Seminari 3

Matematičke metode za informatičare

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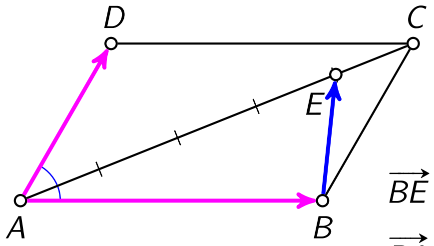
FOI, Varaždin

$\overrightarrow{BE}\cdot\overrightarrow{BA}=-1$



$|AB|=5,\ \ |AD|=3,\ \ \sphericalangle DAB=60^\circ$

$|AE|=\frac{4}{5}|AC| \rightsquigarrow \overrightarrow{AE}=\frac{4}{5}\overrightarrow{AC}$

Zadana baza: $\mathcal{B}=\left(\overrightarrow{AB},\overrightarrow{AD}\right)$

$\overrightarrow{BE}=-\frac{1}{5}\overrightarrow{AB}+\frac{4}{5}\overrightarrow{AD} \qquad \overrightarrow{AB}\cdot\overrightarrow{AD}=\frac{15}{2}$

$\overrightarrow{BA}=-1\cdot\overrightarrow{AB}+0\cdot\overrightarrow{AD}$

b)

$$\overrightarrow{BE}\cdot\overrightarrow{BA}=\left(-\frac{1}{5}\overrightarrow{AB}+\frac{4}{5}\overrightarrow{AD}\right)\cdot\left(-\overrightarrow{AB}\right)=\frac{1}{5}\overrightarrow{AB}^{\,2}-\frac{4}{5}\overrightarrow{AB}\cdot\overrightarrow{AD}=$$

$$=\frac{1}{5}\left|\overrightarrow{AB}\right|^2-\frac{4}{5}\overrightarrow{AB}\cdot\overrightarrow{AD}=\frac{1}{5}\cdot 5^2-\frac{4}{5}\cdot\frac{15}{2}=-1$$

~~$\overrightarrow{BE}\cdot\overrightarrow{BA}=\left(-\frac{1}{5},\frac{4}{5}\right)\cdot(-1,0)=-\frac{1}{5}\cdot(-1)+\frac{4}{5}\cdot 0=\frac{1}{5}$~~

Baza $\mathcal{B}$ nije ortonormirana!

Skalarni produkt vektora

$\vec{a}\,\vec{b}=|\vec{a}|\cdot|\vec{b}|\cdot\cos\left(\vec{a},\vec{b}\,\right)$

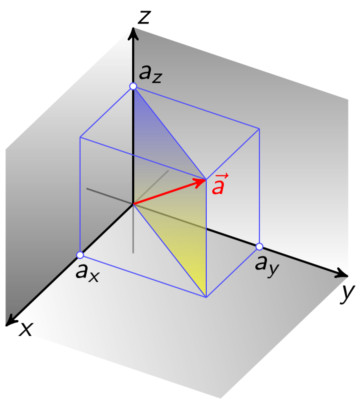
$\vec{a}=\left(a_x,a_y,a_z\right)=a_x\vec{i}+a_y\vec{j}+a_z\vec{k}$

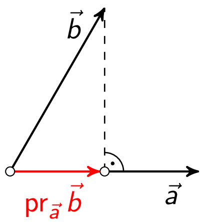
$\vec{b}=\left(b_x,b_y,b_z\right)=b_x\vec{i}+b_y\vec{j}+b_z\vec{k}$

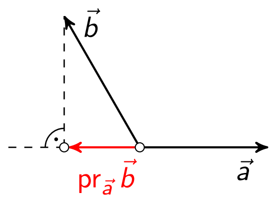
$\vec{a}\,\vec{b}=a_xb_x+a_yb_y+a_zb_z$

$|\vec{a}|=\sqrt{a_x^2+a_y^2+a_z^2}$

$\text{pr}_{\vec{a}}\,\vec{b}=\dfrac{\vec{a}\,\vec{b}}{|\vec{a}|^2}\cdot\vec{a}$







Vektorski produkt vektora

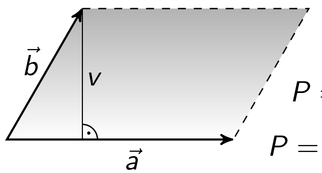
$\left|\vec{a}\times\vec{b}\right|=|\vec{a}|\cdot|\vec{b}|\cdot\sin\left(\vec{a},\vec{b}\,\right)$

