a)  $k_A(\lambda) = \lambda^3 + c_1\lambda^2 + c_2\lambda + c_3$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$c_1 = (-1)^1 \cdot (\Delta_1 + \Delta_2 + \Delta_3) = -(4 + 5 + 2) = -11$$

$$c_2 = (-1)^2 \cdot (\Delta_{1,2} + \Delta_{1,3} + \Delta_{2,3}) = \begin{vmatrix} 4 & 1 \\ 2 & 5 \end{vmatrix} + \begin{vmatrix} 4 & -1 \\ 1 & 2 \end{vmatrix} + \begin{vmatrix} 5 & -2 \\ 1 & 2 \end{vmatrix} = \\ = 18 + 9$$

## Rješenje

a)  $k_A(\lambda) = \lambda^3 + c_1\lambda^2 + c_2\lambda + c_3$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$c_1 = (-1)^1 \cdot (\Delta_1 + \Delta_2 + \Delta_3) = -(4 + 5 + 2) = -11$$

$$c_2 = (-1)^2 \cdot (\Delta_{1,2} + \Delta_{1,3} + \Delta_{2,3}) = \begin{vmatrix} 4 & 1 \\ 2 & 5 \end{vmatrix} + \begin{vmatrix} 4 & -1 \\ 1 & 2 \end{vmatrix} + \begin{vmatrix} 5 & -2 \\ 1 & 2 \end{vmatrix} = \\ = 18 + 9 + 12$$

## Rješenje

a)  $k_A(\lambda) = \lambda^3 + c_1\lambda^2 + c_2\lambda + c_3$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$c_1 = (-1)^1 \cdot (\Delta_1 + \Delta_2 + \Delta_3) = -(4 + 5 + 2) = -11$$

$$c_2 = (-1)^2 \cdot (\Delta_{1,2} + \Delta_{1,3} + \Delta_{2,3}) = \begin{vmatrix} 4 & 1 \\ 2 & 5 \end{vmatrix} + \begin{vmatrix} 4 & -1 \\ 1 & 2 \end{vmatrix} + \begin{vmatrix} 5 & -2 \\ 1 & 2 \end{vmatrix} = \\ = 18 + 9 + 12 = 39$$

## Rješenje

a)  $k_A(\lambda) = \lambda^3 + c_1\lambda^2 + c_2\lambda + c_3$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$c_1 = (-1)^1 \cdot (\Delta_1 + \Delta_2 + \Delta_3) = -(4 + 5 + 2) = -11$$

$$c_2 = (-1)^2 \cdot (\Delta_{1,2} + \Delta_{1,3} + \Delta_{2,3}) = \begin{vmatrix} 4 & 1 \\ 2 & 5 \end{vmatrix} + \begin{vmatrix} 4 & -1 \\ 1 & 2 \end{vmatrix} + \begin{vmatrix} 5 & -2 \\ 1 & 2 \end{vmatrix} =$$

$$= 18 + 9 + 12 = 39$$

$$c_3 =$$

## Rješenje

a)  $k_A(\lambda) = \lambda^3 + c_1\lambda^2 + c_2\lambda + c_3$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$c_1 = (-1)^1 \cdot (\Delta_1 + \Delta_2 + \Delta_3) = -(4 + 5 + 2) = -11$$

$$c_2 = (-1)^2 \cdot (\Delta_{1,2} + \Delta_{1,3} + \Delta_{2,3}) = \begin{vmatrix} 4 & 1 \\ 2 & 5 \end{vmatrix} + \begin{vmatrix} 4 & -1 \\ 1 & 2 \end{vmatrix} + \begin{vmatrix} 5 & -2 \\ 1 & 2 \end{vmatrix} = \\ = 18 + 9 + 12 = 39$$

$$c_3 = (-1)^3 \cdot \Delta_{1,2,3}$$

## Rješenje

a)  $k_A(\lambda) = \lambda^3 + c_1\lambda^2 + c_2\lambda + c_3$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$c_1 = (-1)^1 \cdot (\Delta_1 + \Delta_2 + \Delta_3) = -(4 + 5 + 2) = -11$$

$$c_2 = (-1)^2 \cdot (\Delta_{1,2} + \Delta_{1,3} + \Delta_{2,3}) = \begin{vmatrix} 4 & 1 \\ 2 & 5 \end{vmatrix} + \begin{vmatrix} 4 & -1 \\ 1 & 2 \end{vmatrix} + \begin{vmatrix} 5 & -2 \\ 1 & 2 \end{vmatrix} = \\ = 18 + 9 + 12 = 39$$

$$c_3 = (-1)^3 \cdot \Delta_{1,2,3} = -1 \cdot$$

## Rješenje

a)  $k_A(\lambda) = \lambda^3 + c_1\lambda^2 + c_2\lambda + c_3$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$c_1 = (-1)^1 \cdot (\Delta_1 + \Delta_2 + \Delta_3) = -(4 + 5 + 2) = -11$$

$$c_2 = (-1)^2 \cdot (\Delta_{1,2} + \Delta_{1,3} + \Delta_{2,3}) = \begin{vmatrix} 4 & 1 \\ 2 & 5 \end{vmatrix} + \begin{vmatrix} 4 & -1 \\ 1 & 2 \end{vmatrix} + \begin{vmatrix} 5 & -2 \\ 1 & 2 \end{vmatrix} = \\ = 18 + 9 + 12 = 39$$

$$c_3 = (-1)^3 \cdot \Delta_{1,2,3} = -1 \cdot \begin{vmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{vmatrix}$$

## Rješenje

a)  $k_A(\lambda) = \lambda^3 + c_1\lambda^2 + c_2\lambda + c_3$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$c_1 = (-1)^1 \cdot (\Delta_1 + \Delta_2 + \Delta_3) = -(4 + 5 + 2) = -11$$

$$c_2 = (-1)^2 \cdot (\Delta_{1,2} + \Delta_{1,3} + \Delta_{2,3}) = \begin{vmatrix} 4 & 1 \\ 2 & 5 \end{vmatrix} + \begin{vmatrix} 4 & -1 \\ 1 & 2 \end{vmatrix} + \begin{vmatrix} 5 & -2 \\ 1 & 2 \end{vmatrix} = \\ = 18 + 9 + 12 = 39$$

$$c_3 = (-1)^3 \cdot \Delta_{1,2,3} = -1 \cdot \begin{vmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{vmatrix} = -1 \cdot 45$$

## Rješenje

a)  $k_A(\lambda) = \lambda^3 + c_1\lambda^2 + c_2\lambda + c_3$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$c_1 = (-1)^1 \cdot (\Delta_1 + \Delta_2 + \Delta_3) = -(4 + 5 + 2) = -11$$

$$c_2 = (-1)^2 \cdot (\Delta_{1,2} + \Delta_{1,3} + \Delta_{2,3}) = \begin{vmatrix} 4 & 1 \\ 2 & 5 \end{vmatrix} + \begin{vmatrix} 4 & -1 \\ 1 & 2 \end{vmatrix} + \begin{vmatrix} 5 & -2 \\ 1 & 2 \end{vmatrix} = \\ = 18 + 9 + 12 = 39$$

$$c_3 = (-1)^3 \cdot \Delta_{1,2,3} = -1 \cdot \begin{vmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{vmatrix} = -1 \cdot 45 = -45$$

## Rješenje

a)  $k_A(\lambda) = \lambda^3 + c_1\lambda^2 + c_2\lambda + c_3$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$

$$c_1 = (-1)^1 \cdot (\Delta_1 + \Delta_2 + \Delta_3) = -(4 + 5 + 2) = -11$$

$$c_2 = (-1)^2 \cdot (\Delta_{1,2} + \Delta_{1,3} + \Delta_{2,3}) = \begin{vmatrix} 4 & 1 \\ 2 & 5 \end{vmatrix} + \begin{vmatrix} 4 & -1 \\ 1 & 2 \end{vmatrix} + \begin{vmatrix} 5 & -2 \\ 1 & 2 \end{vmatrix} = \\ = 18 + 9 + 12 = 39$$

$$c_3 = (-1)^3 \cdot \Delta_{1,2,3} = -1 \cdot \begin{vmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{vmatrix} = -1 \cdot 45 = -45$$

$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$

$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$

$$\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$$

$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$

$$\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$$

$$1, -1,$$

$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$

$$\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$$

$$1, -1, 3, -3,$$

$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$

$$\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$$

$$1, -1, 3, -3, 5, -5,$$

$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$

$$\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$$

$$1, -1, 3, -3, 5, -5, 9, -9,$$

$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$

$$\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$$

$$1, -1, 3, -3, 5, -5, 9, -9, 15, -15,$$

$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$

$$\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$$

$$1, -1, 3, -3, 5, -5, 9, -9, 15, -15, 45, -45$$

$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$

$$\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$$

$$1, -1, 3, -3, 5, -5, 9, -9, 15, -15, 45, -45$$



$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$

$$\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$$

$$1, -1, 3, -3, 5, -5, 9, -9, 15, -15, 45, -45$$



$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$

$$\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$$

$$1, -1, 3, -3, 5, -5, 9, -9, 15, -15, 45, -45$$

$$\begin{array}{c|c|c|c} & 1 & -11 & \\ \hline & | & | & | \end{array}$$

$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$

$$\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$$

$$1, -1, 3, -3, 5, -5, 9, -9, 15, -15, 45, -45$$

	1	-11	39	

$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$

$$\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$$

$$1, -1, 3, -3, 5, -5, 9, -9, 15, -15, 45, -45$$

	1	-11	39	-45

$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$

$$\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$$

$$1, -1, 3, -3, 5, -5, 9, -9, 15, -15, 45, -45$$

$$\begin{array}{c|ccccc} & 1 & -11 & 39 & -45 \\ \hline 3 & | & | & | & | \end{array}$$

$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$

$$\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$$

$$1, -1, 3, -3, 5, -5, 9, -9, 15, -15, 45, -45$$

$$\begin{array}{c|ccccc} & 1 & -11 & 39 & -45 \\ \hline 3 & 1 & & & \end{array}$$

$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$

$$\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$$

$$1, -1, 3, -3, 5, -5, 9, -9, 15, -15, 45, -45$$

	1	-11	39	-45
3	1	-8		

$3 \cdot 1 + (-11) = -8$

$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$

$$\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$$

$$1, -1, 3, -3, 5, -5, 9, -9, 15, -15, 45, -45$$

$$\begin{array}{c|ccccc} & 1 & -11 & 39 & -45 \\ \hline 3 & 1 & -8 & 15 & \\ \end{array} \quad 3 \cdot (-8) + 39 = 15$$