$\vec{a}=\left(a_x,a_y,a_z\right)=a_x\vec{i}+a_y\vec{j}+a_z\vec{k}$

$\vec{b}=\left(b_x,b_y,b_z\right)=b_x\vec{i}+b_y\vec{j}+b_z\vec{k}$

$$\vec{a}\times\vec{b}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ a_x & a_y & a_z\\ b_x & b_y & b_z\end{vmatrix}$$

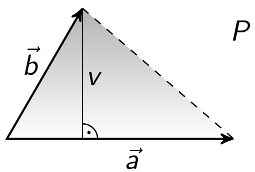
Površina paralelograma



$P=|\vec{a}|\cdot v$

$P=\left|\vec{a}\times\vec{b}\right|$

Površina trokuta



$P=\frac{1}{2}\left|\vec{a}\times\vec{b}\right|$

$P=\frac{1}{2}|\vec{a}|\cdot v$

2

Mješoviti produkt vektora

$$(\vec{a}, \vec{b}, \vec{c}) = (\vec{a} \times \vec{b}) \cdot \vec{c}$$

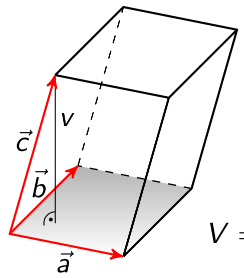
$$\vec{a} = (a_x, a_y, a_z) = a_x\vec{i} + a_y\vec{j} + a_z\vec{k}$$

$$\vec{b} = (b_x, b_y, b_z) = b_x\vec{i} + b_y\vec{j} + b_z\vec{k}$$

$$\vec{c} = (c_x, c_y, c_z) = c_x\vec{i} + c_y\vec{j} + c_z\vec{k}$$

$$(\vec{a}, \vec{b}, \vec{c}) = \begin{vmatrix} a_x & a_y & a_z \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{vmatrix}$$

Volumen paralelepipeda

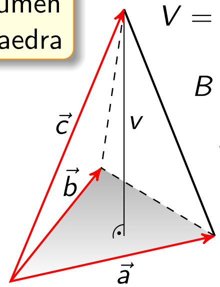


$$B = |\vec{a} \times \vec{b}|$$

$$V = B \cdot v$$

$$V = |(\vec{a}, \vec{b}, \vec{c})|$$

Volumen tetraedra



$$V = \frac{1}{6}|(\vec{a}, \vec{b}, \vec{c})|$$

$$B = \frac{1}{2}|\vec{a} \times \vec{b}|$$

$$V = \frac{1}{3}B \cdot v$$

Zadatak 1

Zadane su točke $A(2,3,-1)$, $B(3,4,2)$ i $C(1,0,-5)$.

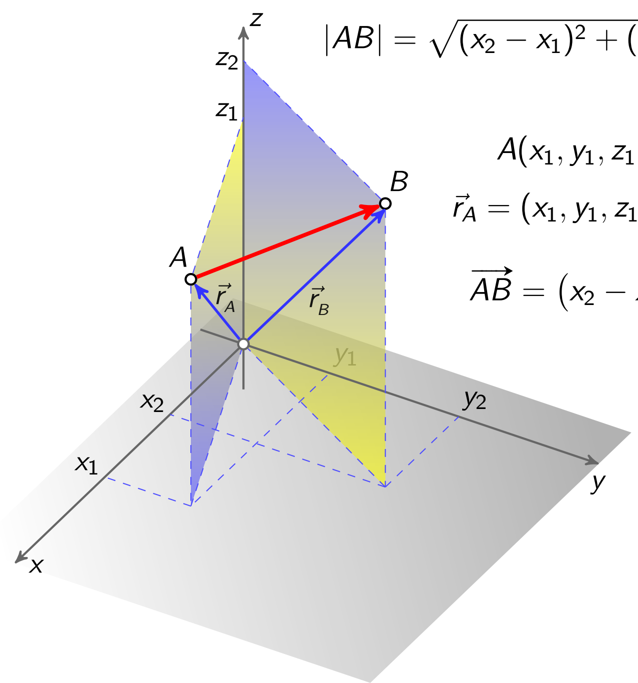
a) *Odredite točku D tako da četverokut $ABCD$ bude paralelogram.*

b) *Odredite unutarnji kut paralelograma $ABCD$ pri vrhu A.*

c) *Izračunajte površinu paralelograma $ABCD$ i duljinu visine paralelograma na stranicu $\overline{AB}$.*

d) *Ispitajte je li vektor $\vec{v} = (1,2,-1)$ paralelan s ravninom paralelograma $ABCD$.*

e) *Odredite ortogonalnu projekciju vektora $\vec{v}$ na ravninu paralelograma $ABCD$.*



$$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

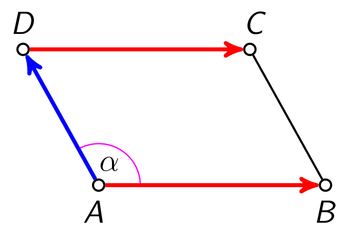
$$A(x_1, y_1, z_1) \qquad B(x_2, y_2, z_2)$$

$$\vec{r}_A = (x_1, y_1, z_1) \qquad \vec{r}_B = (x_2, y_2, z_2)$$

$$\overrightarrow{AB} = (x_2 - x_1,\ y_2 - y_1,\ z_2 - z_1)$$

$$\overrightarrow{AB} = \vec{r}_B - \vec{r}_A$$

Rješenje



$$A(2,3,-1), \quad B(3,4,2), \quad C(1,0,-5)$$

$$\overrightarrow{AB} = (1,1,3) \qquad \overrightarrow{AD} = (-2,-4,-7)$$

$$|\overrightarrow{AD}| = \sqrt{(-2)^2 + (-4)^2 + (-7)^2} = \sqrt{69}$$

$$|\overrightarrow{AB}| = \sqrt{1^2 + 1^2 + 3^2} = \sqrt{11}$$

a)

$$\overrightarrow{AB} = \overrightarrow{DC}$$

$$\vec{r}_B - \vec{r}_A = \vec{r}_C - \vec{r}_D$$

$$\vec{r}_D = \vec{r}_A - \vec{r}_B + \vec{r}_C$$

$$\vec{r}_D = (2,3,-1) - (3,4,2) + (1,0,-5)$$

$$\vec{r}_D = (0,-1,-8)$$

$$\boxed{D(0,-1,-8)}$$

b)

$$\alpha = \sphericalangle(\overrightarrow{AB}, \overrightarrow{AD})$$

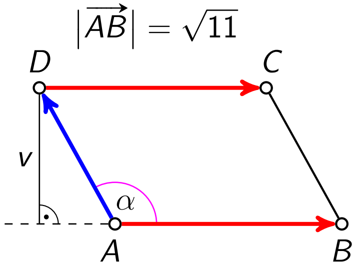
$$\cos\alpha = \frac{\overrightarrow{AB} \cdot \overrightarrow{AD}}{|\overrightarrow{AB}| \cdot |\overrightarrow{AD}|}$$

$$\cos\alpha = \frac{-27}{\sqrt{11} \cdot \sqrt{69}}$$

$$\alpha = \arccos\frac{-27}{\sqrt{11}\sqrt{69}}$$

$$\boxed{\alpha = 168^\circ\, 31'\, 57''}$$

$$\overrightarrow{AB} \cdot \overrightarrow{AD} = 1 \cdot (-2) + 1 \cdot (-4) + 3 \cdot (-7) = -27$$