$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$

$$\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$$

$$1, -1, 3, -3, 5, -5, 9, -9, 15, -15, 45, -45$$

	1	-11	39	-45
3	1	-8	15	0

$3 \cdot 15 + (-45) = 0$

$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$

$$\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$$

$$1, -1, 3, -3, 5, -5, 9, -9, 15, -15, 45, -45$$

$$\begin{array}{c|c|c|c|c} & 1 & -11 & 39 & -45 \\ \hline 3 & 1 & -8 & 15 & 0 \end{array}$$

$$(\lambda - 3)$$

$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$

$$\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$$

$$1, -1, 3, -3, 5, -5, 9, -9, 15, -15, 45, -45$$

$$\begin{array}{c|c|c|c|c} & 1 & -11 & 39 & -45 \\ \hline 3 & 1 & -8 & 15 & 0 \end{array}$$

$$(\lambda - 3)(\quad)$$

$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$

$$\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$$

$$1, -1, 3, -3, 5, -5, 9, -9, 15, -15, 45, -45$$

$$\begin{array}{c|c|c|c|c} & 1 & -11 & 39 & -45 \\ \hline 3 & 1 & -8 & 15 & 0 \end{array}$$

$$(\lambda - 3)(\lambda^2 \quad \quad \quad )$$

$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$

$$\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$$

$$1, -1, 3, -3, 5, -5, 9, -9, 15, -15, 45, -45$$

$$\begin{array}{c|c|c|c|c} & 1 & -11 & 39 & -45 \\ \hline 3 & 1 & -8 & 15 & 0 \end{array}$$

$$(\lambda - 3)(\lambda^2 - 8\lambda \quad \quad )$$

$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$

$$\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$$

$$1, -1, 3, -3, 5, -5, 9, -9, 15, -15, 45, -45$$

$$\begin{array}{c|c|c|c|c} & 1 & -11 & 39 & -45 \\ \hline 3 & 1 & -8 & 15 & 0 \end{array}$$

$$(\lambda - 3)(\lambda^2 - 8\lambda + 15)$$

$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$

$$\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$$

$$1, -1, 3, -3, 5, -5, 9, -9, 15, -15, 45, -45$$

$$\begin{array}{c|c|c|c|c} & 1 & -11 & 39 & -45 \\ \hline 3 & 1 & -8 & 15 & 0 \end{array}$$

$$(\lambda - 3)(\lambda^2 - 8\lambda + 15) = 0$$

$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$

$$\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$$

1, -1, 3, -3, 5, -5, 9, -9, 15, -15, 45, -45

	1	-11	39	-45
3	1	-8	15	0

$$(\lambda - 3)(\lambda^2 - 8\lambda + 15) = 0$$

$$\lambda_1 = 3$$

$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$

$$\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$$

1, -1, 3, -3, 5, -5, 9, -9, 15, -15, 45, -45

	1	-11	39	-45
3	1	-8	15	0

$$(\lambda - 3)(\lambda^2 - 8\lambda + 15) = 0$$

$\lambda_1 = 3$

$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$

$$\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$$

1, -1, 3, -3, 5, -5, 9, -9, 15, -15, 45, -45

	1	-11	39	-45
3	1	-8	15	0

$$(\lambda - 3)(\lambda^2 - 8\lambda + 15) = 0$$

$$\boxed{\lambda_1 = 3}$$

$$\lambda^2 - 8\lambda + 15 = 0$$

$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$

$$\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$$

1, -1, 3, -3, 5, -5, 9, -9, 15, -15, 45, -45

	1	-11	39	-45
3	1	-8	15	0

$$(\lambda - 3)(\lambda^2 - 8\lambda + 15) = 0$$

$$\boxed{\lambda_1 = 3}$$

$$\lambda^2 - 8\lambda + 15 = 0$$

$$\lambda_{2,3} = \frac{-(-8) \pm \sqrt{(-8)^2 - 4 \cdot 1 \cdot 15}}{2 \cdot 1}$$

$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$

$$\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$$

1, -1, 3, -3, 5, -5, 9, -9, 15, -15, 45, -45

	1	-11	39	-45
3	1	-8	15	0

$$(\lambda - 3)(\lambda^2 - 8\lambda + 15) = 0$$

$$\boxed{\lambda_1 = 3}$$

$$\lambda^2 - 8\lambda + 15 = 0$$

$$\lambda_{2,3} = \frac{-(-8) \pm \sqrt{(-8)^2 - 4 \cdot 1 \cdot 15}}{2 \cdot 1}$$

$$\lambda_{2,3} = \frac{8 \pm 2}{2}$$

$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$

$$\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$$

1, -1, 3, -3, 5, -5, 9, -9, 15, -15, 45, -45

	1	-11	39	-45
3	1	-8	15	0

$$(\lambda - 3)(\lambda^2 - 8\lambda + 15) = 0$$

$$\boxed{\lambda_1 = 3}$$

$$\lambda^2 - 8\lambda + 15 = 0$$

$$\lambda_{2,3} = \frac{-(-8) \pm \sqrt{(-8)^2 - 4 \cdot 1 \cdot 15}}{2 \cdot 1}$$

$$\lambda_{2,3} = \frac{8 \pm 2}{2} \quad \lambda_2 = 5$$

$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$

$$\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$$

$$1, -1, 3, -3, 5, -5, 9, -9, 15, -15, 45, -45$$

	1	-11	39	-45
3	1	-8	15	0

$$(\lambda - 3)(\lambda^2 - 8\lambda + 15) = 0$$

$$\boxed{\lambda_1 = 3}$$

$$\lambda^2 - 8\lambda + 15 = 0$$

$$\lambda_{2,3} = \frac{-(-8) \pm \sqrt{(-8)^2 - 4 \cdot 1 \cdot 15}}{2 \cdot 1}$$

$$\lambda_{2,3} = \frac{8 \pm 2}{2} \quad \lambda_2 = 5, \lambda_3 = 3$$

$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$

$$\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$$

$$1, -1, 3, -3, 5, -5, 9, -9, 15, -15, 45, -45$$

	1	-11	39	-45
3	1	-8	15	0

$$(\lambda - 3)(\lambda^2 - 8\lambda + 15) = 0$$

$$\boxed{\lambda_1 = 3}$$

$$\lambda^2 - 8\lambda + 15 = 0$$

$$\lambda_{2,3} = \frac{-(-8) \pm \sqrt{(-8)^2 - 4 \cdot 1 \cdot 15}}{2 \cdot 1}$$

$$\lambda_{2,3} = \frac{8 \pm 2}{2}$$

$$\boxed{\lambda_2 = 5, \lambda_3 = 3}$$

$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$

$$\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$$

$$1, -1, 3, -3, 5, -5, 9, -9, 15, -15, 45, -45$$

$\sigma(A) =$

	1	-11	39	-45
3	1	-8	15	0

$$(\lambda - 3)(\lambda^2 - 8\lambda + 15) = 0$$

$$\boxed{\lambda_1 = 3}$$

$$\lambda^2 - 8\lambda + 15 = 0$$

$$\lambda_{2,3} = \frac{-(-8) \pm \sqrt{(-8)^2 - 4 \cdot 1 \cdot 15}}{2 \cdot 1}$$

$$\lambda_{2,3} = \frac{8 \pm 2}{2}$$

$$\boxed{\lambda_2 = 5, \lambda_3 = 3}$$

$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$

$$\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$$

$$1, -1, 3, -3, 5, -5, 9, -9, 15, -15, 45, -45$$

$$\sigma(A) = \{3, 5\}$$

	1	-11	39	-45
3	1	-8	15	0

$$(\lambda - 3)(\lambda^2 - 8\lambda + 15) = 0$$

$$\boxed{\lambda_1 = 3}$$

$$\lambda^2 - 8\lambda + 15 = 0$$

$$\lambda_{2,3} = \frac{-(-8) \pm \sqrt{(-8)^2 - 4 \cdot 1 \cdot 15}}{2 \cdot 1}$$

$$\lambda_{2,3} = \frac{8 \pm 2}{2}$$

$$\boxed{\lambda_2 = 5, \lambda_3 = 3}$$

$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$

$$\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$$

1, -1, 3, -3, 5, -5, 9, -9, 15, -15, 45, -45

$$\sigma(A) = \{3, 5\}$$

	1	-11	39	-45
3	1	-8	15	0

algebarska kratnost  
jednaka je 2

$$(\lambda - 3)(\lambda^2 - 8\lambda + 15) = 0$$

$$\lambda_1 = 3$$

$$\lambda^2 - 8\lambda + 15 = 0$$

$$\lambda_{2,3} = \frac{-(-8) \pm \sqrt{(-8)^2 - 4 \cdot 1 \cdot 15}}{2 \cdot 1}$$

$$\lambda_{2,3} = \frac{8 \pm 2}{2}$$

$$\lambda_2 = 5, \lambda_3 = 3$$

$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$

algebarska kratnost  
jednaka je 1

$$\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$$

$$1, -1, 3, -3, 5, -5, 9, -9, 15, -15, 45, -45$$

$$\sigma(A) = \{3, 5\}$$

	1	-11	39	-45
3	1	-8	15	0

algebarska kratnost  
jednaka je 2

$$(\lambda - 3)(\lambda^2 - 8\lambda + 15) = 0$$

$$\lambda_1 = 3$$

$$\lambda^2 - 8\lambda + 15 = 0$$

$$\lambda_{2,3} = \frac{-(-8) \pm \sqrt{(-8)^2 - 4 \cdot 1 \cdot 15}}{2 \cdot 1}$$

$$\lambda_{2,3} = \frac{8 \pm 2}{2}$$

$$\lambda_2 = 5, \lambda_3 = 3$$

$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$

algebarska kratnost  
jednaka je 1

$$\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$$

$$1, -1, 3, -3, 5, -5, 9, -9, 15, -15, 45, -45$$

$$\sigma(A) = \{3, 5\}$$

	1	-11	39	-45
3	1	-8	15	0

algebarska kratnost  
jednaka je 2

$$(\lambda - 3)(\lambda^2 - 8\lambda + 15) = 0$$

$$\lambda_1 = 3$$

$$\lambda^2 - 8\lambda + 15 = 0$$

$$\lambda_{2,3} = \frac{-(-8) \pm \sqrt{(-8)^2 - 4 \cdot 1 \cdot 15}}{2 \cdot 1}$$