$A(2,3,-1),\ B(3,4,2),\ C(1,0,-5)$

$\overrightarrow{AB} = (1,1,3) \qquad \overrightarrow{AD} = (-2,-4,-7)$

$|\overrightarrow{AB} \times \overrightarrow{AD}| = \sqrt{5^2 + 1^2 + (-2)^2} = \sqrt{30}$

$P = |\overrightarrow{AB} \times \overrightarrow{AD}|$

$\boxed{P = \sqrt{30}}$

$P = |\overrightarrow{AB}| \cdot v$

$v = \dfrac{P}{|\overrightarrow{AB}|}$

$\boxed{v = \dfrac{\sqrt{30}}{\sqrt{11}}}$

c)

$$\overrightarrow{AB} \times \overrightarrow{AD} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 3 \\ -2 & -4 & -7 \end{vmatrix} = \vec{i} \cdot (-1)^{1+1} \begin{vmatrix} 1 & 3 \\ -4 & -7 \end{vmatrix} +$$

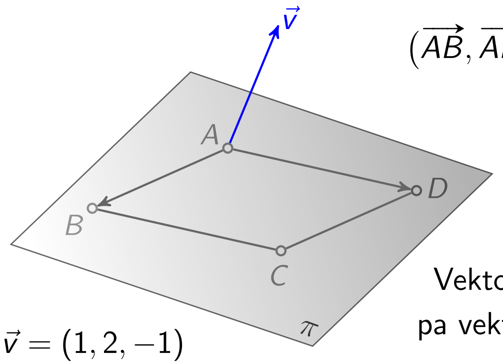
$$+ \vec{j} \cdot (-1)^{1+2} \begin{vmatrix} 1 & 3 \\ -2 & -7 \end{vmatrix} + \vec{k} \cdot (-1)^{1+3} \begin{vmatrix} 1 & 1 \\ -2 & -4 \end{vmatrix} =$$

$$= 5\vec{i} + \vec{j} - 2\vec{k} = (5, 1, -2)$$

$A_{ij} = (-1)^{i+j} M_{ij}$

d)

$\overrightarrow{AB} = (1,1,3) \qquad \overrightarrow{AD} = (-2,-4,-7)$



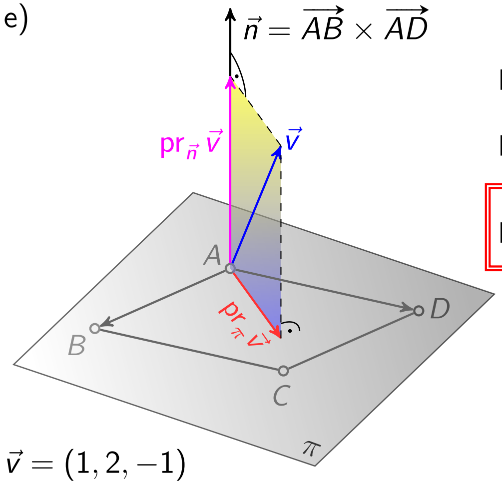
$$(\overrightarrow{AB}, \overrightarrow{AD}, \vec{v}) = \begin{vmatrix} 1 & 1 & 3 \\ -2 & -4 & -7 \\ 1 & 2 & -1 \end{vmatrix} = 9 \neq 0$$

$\vec{v} = (1, 2, -1)$

Vektori $\overrightarrow{AB}, \overrightarrow{AD}$ i $\vec{v}$ su nekomplanarni pa vektor $\vec{v}$ nije paralelan s ravninom π.

Kako je $(\overrightarrow{AB}, \overrightarrow{AD}, \vec{v}) > 0$, vektori $\overrightarrow{AB}, \overrightarrow{AD}$ i $\vec{v}$ u danom poretku čine jednu desnu bazu za V^3.

e)



$\vec{v} = \mathrm{pr}_{\vec{n}}\,\vec{v} + \mathrm{pr}_{\pi}\,\vec{v}$

$\mathrm{pr}_{\pi}\,\vec{v} = \vec{v} - \mathrm{pr}_{\vec{n}}\,\vec{v}$

$\mathrm{pr}_{\pi}\,\vec{v} = (1,2,-1) - \left(\frac{3}{2}, \frac{3}{10}, -\frac{3}{5}\right)$

$\boxed{\mathrm{pr}_{\pi}\,\vec{v} = \left(-\frac{1}{2}, \frac{17}{10}, -\frac{2}{5}\right)}$

$\vec{v} = (1, 2, -1)$

$\vec{n} = (5, 1, -2) \qquad |\vec{n}| = \sqrt{30}$

$\vec{v} \cdot \vec{n} = 1 \cdot 5 + 2 \cdot 1 + (-1) \cdot (-2) = 9$

$\mathrm{pr}_{\vec{n}}\,\vec{v} = \dfrac{\vec{v}\vec{n}}{|\vec{n}|^2} \cdot \vec{n}$

$\mathrm{pr}_{\vec{n}}\,\vec{v} = \dfrac{9}{\sqrt{30}^{\,2}} \cdot (5, 1, -2)$

$\mathrm{pr}_{\vec{n}}\,\vec{v} = \dfrac{3}{10} \cdot (5, 1, -2)$

$\boxed{\mathrm{pr}_{\vec{n}}\,\vec{v} = \left(\frac{3}{2}, \frac{3}{10}, -\frac{3}{5}\right)}$

Zadatak 2

Zadani su vektori $\vec{a} = (2m, 1, 1-m)$, $\vec{b} = (-1, 3, 0)$ i $\vec{c} = (5, -1, 8)$.

a) *Odredite $m \in \mathbb{R}$ tako da vektor $\vec{a}$ zatvara jednake kutove s vektorima $\vec{b}$ i $\vec{c}$.*

b) *Za pronađeni m iz a) dijela zadatka izračunajte volumen tetraedra određenog s vektorima $\vec{a}, \vec{b}, \vec{c}$ i duljinu visine tog tetraedra spuštenu na stranu određenu s vektorima $\vec{b}$ i $\vec{c}$.*

Rješenje

a) $\vec{a}=(2m,1,1-m),\quad \vec{b}=(-1,3,0),\quad \vec{c}=(5,-1,8)$

$$\sphericalangle(\vec{a},\vec{b})=\sphericalangle(\vec{a},\vec{c})$$

$$\cos(\vec{a},\vec{b})=\cos(\vec{a},\vec{c})$$

$$\frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\cdot|\vec{b}|}=\frac{\vec{a}\cdot\vec{c}}{|\vec{a}|\cdot|\vec{c}|}\quad /\cdot|\vec{a}|$$

$$\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}=\frac{\vec{a}\cdot\vec{c}}{|\vec{c}|}$$

$$\vec{a}\cdot\vec{b}=2m\cdot(-1)+1\cdot 3+(1-m)\cdot 0=-2m+3$$

$$\vec{a}\cdot\vec{c}=2m\cdot 5+1\cdot(-1)+(1-m)\cdot 8=2m+7$$

$$|\vec{b}|=\sqrt{(-1)^2+3^2+0^2}=\sqrt{10}$$

$$|\vec{c}|=\sqrt{5^2+(-1)^2+8^2}=\sqrt{90}=3\sqrt{10}$$

$$\frac{-2m+3}{\sqrt{10}}=\frac{2m+7}{3\sqrt{10}}\quad /\cdot 3\sqrt{10}$$

$$-6m+9=2m+7$$

$$-8m=-2$$

$$\boxed{m=\frac{1}{4}}$$

$$\boxed{\vec{a}=\left(\frac{1}{2},1,\frac{3}{4}\right)}$$

Zadatak 3

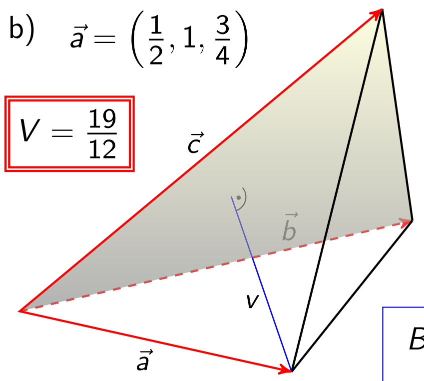
Zadane su točke $A(1,2,1)$, $B(2,3,1)$ i $C(-2,5,3)$.