$$k_A(\lambda) =$$

$$\lambda_{2,3} = \frac{8 \pm 2}{2}$$

$$\lambda_2 = 5, \lambda_3 = 3$$

$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$

algebarska kratnost  
jednaka je 1

$$\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$$

$$1, -1, 3, -3, 5, -5, 9, -9, 15, -15, 45, -45$$

$$\sigma(A) = \{3, 5\}$$

	1	-11	39	-45
3	1	-8	15	0

algebarska kratnost  
jednaka je 2

$$(\lambda - 3)(\lambda^2 - 8\lambda + 15) = 0$$

$$\lambda_1 = 3$$

$$\lambda^2 - 8\lambda + 15 = 0$$

$$\lambda_{2,3} = \frac{-(-8) \pm \sqrt{(-8)^2 - 4 \cdot 1 \cdot 15}}{2 \cdot 1}$$

$$k_A(\lambda) = (\lambda - 3)^2$$

$$\lambda_{2,3} = \frac{8 \pm 2}{2}$$

$$\lambda_2 = 5, \lambda_3 = 3$$

$$k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$

algebarska kratnost  
jednaka je 1

$$\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$$

$$1, -1, 3, -3, 5, -5, 9, -9, 15, -15, 45, -45$$

$$\sigma(A) = \{3, 5\}$$

	1	-11	39	-45
3	1	-8	15	0

algebarska kratnost  
jednaka je 2

$$(\lambda - 3)(\lambda^2 - 8\lambda + 15) = 0$$

$$\lambda_1 = 3$$

$$\lambda^2 - 8\lambda + 15 = 0$$

$$\lambda_{2,3} = \frac{-(-8) \pm \sqrt{(-8)^2 - 4 \cdot 1 \cdot 15}}{2 \cdot 1}$$

$$k_A(\lambda) = (\lambda - 3)^2 \cdot (\lambda - 5)$$

$$\lambda_{2,3} = \frac{8 \pm 2}{2}$$

$$\lambda_2 = 5, \lambda_3 = 3$$

b)

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$S(\lambda) = \{x \in V : f(x) = \lambda x\}$$

$$(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O$$

b)

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$(A - 3I)X = O$$

$$S(\lambda) = \{x \in V : f(x) = \lambda x\}$$

$$(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O$$

b)

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$(A - 3I)X = O$$

$$\left[ \quad \quad \quad \right]$$

$$S(\lambda) = \{x \in V : f(x) = \lambda x\}$$

$$(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O$$

b)

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$(A - 3I)X = O$$

$$\begin{bmatrix} 1 \\ \\ \\ \end{bmatrix}$$

$$S(\lambda) = \{x \in V : f(x) = \lambda x\}$$

$$(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O$$

b)

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$(A - 3I)X = O$$

$$\begin{bmatrix} 1 & & \\ & 2 & \\ & & \end{bmatrix}$$

$$S(\lambda) = \{x \in V : f(x) = \lambda x\}$$

$$(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O$$

b)

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$(A - 3I)X = O$$

$$\begin{bmatrix} 1 & & \\ & 2 & \\ & & -1 \end{bmatrix}$$

$$S(\lambda) = \{x \in V : f(x) = \lambda x\}$$

$$(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O$$

b)

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$(A - 3I)X = O$$

$$\begin{bmatrix} 1 & 1 & -1 \\ & 2 & -2 \\ & & -1 \end{bmatrix}$$

$$S(\lambda) = \{x \in V : f(x) = \lambda x\}$$

$$(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O$$

b)

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$(A - 3I)X = O$$

$$\begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & -2 \\ 1 & 1 & -1 \end{bmatrix}$$

$$S(\lambda) = \{x \in V : f(x) = \lambda x\}$$

$$(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O$$

b)

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$(A - 3I)X = O$$

$$\begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & -2 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$S(\lambda) = \{x \in V : f(x) = \lambda x\}$$

$$(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O$$

b)

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

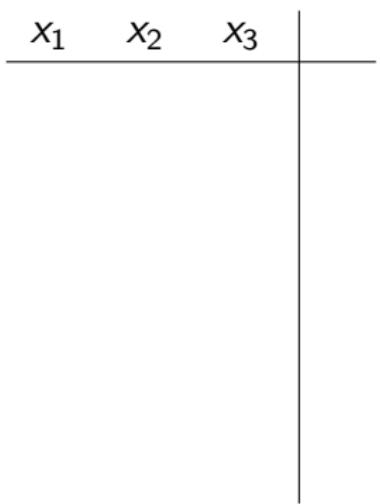
$$(A - 3I)X = O$$

$$\begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & -2 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$S(\lambda) = \{x \in V : f(x) = \lambda x\}$$

$$(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O$$

b)



$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

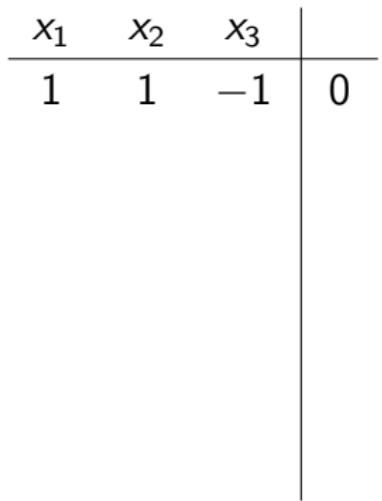
$$(A - 3I)X = O$$

$$\begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & -2 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$S(\lambda) = \{x \in V : f(x) = \lambda x\}$$

$$(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O$$

b)



$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$(A - 3I)X = O$$

$$\begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & -2 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$S(\lambda) = \{x \in V : f(x) = \lambda x\}$$

$$(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O$$

b)

$x_1$	$x_2$	$x_3$	
1	1	-1	0
2	2	-2	0

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$(A - 3I)X = O$$

$$\begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & -2 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$S(\lambda) = \{x \in V : f(x) = \lambda x\}$$

$$(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O$$

b)

$x_1$	$x_2$	$x_3$	
1	1	-1	0
2	2	-2	0
1	1	-1	0

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$(A - 3I)X = O$$

$$\begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & -2 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$S(\lambda) = \{x \in V : f(x) = \lambda x\}$$

$$(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O$$

b)

$x_1$	$x_2$	$x_3$	
1	1	-1	0
2	2	-2	0
1	1	-1	0

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$(A - 3I)X = O$$

$$\begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & -2 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$S(\lambda) = \{x \in V : f(x) = \lambda x\}$$

$$(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O$$

b)

$$\begin{array}{ccc|c} x_1 & x_2 & x_3 & \\ \hline 1 & 1 & -1 & 0 \\ 2 & 2 & -2 & 0 \\ 1 & 1 & -1 & 0 \\ \hline & & & \end{array} \quad / : 2$$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$(A - 3I)X = O$$

$$\begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & -2 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$S(\lambda) = \{x \in V : f(x) = \lambda x\}$$

$$(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O$$

b)

$$\begin{array}{ccc|c} x_1 & x_2 & x_3 & \\ \hline 1 & 1 & -1 & 0 \\ 2 & 2 & -2 & 0 \\ 1 & 1 & -1 & 0 \\ \hline 1 & 1 & -1 & 0 \end{array} \quad /: 2$$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$(A - 3I)X = O$$

$$\begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & -2 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$S(\lambda) = \{x \in V : f(x) = \lambda x\}$$

$$(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O$$

b)

$x_1$	$x_2$	$x_3$	
1	1	-1	0
2	2	-2	0
1	1	-1	0
1	1	-1	0
1	1	-1	0

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$(A - 3I)X = O$$

$$\begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & -2 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$S(\lambda) = \{x \in V : f(x) = \lambda x\}$$

$$(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O$$

b)

$x_1$	$x_2$	$x_3$	
1	1	-1	0
2	2	-2	0
1	1	-1	0
1	1	-1	0
1	1	-1	0
1	1	-1	0

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$(A - 3I)X = O$$

$$\begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & -2 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$S(\lambda) = \{x \in V : f(x) = \lambda x\}$$

$$(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O$$

b)

$x_1$	$x_2$	$x_3$	
1	1	-1	0
2	2	-2	0
1	1	-1	0
1	1	-1	0
1	1	-1	0
1	1	-1	0

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$(A - 3I)X = O$$

$$\begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & -2 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$S(\lambda) = \{x \in V : f(x) = \lambda x\}$$

$$(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O$$

b)

$x_1$	$x_2$	$x_3$	
1	1	-1	0
2	2	-2	0
1	1	-1	0
1	1	-1	0
1	1	-1	0
1	1	-1	0
1	1	-1	0

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$(A - 3I)X = O$$

$$\begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & -2 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$S(\lambda) = \{x \in V : f(x) = \lambda x\}$$

$$(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O$$

b)

$$\begin{array}{ccc|c} x_1 & x_2 & x_3 & \\ \hline 1 & 1 & -1 & 0 \\ 2 & 2 & -2 & 0 \end{array} \quad /: 2$$

$$\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ \hline 1 & 1 & -1 & 0 \\ 1 & 1 & -1 & 0 \\ 1 & 1 & -1 & 0 \end{array}$$

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$$x_1 + x_2 - x_3 = 0 \quad \text{--->} \quad x_3 = x_1 + x_2$$

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$$(x_1, x_2, x_1 + x_2) = x_1 \cdot (1,$$

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$S(\lambda) = \{x \in V : f(x) = \lambda x\}$

$(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O$

b)  $\mathcal{B}_{S(3)} = \{(1, 0, 1), (0, 1, 1)\}$      $\dim S(3) = 2$      $A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$