a) *Pokažite da je ABC pravokutni trokut s pravim kutom kod vrha A.*

b) *Odredite točku D za koju je $|AD|=\sqrt{11}$ tako da vektori $\overrightarrow{AB}$, $\overrightarrow{AC}$, $\overrightarrow{AD}$ budu međusobno okomiti i u danom poretku čine desnu bazu za V^3.*

b) $\vec{a}=\left(\frac{1}{2},1,\frac{3}{4}\right)$

$$\boxed{V=\frac{19}{12}}$$



$\vec{b}=(-1,3,0)\qquad \vec{c}=(5,-1,8)$

$$V=\frac{1}{6}\left|(\vec{a},\vec{b},\vec{c})\right|=\frac{1}{6}\cdot\left|\frac{19}{2}\right|=\frac{19}{12}$$

$$(\vec{a},\vec{b},\vec{c})=\begin{vmatrix}\frac{1}{2} & 1 & \frac{3}{4}\\ -1 & 3 & 0\\ 5 & -1 & 8\end{vmatrix}=\frac{19}{2}$$

$$V=\frac{1}{3}Bv \rightsquigarrow \boxed{v=\frac{3V}{B}}$$

$$B=\frac{1}{2}|\vec{b}\times\vec{c}|$$

$$B=\frac{1}{2}\cdot 2\sqrt{209}$$

$$B=\sqrt{209}$$

$$v=\frac{3V}{B}$$

$$v=\frac{3\cdot\frac{19}{12}}{\sqrt{209}}$$

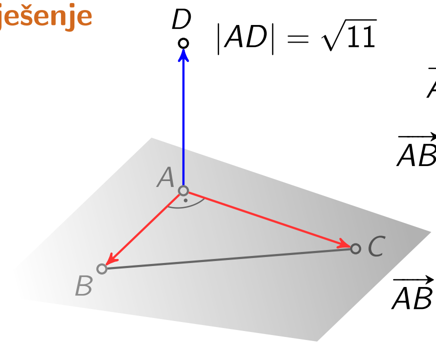
$$\boxed{v=\frac{19}{4\sqrt{209}}}$$

$$\vec{b}\times\vec{c}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ -1 & 3 & 0\\ 5 & -1 & 8\end{vmatrix}=(24,8,-14)$$

$$|\vec{b}\times\vec{c}|=\sqrt{24^2+8^2+(-14)^2}=\sqrt{836}=2\sqrt{209}$$

Rješenje

a)



$|AD|=\sqrt{11}$

$A(1,2,1),\quad B(2,3,1),\quad C(-2,5,3)$

$$\overrightarrow{AB}=(1,1,0)\qquad \overrightarrow{AC}=(-3,3,2)$$

$$\overrightarrow{AB}\cdot\overrightarrow{AC}=1\cdot(-3)+1\cdot 3+0\cdot 2=0$$

$$\overrightarrow{AB}\perp\overrightarrow{AC}$$

b)

$$\overrightarrow{AB}\times\overrightarrow{AC}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ 1 & 1 & 0\\ -3 & 3 & 2\end{vmatrix}=(2,-2,6)$$

$$\overrightarrow{AD}=\lambda\cdot\left(\overrightarrow{AB}\times\overrightarrow{AC}\right),\quad \lambda>0$$

$$\left|\overrightarrow{AB}\times\overrightarrow{AC}\right|=\sqrt{4+4+36}=2\sqrt{11}$$

$$\overrightarrow{AD}=\frac{\sqrt{11}}{\left|\overrightarrow{AB}\times\overrightarrow{AC}\right|}\cdot\left(\overrightarrow{AB}\times\overrightarrow{AC}\right)$$