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$$(x_1, x_2, x_1 + x_2) = x_1 \cdot (1, 0, 1) + x_2 \cdot (0, 1, 1)$$

$$S(\lambda) = \{x \in V : f(x) = \lambda x\}$$

$$(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O$$

b)  $\mathcal{B}_{S(3)} = \{(1, 0, 1), (0, 1, 1)\}$   $\dim S(3) = 2$

$$\begin{array}{ccc|c} x_1 & x_2 & x_3 & \\ \hline 1 & 1 & -1 & 0 \\ 2 & 2 & -2 & 0 \end{array} \quad / : 2$$

$$\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ \hline 1 & 1 & -1 & 0 \\ 1 & 1 & -1 & 0 \\ 1 & 1 & -1 & 0 \end{array}$$

$$\begin{array}{ccc|c} 1 & 1 & -1 & 0 \end{array}$$

geometrijska kratnost  
svojstvene vrijednosti  $\lambda = 3$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$(A - 3I)X = O$$

$$\begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & -2 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$x_1 + x_2 - x_3 = 0 \quad \rightsquigarrow x_3 = x_1 + x_2$$

$$S(3) = \{(x_1, x_2, x_1 + x_2) : x_1, x_2 \in \mathbb{R}\}$$

$$(x_1, x_2, x_1 + x_2) = x_1 \cdot (1, 0, 1) + x_2 \cdot (0, 1, 1)$$

$S(\lambda) = \{x \in V : f(x) = \lambda x\}$

$(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$S(\lambda) = \{x \in V : f(x) = \lambda x\}$$

$$(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O$$

$$(A - 5I)X = O \quad A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$S(\lambda) = \{x \in V : f(x) = \lambda x\}$$

$$(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O$$

$$(A - 5I)X = O \quad A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\left[ \quad \right]$$

$$S(\lambda) = \{x \in V : f(x) = \lambda x\}$$

$$(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O$$

$$(A - 5I)X = O \quad A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} -1 \\ \\ \end{bmatrix}$$

$$S(\lambda) = \{x \in V : f(x) = \lambda x\}$$

$$(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O$$

$$(A - 5I)X = O$$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} -1 \\ 0 \end{bmatrix}$$

$$S(\lambda) = \{x \in V : f(x) = \lambda x\}$$

$$(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O$$

$$(A - 5I)X = O \quad A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} -1 & & \\ & 0 & \\ & & -3 \end{bmatrix}$$

$$S(\lambda) = \{x \in V : f(x) = \lambda x\}$$

$$(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O$$

$$(A - 5I)X = O \quad A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 & -1 \\ 0 & -2 \\ -3 \end{bmatrix}$$

$$S(\lambda) = \{x \in V : f(x) = \lambda x\}$$

$$(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O$$

$$(A - 5I)X = O \quad A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix}$$

$$S(\lambda) = \{x \in V : f(x) = \lambda x\}$$

$$(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O$$

$$(A - 5I)X = O \quad A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$S(\lambda) = \{x \in V : f(x) = \lambda x\}$$

$$(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O$$

$$(A - 5I)X = O \quad A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$S(\lambda) = \{x \in V : f(x) = \lambda x\}$$

$$(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O$$

$x_1 \quad x_2 \quad x_3$ 

$$(A - 5I)X = O$$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$S(\lambda) = \{x \in V : f(x) = \lambda x\}$$

$$(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O$$

$x_1$	$x_2$	$x_3$	
-1	1	-1	0

$$(A - 5I)X = O$$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$S(\lambda) = \{x \in V : f(x) = \lambda x\}$$

$$(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O$$

$x_1$	$x_2$	$x_3$	
-1	1	-1	0
2	0	-2	0

$$(A - 5I)X = O$$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$S(\lambda) = \{x \in V : f(x) = \lambda x\}$$

$$(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O$$

$x_1$	$x_2$	$x_3$	
-1	1	-1	0
2	0	-2	0
1	1	-3	0

$$(A - 5I)X = O \quad A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$S(\lambda) = \{x \in V : f(x) = \lambda x\}$$

$$(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O$$

$x_1$	$x_2$	$x_3$	
-1	1	-1	0
2	0	-2	0
1	1	-3	0

$$(A - 5I)X = O \quad A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$S(\lambda) = \{x \in V : f(x) = \lambda x\}$$

$$(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O$$

$x_1$	$x_2$	$x_3$	
-1	1	-1	0
2	0	-2	0
1	1	-3	0

/: 2

$$(A - 5I)X = O \quad A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$S(\lambda) = \{x \in V : f(x) = \lambda x\}$$

$$(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O$$

$x_1$	$x_2$	$x_3$	
-1	1	-1	0
2	0	-2	0
1	1	-3	0
-1	1	-1	0

/ : 2

$$(A - 5I)X = O \quad A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$S(\lambda) = \{x \in V : f(x) = \lambda x\}$$

$$(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O$$

$x_1$	$x_2$	$x_3$	
-1	1	-1	0
2	0	-2	0
1	1	-3	0
/ : 2			
-1	1	-1	0
1	0	-1	0

$$(A - 5I)X = O$$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$S(\lambda) = \{x \in V : f(x) = \lambda x\}$$

$$(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O$$

$x_1$	$x_2$	$x_3$	
-1	1	-1	0
2	0	-2	0
1	1	-3	0
<hr/>			
-1	1	-1	0
1	0	-1	0
1	1	-3	0

$/: 2$

$$(A - 5I)X = O \quad A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$S(\lambda) = \{x \in V : f(x) = \lambda x\}$$

$$(F_B - \lambda I)X_B = O$$

$x_1$	$x_2$	$x_3$	
-1	1	-1	0
2	0	-2	0
1	1	-3	0
-1	1	-1	0
1	0	-1	0
1	1	-3	0

$/: 2$

$$(A - 5I)X = O \quad A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$S(\lambda) = \{x \in V : f(x) = \lambda x\}$$

$$(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O$$

$x_1$	$x_2$	$x_3$	
-1	1	-1	0
2	0	-2	0
1	1	-3	0
-1	1	-1	0
1	0	-1	0
1	1	-3	0

$/: 2$

$$(A - 5I)X = O \quad A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$S(\lambda) = \{x \in V : f(x) = \lambda x\}$$

$$(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O$$

$$\begin{array}{ccc|c}
 x_1 & x_2 & x_3 & \\
 \hline
 -1 & 1 & -1 & 0 \\
 2 & 0 & -2 & 0 \\
 1 & 1 & -3 & 0 \\
 \hline
 -1 & \textcircled{1} & -1 & 0 \\
 1 & 0 & -1 & 0 \\
 1 & 1 & -3 & 0
 \end{array}
 \quad /:2 \quad (A - 5I)X = O \quad A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$S(\lambda) = \{x \in V : f(x) = \lambda x\}$$

$$(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O$$

$x_1$	$x_2$	$x_3$	
-1	1	-1	0
2	0	-2	0
1	1	-3	0
-1	1	-1	0
1	0	-1	0
1	1	-3	0

$$(A - 5I)X = O \quad A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$S(\lambda) = \{x \in V : f(x) = \lambda x\}$$

$$(F_B - \lambda I)X_B = O$$

$x_1$	$x_2$	$x_3$	
-1	1	-1	0
2	0	-2	0
1	1	-3	0
-1	1	-1	0
1	0	-1	0
1	1	-3	0
-1	1	-1	0

$$(A - 5I)X = O \quad A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$S(\lambda) = \{x \in V : f(x) = \lambda x\}$$

$$(F_B - \lambda I)X_B = O$$

$$\begin{array}{ccc|c}
 x_1 & x_2 & x_3 & \\
 \hline
 -1 & 1 & -1 & 0 \\
 2 & 0 & -2 & 0 \\
 1 & 1 & -3 & 0 \\
 \hline
 -1 & \textcircled{1} & -1 & 0 \\
 1 & 0 & -1 & 0 \\
 1 & 1 & -3 & 0 \\
 \hline
 -1 & 1 & -1 & 0 \\
 1 & 0 & -1 & 0
 \end{array}
 \quad /:2 \quad / \cdot (-1) \quad + \quad
 \begin{matrix}
 (A - 5I)X = O \\
 A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix} \\
 \begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}
 \end{matrix}$$

$$S(\lambda) = \{x \in V : f(x) = \lambda x\}$$

$$(F_B - \lambda I)X_B = O$$

$x_1$	$x_2$	$x_3$	
-1	1	-1	0
2	0	-2	0
1	1	-3	0
<hr/>			
-1	1	-1	0
1	0	-1	0
1	1	-3	0
<hr/>			
-1	1	-1	0
1	0	-1	0
2			

$$(A - 5I)X = O$$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$S(\lambda) = \{x \in V : f(x) = \lambda x\}$$

$$(F_B - \lambda I)X_B = O$$

$x_1$	$x_2$	$x_3$	
-1	1	-1	0
2	0	-2	0
1	1	-3	0
<hr/>			
-1	1	-1	0
1	0	-1	0
1	1	-3	0
<hr/>			
-1	1	-1	0
1	0	-1	0
2	0		

$$(A - 5I)X = O \quad A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$S(\lambda) = \{x \in V : f(x) = \lambda x\}$$

$$(F_B - \lambda I)X_B = O$$

$$\begin{array}{ccc|c}
 x_1 & x_2 & x_3 & \\
 \hline
 -1 & 1 & -1 & 0 \\
 2 & 0 & -2 & 0 \\
 1 & 1 & -3 & 0 \\
 \hline
 -1 & \textcircled{1} & -1 & 0 \\
 1 & 0 & -1 & 0 \\
 1 & 1 & -3 & 0 \\
 \hline
 -1 & 1 & -1 & 0 \\
 1 & 0 & -1 & 0 \\
 2 & 0 & -2 &
 \end{array}
 \quad /:2 \quad / \cdot (-1) \quad$$

$(A - 5I)X = O$        $A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$

$\begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$$S(\lambda) = \{x \in V : f(x) = \lambda x\}$$

$$(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O$$

$$\begin{array}{ccc|c}
 x_1 & x_2 & x_3 & \\
 \hline
 -1 & 1 & -1 & 0 \\
 2 & 0 & -2 & 0 \\
 1 & 1 & -3 & 0 \\
 \hline
 -1 & \textcircled{1} & -1 & 0 \\
 1 & 0 & -1 & 0 \\
 1 & 1 & -3 & 0 \\
 \hline
 -1 & 1 & -1 & 0 \\
 1 & 0 & -1 & 0 \\
 2 & 0 & -2 & 0
 \end{array}
 \quad /:2 \quad / \cdot (-1) \quad
 \begin{matrix}
 \left[ \begin{matrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{matrix} \right] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}
 \end{matrix}$$

$$S(\lambda) = \{x \in V : f(x) = \lambda x\}$$

$$(F_B - \lambda I)X_B = O$$

$x_1$	$x_2$	$x_3$	
-1	1	-1	0
2	0	-2	0
1	1	-3	0
<hr/>			
-1	1	-1	0
1	0	-1	0
1	1	-3	0
<hr/>			
-1	1	-1	0
1	0	-1	0
2	0	-2	0
<hr/>			

$$(A - 5I)X = O$$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$S(\lambda) = \{x \in V : f(x) = \lambda x\}$$

$$(F_B - \lambda I)X_B = O$$

$$\begin{array}{ccc|c} x_1 & x_2 & x_3 & \\ \hline -1 & 1 & -1 & 0 \\ 2 & 0 & -2 & 0 \\ 1 & 1 & -3 & 0 \\ \hline -1 & \textcircled{1} & -1 & 0 \\ 1 & 0 & -1 & 0 \\ 1 & 1 & -3 & 0 \\ \hline -1 & 1 & -1 & 0 \\ 1 & 0 & -1 & 0 \\ 2 & 0 & -2 & 0 \\ \hline -1 & 1 & -1 & 0 \end{array} \quad /:2 \quad (A - 5I)X = O \quad A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$\begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$$S(\lambda) = \{x \in V : f(x) = \lambda x\}$$

$$(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O$$

$x_1$	$x_2$	$x_3$	
-1	1	-1	0
2	0	-2	0
1	1	-3	0
<hr/>			
-1	1	-1	0
1	0	-1	0
1	1	-3	0
<hr/>			
-1	1	-1	0
1	0	-1	0
2	0	-2	0
<hr/>			
-1	1	-1	0
1	0	-1	0

$$(A - 5I)X = O$$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$S(\lambda) = \{x \in V : f(x) = \lambda x\}$$

$$(F_B - \lambda I)X_B = O$$

$x_1$	$x_2$	$x_3$	
-1	1	-1	0
2	0	-2	0
1	1	-3	0
<hr/>			
-1	1	-1	0
1	0	-1	0
1	1	-3	0
<hr/>			
-1	1	-1	0
1	0	-1	0
2	0	-2	0
<hr/>			
-1	1	-1	0
1	0	-1	0
<hr/>			

$$(A - 5I)X = O$$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$S(\lambda) = \{x \in V : f(x) = \lambda x\}$$

$$(F_B - \lambda I)X_B = O$$

$x_1$	$x_2$	$x_3$	
-1	1	-1	0
2	0	-2	0
1	1	-3	0
<hr/>			
-1	1	-1	0
1	0	-1	0
1	1	-3	0
<hr/>			
-1	1	-1	0
1	0	-1	0
2	0	-2	0
<hr/>			
-1	1	-1	0
1	0	-1	0
<hr/>			

$$(A - 5I)X = O \quad A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$S(\lambda) = \{x \in V : f(x) = \lambda x\}$$

$$(F_B - \lambda I)X_B = O$$

$x_1$	$x_2$	$x_3$	
-1	1	-1	0
2	0	-2	0
1	1	-3	0
<hr/>			
-1	1	-1	0
1	0	-1	0
1	1	-3	0
<hr/>			
-1	1	-1	0
1	0	-1	0
2	0	-2	0
<hr/>			
-1	1	-1	0
1	0	-1	0

$$(A - 5I)X = O \quad A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$S(\lambda) = \{x \in V : f(x) = \lambda x\}$$

$$(F_B - \lambda I)X_B = O$$

$x_1$	$x_2$	$x_3$	
-1	1	-1	0
2	0	-2	0
1	1	-3	0
-1	1	-1	0
1	0	-1	0
1	1	-3	0
-1	1	-1	0
1	0	-1	0
2	0	-2	0
-1	1	-1	0
1	0	-1	0

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-1	1	-1	0
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1	1	-3	0
-1	1	-1	0
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1	1	-3	0
-1	1	-1	0
1	0	-1	0
2	0	-2	0
-1	1	-1	0
1	0	-1	0
1	0	-1	0

$$(A - 5I)X = O$$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

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$$(F_B - \lambda I)X_B = O$$

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-1	1	-1	0
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-1	1	-1	0
1	0	-1	0
1	1	-3	0
-1	1	-1	0
1	0	-1	0
2	0	-2	0
-1	1	-1	0
1	0	-1	0
0			
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$$(F_B - \lambda I)X_B = O$$

$x_1$	$x_2$	$x_3$	
-1	1	-1	0
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1	1	-3	0
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1	0	-1	0
1	1	-3	0
-1	1	-1	0
1	0	-1	0
2	0	-2	0
-1	1	-1	0
1	0	-1	0
0	1		
1	0	-1	0

$$(A - 5I)X = O$$

$$\begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

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$$(F_B - \lambda I)X_B = O$$

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-1	1	-1	0
2	0	-2	0
1	1	-3	0
-1	1	-1	0
1	0	-1	0
1	1	-3	0
-1	1	-1	0
1	0	-1	0
2	0	-2	0
-1	1	-1	0
1	0	-1	0
0	1	-2	
1	0	-1	0

$$(A - 5I)X = O$$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$S(\lambda) = \{x \in V : f(x) = \lambda x\}$$

$$(F_B - \lambda I)X_B = O$$

$x_1$	$x_2$	$x_3$	
-1	1	-1	0
2	0	-2	0
1	1	-3	0
-1	1	-1	0
1	0	-1	0
1	1	-3	0
-1	1	-1	0
1	0	-1	0
2	0	-2	0
-1	1	-1	0
1	0	-1	0
0	1	-2	0
1	0	-1	0

$$(A - 5I)X = O$$

$$\begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$S(\lambda) = \{x \in V : f(x) = \lambda x\}$$

$$(F_B - \lambda I)X_B = O$$

$$\begin{array}{ccc|c} x_1 & x_2 & x_3 & \\ \hline -1 & 1 & -1 & 0 \\ 2 & 0 & -2 & 0 \\ 1 & 1 & -3 & 0 \\ \hline -1 & 1 & -1 & 0 \\ 1 & 0 & -1 & 0 \\ 1 & 1 & -3 & 0 \\ \hline -1 & 1 & -1 & 0 \\ 1 & 0 & -1 & 0 \\ 2 & 0 & -2 & 0 \\ \hline -1 & 1 & -1 & 0 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & -2 & 0 \\ 1 & 0 & -1 & 0 \end{array}$$

$(A - 5I)X = O$

$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$

$\begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$x_2 - 2x_3 = 0$

$$S(\lambda) = \{x \in V : f(x) = \lambda x\}$$

$$(F_B - \lambda I)X_B = O$$

$$\begin{array}{ccc|c}
 x_1 & x_2 & x_3 & \\
 \hline
 -1 & 1 & -1 & 0 \\
 2 & 0 & -2 & 0 \\
 1 & 1 & -3 & 0 \\
 \hline
 -1 & 1 & -1 & 0 \\
 1 & 0 & -1 & 0 \\
 1 & 1 & -3 & 0 \\
 \hline
 -1 & 1 & -1 & 0 \\
 1 & 0 & -1 & 0 \\
 2 & 0 & -2 & 0 \\
 \hline
 -1 & 1 & -1 & 0 \\
 1 & 0 & -1 & 0 \\
 0 & 1 & -2 & 0 \\
 1 & 0 & -1 & 0
 \end{array}$$

$(A - 5I)X = O$

$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$

$\begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$x_2 - 2x_3 = 0$

$x_1 - x_3 = 0$

$\begin{array}{ccc|c} & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \end{array}$

$$S(\lambda) = \{x \in V : f(x) = \lambda x\}$$

$$(F_B - \lambda I)X_B = O$$

$x_1$	$x_2$	$x_3$	
-1	1	-1	0
2	0	-2	0
1	1	-3	0
-1	1	-1	0
1	0	-1	0
1	1	-3	0
-1	1	-1	0
1	0	-1	0
2	0	-2	0
-1	1	-1	0
1	0	-1	0
0	1	-2	0
1	0	-1	0

$$(A - 5I)X = O \quad A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$x_2 - 2x_3 = 0 \} \\ x_1 - x_3 = 0 \}$$

$$S(\lambda) = \{x \in V : f(x) = \lambda x\}$$

$$(F_B - \lambda I)X_B = O$$

$x_1$	$x_2$	$x_3$	
-1	1	-1	0
2	0	-2	0
1	1	-3	0
-1	1	-1	0
1	0	-1	0
1	1	-3	0
-1	1	-1	0
1	0	-1	0
2	0	-2	0
-1	1	-1	0
1	0	-1	0
0	1	-2	0
1	0	-1	0

$$(A - 5I)X = O \quad A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$x_2 - 2x_3 = 0 \quad x_2 = 2x_3$$

$$x_1 - x_3 = 0 \quad x_1 = x_3$$

$$S(\lambda) = \{x \in V : f(x) = \lambda x\}$$

$$(F_B - \lambda I)X_B = O$$

$x_1$	$x_2$	$x_3$	
-1	1	-1	0
2	0	-2	0
1	1	-3	0
-1	1	-1	0
1	0	-1	0
1	1	-3	0
-1	1	-1	0
1	0	-1	0
2	0	-2	0
-1	1	-1	0
1	0	-1	0
0	1	-2	0
1	0	-1	0

$$(A - 5I)X = O \quad A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$x_2 - 2x_3 = 0 \quad \left. \begin{array}{l} \xrightarrow{\text{---}} x_2 = 2x_3 \\ x_1 - x_3 = 0 \quad \left. \begin{array}{l} \xrightarrow{\text{---}} x_1 = x_3 \end{array} \right. \end{array} \right\}$$

$$S(\lambda) = \{x \in V : f(x) = \lambda x\}$$

$$(F_B - \lambda I)X_B = O$$

$x_1$	$x_2$	$x_3$	
-1	1	-1	0
2	0	-2	0
1	1	-3	0
-1	1	-1	0
1	0	-1	0
1	1	-3	0
-1	1	-1	0
1	0	-1	0
2	0	-2	0
-1	1	-1	0
1	0	-1	0
0	1	-2	0
1	0	-1	0

$$(A - 5I)X = O \quad A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\left. \begin{array}{l} x_2 - 2x_3 = 0 \\ x_1 - x_3 = 0 \end{array} \right\} \begin{array}{l} \rightsquigarrow x_2 = 2x_3 \\ \rightsquigarrow x_1 = x_3 \end{array}$$

$$S(5) =$$

$$S(\lambda) = \{x \in V : f(x) = \lambda x\}$$

$$(F_B - \lambda I)X_B = O$$

$x_1$	$x_2$	$x_3$	
-1	1	-1	0
2	0	-2	0
1	1	-3	0
-1	1	-1	0
1	0	-1	0
1	1	-3	0
-1	1	-1	0
1	0	-1	0
2	0	-2	0
-1	1	-1	0
1	0	-1	0
0	1	-2	0
1	0	-1	0

$$(A - 5I)X = O \quad A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$x_2 - 2x_3 = 0 \quad \left. \begin{array}{l} \xrightarrow{\text{---}} x_2 = 2x_3 \\ x_1 - x_3 = 0 \quad \left. \begin{array}{l} \xrightarrow{\text{---}} x_1 = x_3 \end{array} \right. \end{array} \right\}$$

$$S(5) = \{($$

$$S(\lambda) = \{x \in V : f(x) = \lambda x\}$$

$$(F_B - \lambda I)X_B = O$$

$x_1$	$x_2$	$x_3$	
-1	1	-1	0
2	0	-2	0
1	1	-3	0
-1	1	-1	0
1	0	-1	0
1	1	-3	0
-1	1	-1	0
1	0	-1	0
2	0	-2	0
-1	1	-1	0
1	0	-1	0
0	1	-2	0
1	0	-1	0