$$\overrightarrow{AD}=\frac{\sqrt{11}}{2\sqrt{11}}\cdot(2,-2,6)$$

$$\overrightarrow{AD}=\frac{1}{2}\cdot(2,-2,6)$$

$$\overrightarrow{AD}=(1,-1,3)$$

$$\overrightarrow{AD}=(1,-1,3)$$

$$\vec{r}_D-\vec{r}_A=(1,-1,3)$$

$$\vec{r}_D=\vec{r}_A+(1,-1,3)$$

$$\vec{r}_D=(1,2,1)+(1,-1,3)$$

$$\vec{r}_D=(2,1,4)$$

$$\boxed{D(2,1,4)}$$

Zadatak 4

Zadana je dužina $\overline{AB}$ s koordinatama svojih krajeva $A(3,4,1)$ i $B(-5,2,-3)$.

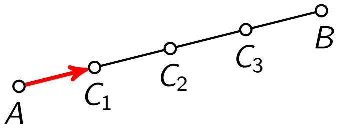
a) *Točkama C_1, C_2 i C_3 dužina $\overline{AB}$ je podijeljena na četiri jednaka dijela. Odredite koordinate točaka C_1, C_2 i C_3.*

b) *Odredite na pravcu AB točku D za koju je točka A polovište dužine $\overline{C_1D}$.*

Rješenje

$A(3,4,1)$ $\quad B(-5,2,-3)$

a)



$$\overrightarrow{AC_1} = \frac{1}{4}\overrightarrow{AB}$$

$$\vec{r}_{C_1} - \vec{r}_A = \frac{1}{4}\overrightarrow{AB}$$

$$\boxed{\vec{r}_{C_1} = \vec{r}_A + \frac{1}{4}\overrightarrow{AB}}$$

$$\overrightarrow{AB} = (-8,-2,-4)$$

$$\vec{r}_{C_1} = (3,4,1) + \frac{1}{4}\cdot(-8,-2,-4)$$

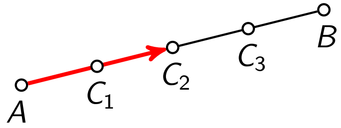
$$\vec{r}_{C_1} = (3,4,1) + \left(-2,-\frac{1}{2},-1\right)$$

$$\vec{r}_{C_1} = \left(1,\frac{7}{2},0\right)$$

$$C_1\left(1,\frac{7}{2},0\right)$$

$A(3,4,1)$ $\quad B(-5,2,-3)$

a)



$$\overrightarrow{AC_2} = \frac{1}{2}\overrightarrow{AB}$$

$$\vec{r}_{C_2} - \vec{r}_A = \frac{1}{2}\overrightarrow{AB}$$

$$\boxed{\vec{r}_{C_2} = \vec{r}_A + \frac{1}{2}\overrightarrow{AB}}$$

$$\overrightarrow{AB} = (-8,-2,-4)$$

$$C_1\left(1,\frac{7}{2},0\right)$$

$$C_2(-1,3,-1)$$

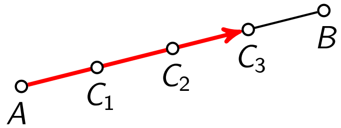
$$\vec{r}_{C_2} = (3,4,1) + \frac{1}{2}\cdot(-8,-2,-4)$$

$$\vec{r}_{C_2} = (3,4,1) + (-4,-1,-2)$$

$$\vec{r}_{C_2} = (-1,3,-1)$$

$A(3,4,1)$ $\quad B(-5,2,-3)$

a)



$$\overrightarrow{AC_3} = \frac{3}{4}\overrightarrow{AB}$$

$$\vec{r}_{C_3} - \vec{r}_A = \frac{3}{4}\overrightarrow{AB}$$

$$\boxed{\vec{r}_{C_3} = \vec{r}_A + \frac{3}{4}\overrightarrow{AB}}$$

$$\overrightarrow{AB} = (-8,-2,-4)$$

$$C_1\left(1,\frac{7}{2},0\right)$$

$$C_2(-1,3,-1)$$

$$C_3\left(-3,\frac{5}{2},-2\right)$$

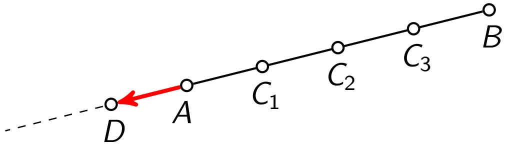
$$\vec{r}_{C_3} = (3,4,1) + \frac{3}{4}\cdot(-8,-2,-4)$$