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$$\left. \begin{array}{l} x_2 - 2x_3 = 0 \\ x_1 - x_3 = 0 \end{array} \right\} \begin{array}{l} \rightsquigarrow x_2 = 2x_3 \\ \rightsquigarrow x_1 = x_3 \end{array}$$

$$S(5) = \{(x_3,$$

$$S(\lambda) = \{x \in V : f(x) = \lambda x\}$$

$$(F_B - \lambda I)X_B = O$$

$x_1$	$x_2$	$x_3$	
-1	1	-1	0
2	0	-2	0
1	1	-3	0
-1	1	-1	0
1	0	-1	0
1	1	-3	0
-1	1	-1	0
1	0	-1	0
2	0	-2	0
-1	1	-1	0
1	0	-1	0
0	1	-2	0
1	0	-1	0

$$(A - 5I)X = O \quad A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\left. \begin{array}{l} x_2 - 2x_3 = 0 \\ x_1 - x_3 = 0 \end{array} \right\} \xrightarrow{\text{wavy}} \begin{array}{l} x_2 = 2x_3 \\ x_1 = x_3 \end{array}$$

$$S(5) = \{(x_3, 2x_3,$$

$$S(\lambda) = \{x \in V : f(x) = \lambda x\}$$

$$(F_B - \lambda I)X_B = O$$

$x_1$	$x_2$	$x_3$	
-1	1	-1	0
2	0	-2	0
1	1	-3	0
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1	0	-1	0
1	1	-3	0
-1	1	-1	0
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1	0	-1	0
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$$(A - 5I)X = O$$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

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$$\left. \begin{array}{l} x_2 - 2x_3 = 0 \\ x_1 - x_3 = 0 \end{array} \right\} \begin{array}{l} \rightsquigarrow x_2 = 2x_3 \\ \rightsquigarrow x_1 = x_3 \end{array}$$

$$S(5) = \{(x_3, 2x_3, x_3)$$

$$S(\lambda) = \{x \in V : f(x) = \lambda x\}$$

$$(F_B - \lambda I)X_B = O$$

$x_1$	$x_2$	$x_3$	
-1	1	-1	0
2	0	-2	0
1	1	-3	0
-1	1	-1	0
1	0	-1	0
1	1	-3	0
-1	1	-1	0
1	0	-1	0
2	0	-2	0
-1	1	-1	0
1	0	-1	0
0	1	-2	0
1	0	-1	0

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$$\begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$x_2 - 2x_3 = 0 \quad \left. \begin{array}{l} \xrightarrow{\text{---}} x_2 = 2x_3 \\ x_1 - x_3 = 0 \quad \left. \begin{array}{l} \xrightarrow{\text{---}} x_1 = x_3 \end{array} \right. \end{array} \right\}$$

$$S(5) = \{(x_3, 2x_3, x_3) : x_3 \in \mathbb{R}\}$$

$$S(\lambda) = \{x \in V : f(x) = \lambda x\}$$

$$(F_B - \lambda I)X_B = O$$

$x_1$	$x_2$	$x_3$	
-1	1	-1	0
2	0	-2	0
1	1	-3	0
-1	1	-1	0
1	0	-1	0
1	1	-3	0
-1	1	-1	0
1	0	-1	0
2	0	-2	0
-1	1	-1	0
1	0	-1	0
0	1	-2	0
1	0	-1	0

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$$\begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\left. \begin{array}{l} x_2 - 2x_3 = 0 \\ x_1 - x_3 = 0 \end{array} \right\} \begin{array}{l} \rightsquigarrow x_2 = 2x_3 \\ \rightsquigarrow x_1 = x_3 \end{array}$$

$$S(5) = \{(x_3, 2x_3, x_3) : x_3 \in \mathbb{R}\}$$

$$(x_3, 2x_3, x_3) =$$

$$S(\lambda) = \{x \in V : f(x) = \lambda x\}$$

$$(F_B - \lambda I)X_B = O$$

$x_1$	$x_2$	$x_3$	
-1	1	-1	0
2	0	-2	0
1	1	-3	0
-1	1	-1	0
1	0	-1	0
1	1	-3	0
-1	1	-1	0
1	0	-1	0
2	0	-2	0
-1	1	-1	0
1	0	-1	0
0	1	-2	0
1	0	-1	0

$$(A - 5I)X = O$$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{aligned} x_2 - 2x_3 &= 0 \\ x_1 - x_3 &= 0 \end{aligned} \left. \begin{array}{l} \xrightarrow{\text{---}} x_2 = 2x_3 \\ \xrightarrow{\text{---}} x_1 = x_3 \end{array} \right.$$

$$S(5) = \{(x_3, 2x_3, x_3) : x_3 \in \mathbb{R}\}$$

$$(x_3, 2x_3, x_3) = x_3 \cdot ($$

$$S(\lambda) = \{x \in V : f(x) = \lambda x\}$$

$$(F_B - \lambda I)X_B = O$$

$x_1$	$x_2$	$x_3$	
-1	1	-1	0
2	0	-2	0
1	1	-3	0
-1	1	-1	0
1	0	-1	0
1	1	-3	0
-1	1	-1	0
1	0	-1	0
2	0	-2	0
-1	1	-1	0
1	0	-1	0
0	1	-2	0
1	0	-1	0

$$(A - 5I)X = O \quad A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$x_2 - 2x_3 = 0 \quad \left. \begin{array}{l} \xrightarrow{\text{---}} x_2 = 2x_3 \\ x_1 - x_3 = 0 \quad \left. \begin{array}{l} \xrightarrow{\text{---}} x_1 = x_3 \end{array} \right. \end{array} \right\}$$

$$S(5) = \{(x_3, 2x_3, x_3) : x_3 \in \mathbb{R}\}$$

$$(x_3, 2x_3, x_3) = x_3 \cdot (1,$$

$$S(\lambda) = \{x \in V : f(x) = \lambda x\}$$

$$(F_B - \lambda I)X_B = O$$

$x_1$	$x_2$	$x_3$	
-1	1	-1	0
2	0	-2	0
1	1	-3	0
-1	1	-1	0
1	0	-1	0
1	1	-3	0
-1	1	-1	0
1	0	-1	0
2	0	-2	0
-1	1	-1	0
1	0	-1	0
0	1	-2	0
1	0	-1	0

$$(A - 5I)X = O$$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{aligned} x_2 - 2x_3 &= 0 \\ x_1 - x_3 &= 0 \end{aligned} \left\{ \begin{array}{l} \rightsquigarrow x_2 = 2x_3 \\ \rightsquigarrow x_1 = x_3 \end{array} \right.$$

$$S(5) = \{(x_3, 2x_3, x_3) : x_3 \in \mathbb{R}\}$$

$$(x_3, 2x_3, x_3) = x_3 \cdot (1, 2,$$

$$S(\lambda) = \{x \in V : f(x) = \lambda x\}$$

$$(F_B - \lambda I)X_B = O$$

$x_1$	$x_2$	$x_3$	
-1	1	-1	0
2	0	-2	0
1	1	-3	0
-1	1	-1	0
1	0	-1	0
1	1	-3	0
-1	1	-1	0
1	0	-1	0
2	0	-2	0
-1	1	-1	0
1	0	-1	0
0	1	-2	0
1	0	-1	0

$$(A - 5I)X = O$$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{aligned} x_2 - 2x_3 &= 0 \\ x_1 - x_3 &= 0 \end{aligned} \left\{ \begin{array}{l} \rightsquigarrow x_2 = 2x_3 \\ \rightsquigarrow x_1 = x_3 \end{array} \right.$$

$$S(5) = \{(x_3, 2x_3, x_3) : x_3 \in \mathbb{R}\}$$

$$(x_3, 2x_3, x_3) = x_3 \cdot (1, 2, 1)$$

$$S(\lambda) = \{x \in V : f(x) = \lambda x\}$$

$$(F_B - \lambda I)X_B = O$$

$x_1$	$x_2$	$x_3$				
-1	1	-1	0			
2	0	-2	0	/ : 2		
1	1	-3	0			
-1	1	-1	0	/ \cdot (-1)		
1	0	-1	0			
1	1	-3	0	+ ↪		
-1	1	-1	0			
1	0	-1	0			
2	0	-2	0			
-1	1	-1	0	+ ↪		
1	0	-1	0	/ \cdot 1		
0	1	-2	0			
1	0	-1	0			

$(A - 5I)X = O$

$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$

$\begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$x_2 - 2x_3 = 0 \quad \left. \begin{array}{l} \xrightarrow{\hspace{1cm}} \\ \xrightarrow{\hspace{1cm}} \end{array} \right\} x_2 = 2x_3$

$x_1 - x_3 = 0 \quad \left. \begin{array}{l} \xrightarrow{\hspace{1cm}} \\ \xrightarrow{\hspace{1cm}} \end{array} \right\} x_1 = x_3$

$S(5) = \{(x_3, 2x_3, x_3) : x_3 \in \mathbb{R}\}$

$(x_3, 2x_3, x_3) = x_3 \cdot (1, 2, 1)$

$\mathcal{B}_{S(5)} = \{(1, 2, 1)\}$

$$S(\lambda) = \{x \in V : f(x) = \lambda x\}$$

$$(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O$$

$x_1$	$x_2$	$x_3$				
-1	1	-1	0			
2	0	-2	0	/: 2		
1	1	-3	0			
-1	1	-1	0	/·(-1)		
1	0	-1	0			
1	1	-3	0	+ ↪		
-1	1	-1	0			
1	0	-1	0			
2	0	-2	0			
-1	1	-1	0	+ ↪		
1	0	-1	0	/·1		
0	1	-2	0			
1	0	-1	0			

$(A - 5I)X = O$

$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$

$\begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$x_2 - 2x_3 = 0 \quad \left. \begin{array}{l} \xrightarrow{\hspace{1cm}} \\ \xrightarrow{\hspace{1cm}} \end{array} \right\} x_2 = 2x_3$

$x_1 - x_3 = 0 \quad \left. \begin{array}{l} \xrightarrow{\hspace{1cm}} \\ \xrightarrow{\hspace{1cm}} \end{array} \right\} x_1 = x_3$

$S(5) = \{(x_3, 2x_3, x_3) : x_3 \in \mathbb{R}\}$

$(x_3, 2x_3, x_3) = x_3 \cdot (1, 2, 1)$

$\dim S(5) = 1$

$\mathcal{B}_{S(5)} = \{(1, 2, 1)\}$

$$S(\lambda) = \{x \in V : f(x) = \lambda x\}$$

$$(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O$$

$x_1$	$x_2$	$x_3$	
-1	1	-1	0
2	0	-2	0
1	1	-3	0
-1	1	-1	0
1	0	-1	0
1	1	-3	0
-1	1	-1	0
1	0	-1	0
2	0	-2	0
-1	1	-1	0
1	0	-1	0
0	1	-2	0
1	0	-1	0

$$(A - 5I)X = O \quad A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{aligned} x_2 - 2x_3 &= 0 \\ x_1 - x_3 &= 0 \end{aligned} \left\{ \begin{array}{l} \rightsquigarrow x_2 = 2x_3 \\ \rightsquigarrow x_1 = x_3 \end{array} \right.$$

$$S(5) = \{(x_3, 2x_3, x_3) : x_3 \in \mathbb{R}\}$$

$$(x_3, 2x_3, x_3) = x_3 \cdot (1, 2, 1) \quad \dim S(5) = 1$$

$$\mathcal{B}_{S(5)} = \{(1, 2, 1)\}$$

geometrijska kratnost  
svojstvene vrijednosti  $\lambda = 5$

$$S(\lambda) = \{x \in V : f(x) = \lambda x\}$$

$$(F_{\mathcal{B}} - \lambda I)X_{\mathcal{B}} = O$$

c)

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$c) \quad k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$c) \quad k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$

$$k_A(\lambda) = (\lambda - 3)^2 \cdot (\lambda - 5)$$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$c) \quad k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$

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$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$(A - 3I) \cdot (A - 5I) =$$

$$c) \quad k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$

$$k_A(\lambda) = (\lambda - 3)^2 \cdot (\lambda - 5)$$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$(A - 3I) \cdot (A - 5I) = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$$

$$c) \quad k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$

$$k_A(\lambda) = (\lambda - 3)^2 \cdot (\lambda - 5)$$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$(A - 3I) \cdot (A - 5I) = \begin{bmatrix} 1 \\ & \\ & \end{bmatrix}$$

$$c) \quad k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$

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$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$(A - 3I) \cdot (A - 5I) = \begin{bmatrix} 1 & & \\ & 2 & \\ & & \end{bmatrix}$$

$$c) \quad k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$

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$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$(A - 3I) \cdot (A - 5I) = \begin{bmatrix} 1 & & \\ & 2 & \\ & & -1 \end{bmatrix}$$

$$c) \quad k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$

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$$c) \quad k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$

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$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$(A - 3I) \cdot (A - 5I) = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & -2 \\ 1 & 1 & -1 \end{bmatrix}$$

$$c) \quad k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$

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$$c) \quad k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$

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$$(A - 3I) \cdot (A - 5I) = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & -2 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} -1 \\ \\ \end{bmatrix}$$

$$c) \quad k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$

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$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

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$$c) \quad k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$

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$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$(A - 3I) \cdot (A - 5I) = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & -2 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} -1 & & \\ & 0 & \\ & & -3 \end{bmatrix}$$

$$c) \quad k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$

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$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

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$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

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$$c) \quad k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$

$$k_A(\lambda) = (\lambda - 3)^2 \cdot (\lambda - 5)$$

$$m_A(\lambda) = (\lambda - 3) \cdot (\lambda - 5)$$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$(A - 3I) \cdot (A - 5I) = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & -2 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

c)  $k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$

$$k_A(\lambda) = (\lambda - 3)^2 \cdot (\lambda - 5)$$
$$m_A(\lambda) = (\lambda - 3) \cdot (\lambda - 5) \quad m_A(\lambda) = \lambda^2 - 8\lambda + 15$$
$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$(A - 3I) \cdot (A - 5I) = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & -2 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$c) \quad k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$

$$k_A(\lambda) = (\lambda - 3)^2 \cdot (\lambda - 5)$$

$$m_A(\lambda) = (\lambda - 3) \cdot (\lambda - 5)$$

$$m_A(\lambda) = \lambda^2 - 8\lambda + 15$$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$(A - 3I) \cdot (A - 5I) = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & -2 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

c)  $k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$

$$k_A(\lambda) = (\lambda - 3)^2 \cdot (\lambda - 5)$$

$$m_A(\lambda) = (\lambda - 3) \cdot (\lambda - 5)$$

$$m_A(\lambda) = \lambda^2 - 8\lambda + 15$$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$(A - 3I) \cdot (A - 5I) = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & -2 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

---

d)

c)  $k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$

$$k_A(\lambda) = (\lambda - 3)^2 \cdot (\lambda - 5)$$
$$m_A(\lambda) = (\lambda - 3) \cdot (\lambda - 5)$$
$$m_A(\lambda) = \lambda^2 - 8\lambda + 15$$
$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$(A - 3I) \cdot (A - 5I) = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & -2 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

---

d)  
 $k_A(A) = O$

c)  $k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$

$$k_A(\lambda) = (\lambda - 3)^2 \cdot (\lambda - 5)$$
$$m_A(\lambda) = (\lambda - 3) \cdot (\lambda - 5)$$
$$m_A(\lambda) = \lambda^2 - 8\lambda + 15$$

$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$

$$(A - 3I) \cdot (A - 5I) = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & -2 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

---

d)  $k_A(A) = O$

$$A^3 - 11A^2 + 39A - 45I = O$$

c)  $k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$

$$k_A(\lambda) = (\lambda - 3)^2 \cdot (\lambda - 5)$$
$$m_A(\lambda) = (\lambda - 3) \cdot (\lambda - 5)$$
$$m_A(\lambda) = \lambda^2 - 8\lambda + 15$$

$$(A - 3I) \cdot (A - 5I) = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & -2 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

---

d)  $k_A(A) = O$

$$A^3 - 11A^2 + 39A - 45I = O$$

$$45I =$$

c)  $k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$

$$k_A(\lambda) = (\lambda - 3)^2 \cdot (\lambda - 5)$$

$$m_A(\lambda) = (\lambda - 3) \cdot (\lambda - 5)$$

$$m_A(\lambda) = \lambda^2 - 8\lambda + 15$$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$(A - 3I) \cdot (A - 5I) = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & -2 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

---

d)

$$k_A(A) = O$$

$$A^3 - 11A^2 + 39A - 45I = O$$

$$45I = A^3 - 11A^2 + 39A$$

c)  $k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$

$$k_A(\lambda) = (\lambda - 3)^2 \cdot (\lambda - 5)$$

$$m_A(\lambda) = (\lambda - 3) \cdot (\lambda - 5)$$

$$m_A(\lambda) = \lambda^2 - 8\lambda + 15$$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$(A - 3I) \cdot (A - 5I) = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & -2 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

---

d)

$$k_A(A) = O$$

$$A^3 - 11A^2 + 39A - 45I = O$$

$$45I = A^3 - 11A^2 + 39A \quad / \cdot A^{-1}$$

$$c) \quad k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$

$$k_A(\lambda) = (\lambda - 3)^2 \cdot (\lambda - 5)$$

$$m_A(\lambda) = (\lambda - 3) \cdot (\lambda - 5)$$

$$m_A(\lambda) = \lambda^2 - 8\lambda + 15$$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$(A - 3I) \cdot (A - 5I) = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & -2 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

d)

$$k_A(A) = O$$

$$A^3 - 11A^2 + 39A - 45I = O$$

$$45I = A^3 - 11A^2 + 39A \quad / \cdot A^{-1}$$

$$45A^{-1} =$$

$$c) \quad k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$$

$$k_A(\lambda) = (\lambda - 3)^2 \cdot (\lambda - 5)$$

$$m_A(\lambda) = (\lambda - 3) \cdot (\lambda - 5)$$

$$m_A(\lambda) = \lambda^2 - 8\lambda + 15$$

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$(A - 3I) \cdot (A - 5I) = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & -2 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

d)

$$k_A(A) = O$$

$$A^3 - 11A^2 + 39A - 45I = O$$

$$45I = A^3 - 11A^2 + 39A \quad / \cdot A^{-1}$$

$$45A^{-1} = A^2$$

c)  $k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$

$$k_A(\lambda) = (\lambda - 3)^2 \cdot (\lambda - 5)$$
$$m_A(\lambda) = (\lambda - 3) \cdot (\lambda - 5)$$
$$m_A(\lambda) = \lambda^2 - 8\lambda + 15$$
$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$(A - 3I) \cdot (A - 5I) = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & -2 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

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d)  $k_A(A) = O$

$$A^3 - 11A^2 + 39A - 45I = O$$

$$45I = A^3 - 11A^2 + 39A \quad / \cdot A^{-1}$$

$$45A^{-1} = A^2 - 11A$$

c)  $k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$

$$k_A(\lambda) = (\lambda - 3)^2 \cdot (\lambda - 5)$$
$$m_A(\lambda) = (\lambda - 3) \cdot (\lambda - 5)$$
$$m_A(\lambda) = \lambda^2 - 8\lambda + 15$$
$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$(A - 3I) \cdot (A - 5I) = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & -2 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

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d)  $k_A(A) = O$

$$A^3 - 11A^2 + 39A - 45I = O$$

$$45I = A^3 - 11A^2 + 39A \quad / \cdot A^{-1}$$

$$45A^{-1} = A^2 - 11A + 39I$$

c)  $k_A(\lambda) = \lambda^3 - 11\lambda^2 + 39\lambda - 45$

$$k_A(\lambda) = (\lambda - 3)^2 \cdot (\lambda - 5)$$
$$m_A(\lambda) = (\lambda - 3) \cdot (\lambda - 5)$$
$$m_A(\lambda) = \lambda^2 - 8\lambda + 15$$

$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$

$$(A - 3I) \cdot (A - 5I) = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & -2 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

---

d)  $k_A(A) = O$

$$A^3 - 11A^2 + 39A - 45I = O$$

$$45I = A^3 - 11A^2 + 39A \quad / \cdot A^{-1}$$

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## **treći zadatak**

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### Zadatak 3

Postoji li linearni operator  $f : \mathbb{R}^3 \rightarrow \mathbb{R}^2$  za kojeg vrijedi

$$f(1, 0, 0) = (1, 0), \quad f(0, 1, 0) = (1, 3), \quad f(1, 1, 1) = (2, 4)?$$

Ako postoji, odredite u tom slučaju  $f(0, 0, 1)$  i njegovu matricu u paru kanonskih baza.

Rješenje

$$f : \mathbb{R}^3 \rightarrow \mathbb{R}^2$$

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## Rješenje

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## Rješenje

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$$f(1, 0, 0) = (1, 0), \quad f(0, 1, 0) = (1, 3), \quad f(1, 1, 1) = (2, 4)$$

$\mathcal{B} = \{(1, 0, 0), (0, 1, 0), (1, 1, 1)\}$  je baza za  $\mathbb{R}^3$ .

Svaki linearni operator zadan je svojim djelovanjem na nekoj bazi. Postoji jedinstveni linearni operator  $f$  koji zadovoljava zadane uvjete.

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$f(0, 0, 1) = ?$$

$$(0, 0, 1) = \alpha_1 \cdot (1, 0, 0) + \alpha_2 \cdot (0, 1, 0) + \alpha_3 \cdot (1, 1, 1)$$

$$\left. \begin{array}{l} \alpha_1 + \alpha_3 = 0 \\ \alpha_2 + \alpha_3 = 0 \\ \alpha_3 = 1 \end{array} \right\} \begin{array}{l} \alpha_2 = -1 \\ \alpha_3 = 1 \end{array} \left. \begin{array}{l} \alpha_1 = -1 \\ \alpha_3 = 1 \end{array} \right\}$$

$$f(0, 0, 1) = f(-1 \cdot (1, 0, 0) + (-1) \cdot (0, 1, 0) + 1 \cdot (1, 1, 1))$$

## Rješenje

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$f$  je linearni operator

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$$\begin{aligned} f(0, 0, 1) &= f(-1 \cdot (1, 0, 0) + (-1) \cdot (0, 1, 0) + 1 \cdot (1, 1, 1)) = \\ &= -1 \cdot f(1, 0, 0) + (-1) \cdot f(0, 1, 0) + 1 \cdot f(1, 1, 1) = \\ &= -(1, 0) \end{aligned}$$

## Rješenje

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## Rješenje

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## Rješenje

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$$\begin{aligned} f(0, 0, 1) &= f(-1 \cdot (1, 0, 0) + (-1) \cdot (0, 1, 0) + 1 \cdot (1, 1, 1)) = \\ &= -1 \cdot f(1, 0, 0) + (-1) \cdot f(0, 1, 0) + 1 \cdot f(1, 1, 1) = \\ &= -(1, 0) - (1, 3) + (2, 4) = (0, 1) \end{aligned}$$

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$$F_{(\mathcal{B},\,\mathcal{A}_{\texttt{kan}})}=\left[\begin{array}{c}\\ \\ \end{array}\right]$$

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$$F_{(\mathcal{B},\,\mathcal{A}_{\texttt{kan}})}=\begin{bmatrix}1\\0\end{bmatrix}$$

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$$F_{(\mathcal{B},\,\mathcal{A}_{\texttt{kan}})}=\begin{bmatrix}1&1\\0&3\end{bmatrix}$$

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$$Y_{\mathcal{A}_{\text{kan}}} = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 3 & 4 \end{bmatrix} \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$X_{\mathcal{B}} = \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$$

$$\begin{aligned} f(0, 0, 1) &= f(-1 \cdot (1, 0, 0) + (-1) \cdot (0, 1, 0) + 1 \cdot (1, 1, 1)) = \\ &= -1 \cdot f(1, 0, 0) + (-1) \cdot f(0, 1, 0) + 1 \cdot f(1, 1, 1) = \\ &= -(1, 0) - (1, 3) + (2, 4) = (0, 1) \end{aligned}$$

$$f:\mathbb{R}^3\rightarrow \mathbb{R}^2$$

$$f(1,0,0) = (1,0), \quad f(0,1,0) = (1,3), \quad f(1,1,1) = (2,4)$$

$$\mathcal{B} = \{(1,0,0), (0,1,0), (1,1,1)\} \qquad \mathcal{A}_{\text{kan}} = \{(1,0), (0,1)\}$$

$$F_{(\mathcal{B}, \mathcal{A}_{\text{kan}})} = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 3 & 4 \end{bmatrix}$$

$$f : \mathbb{R}^3 \rightarrow \mathbb{R}^2$$

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$$F_{(\mathcal{B}_{\text{kan}}, \mathcal{A}_{\text{kan}})} = T^{-1} F_{(\mathcal{B}, \mathcal{A}_{\text{kan}})} S$$

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$$\mathcal{B} \xrightarrow{S} \mathcal{B}_{\text{kan}} \quad \mathcal{A}_{\text{kan}} \xrightarrow{T} \mathcal{A}_{\text{kan}}$$

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$$F_{(\mathcal{B}_{\text{kan}}, \mathcal{A}_{\text{kan}})} = T^{-1} F_{(\mathcal{B}, \mathcal{A}_{\text{kan}})} S$$

$$\begin{array}{ccc} \mathcal{B} & \xrightarrow{S} & \mathcal{B}_{\text{kan}} \\ & \searrow & \downarrow S^{-1} \\ & & \mathcal{A}_{\text{kan}} \xrightarrow{T} \mathcal{A}_{\text{kan}} \end{array}$$

$$f : \mathbb{R}^3 \rightarrow \mathbb{R}^2$$

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$$S^{-1} = \left[ \quad \right]$$

$$\mathcal{B} \xrightarrow[S]{\quad} \mathcal{B}_{\text{kan}} \quad \mathcal{A}_{\text{kan}} \xrightarrow[T]{\quad} \mathcal{A}_{\text{kan}}$$

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$$F_{(\mathcal{B}_{\text{kan}}, \mathcal{A}_{\text{kan}})} = T^{-1} F_{(\mathcal{B}, \mathcal{A}_{\text{kan}})} S \quad S^{-1} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\mathcal{B} \xrightarrow[S]{\quad} \mathcal{B}_{\text{kan}} \quad \mathcal{A}_{\text{kan}} \xrightarrow[T]{\quad} \mathcal{A}_{\text{kan}}$$

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$$S^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$\mathcal{B} \xrightarrow[S]{\quad\quad\quad} \mathcal{B}_{\text{kan}} \quad \mathcal{A}_{\text{kan}} \xrightarrow[T]{\quad\quad\quad} \mathcal{A}_{\text{kan}}$$

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$$F_{(\mathcal{B}_{\text{kan}}, \mathcal{A}_{\text{kan}})} = T^{-1} F_{(\mathcal{B}, \mathcal{A}_{\text{kan}})} S$$

$$S^{-1} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathcal{B} \xrightarrow[S]{\quad\quad\quad} \mathcal{B}_{\text{kan}} \quad \mathcal{A}_{\text{kan}} \xrightarrow[T]{\quad\quad\quad} \mathcal{A}_{\text{kan}}$$

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$$F_{(\mathcal{B}_{\text{kan}}, \mathcal{A}_{\text{kan}})} = T^{-1} F_{(\mathcal{B}, \mathcal{A}_{\text{kan}})} S$$

$$S^{-1} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \quad S = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathcal{B} \xrightarrow[S]{\quad\quad\quad} \mathcal{B}_{\text{kan}} \quad \mathcal{A}_{\text{kan}} \xrightarrow[T]{\quad\quad\quad} \mathcal{A}_{\text{kan}}$$

$\swarrow S^{-1}$

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$$F_{(\mathcal{B}_{\text{kan}}, \mathcal{A}_{\text{kan}})} = T^{-1} F_{(\mathcal{B}, \mathcal{A}_{\text{kan}})} S$$

$$S^{-1} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \quad \boxed{\text{DZ}} \quad S = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathcal{B} \xrightarrow[S]{\quad\quad\quad} \mathcal{B}_{\text{kan}} \quad \mathcal{A}_{\text{kan}} \xrightarrow[T]{\quad\quad\quad} \mathcal{A}_{\text{kan}}$$

$\swarrow$   
 $S^{-1}$

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$$\mathcal{B} \xrightarrow[S]{\quad\quad\quad} \mathcal{B}_{\text{kan}} \quad \mathcal{A}_{\text{kan}} \xrightarrow[T]{\quad\quad\quad} \mathcal{A}_{\text{kan}}$$

$$T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

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$$S^{-1} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \quad \boxed{\text{DZ}} \quad S = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathcal{B} \xrightarrow[S]{\quad} \mathcal{B}_{\text{kan}} \quad \mathcal{A}_{\text{kan}} \xrightarrow[T]{\quad} \mathcal{A}_{\text{kan}}$$
$$T^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

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$$f(0, 0, 1) = ?$$

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$$F_{(\mathcal{B}_{\text{kan}}, \mathcal{A}_{\text{kan}})} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 3 & 1 \end{bmatrix}$$

$$f(0, 0, 1) = ?$$

$$Y_{\mathcal{A}_{\text{kan}}} = F_{(\mathcal{B}_{\text{kan}}, \mathcal{A}_{\text{kan}})} X_{\mathcal{B}_{\text{kan}}}$$

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$$F_{(\mathcal{B}, \mathcal{A}_{\text{kan}})} = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 3 & 4 \end{bmatrix} \quad \mathcal{B}_{\text{kan}} = \{(1, 0, 0), (0, 1, 0), (0, 0, 1)\}$$
$$F_{(\mathcal{B}_{\text{kan}}, \mathcal{A}_{\text{kan}})} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 3 & 1 \end{bmatrix}$$

$$f(0, 0, 1) = ?$$

$$Y_{\mathcal{A}_{\text{kan}}} = F_{(\mathcal{B}_{\text{kan}}, \mathcal{A}_{\text{kan}})} X_{\mathcal{B}_{\text{kan}}}$$

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$$f : \mathbb{R}^3 \rightarrow \mathbb{R}^2$$

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