$$\vec{r}_{C_3} = (3,4,1) + \left(-6,-\frac{3}{2},-3\right)$$

$$\vec{r}_{C_3} = \left(-3,\frac{5}{2},-2\right)$$

b)

$A(3,4,1) \qquad B(-5,2,-3)$



$$\overrightarrow{AD} = -\tfrac{1}{4}\overrightarrow{AB}$$

$$\vec{r}_D - \vec{r}_A = -\tfrac{1}{4}\overrightarrow{AB}$$

$$\boxed{\vec{r}_D = \vec{r}_A - \tfrac{1}{4}\overrightarrow{AB}}$$

$$\overrightarrow{AB} = (-8,-2,-4)$$

$$\vec{r}_D = (3,4,1) - \tfrac{1}{4}\cdot(-8,-2,-4)$$

$$\vec{r}_D = (3,4,1) + \left(2,\tfrac{1}{2},1\right)$$

$$\vec{r}_D = \left(5,\tfrac{9}{2},2\right)$$

$$C_1\left(1,\tfrac{7}{2},0\right)$$

$$C_2(-1,3,-1)$$

$$C_3\left(-3,\tfrac{5}{2},-2\right)$$

$$D\left(5,\tfrac{9}{2},2\right)$$

Koordinate djelišne točke – 1. pristup

$$\overrightarrow{AD} = \lambda\overrightarrow{BD}$$

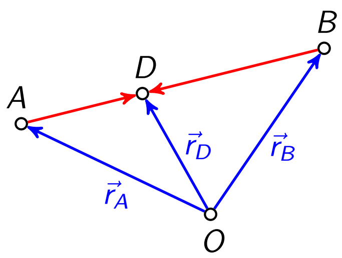
$$\vec{r}_D - \vec{r}_A = \lambda(\vec{r}_D - \vec{r}_B)$$

$$\vec{r}_D - \vec{r}_A = \lambda\vec{r}_D - \lambda\vec{r}_B$$

$$\vec{r}_D - \lambda\vec{r}_D = \vec{r}_A - \lambda\vec{r}_B$$

$$(1-\lambda)\vec{r}_D = \vec{r}_A - \lambda\vec{r}_B$$

$\lambda \neq 1$ $$\boxed{\vec{r}_D = \frac{\vec{r}_A - \lambda\vec{r}_B}{1-\lambda}}$$



$A(x_A, y_A, z_A) \qquad B(x_B, y_B, z_B)$

polovište $\rightsquigarrow$ $\lambda = -1$

$$D\left(\frac{x_A-\lambda x_B}{1-\lambda}, \frac{y_A-\lambda y_B}{1-\lambda}, \frac{z_A-\lambda z_B}{1-\lambda}\right) \qquad P\left(\frac{x_A+x_B}{2}, \frac{y_A+y_B}{2}, \frac{z_A+z_B}{2}\right)$$

Beskonačno daleku točku možemo *uhvatiti* s homogenim koordinatama.

Koordinate djelišne točke – 2. pristup

$$\overrightarrow{AD} = \lambda\overrightarrow{DB}$$

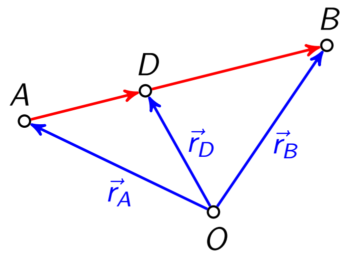
$$\vec{r}_D - \vec{r}_A = \lambda(\vec{r}_B - \vec{r}_D)$$

$$\vec{r}_D - \vec{r}_A = \lambda\vec{r}_B - \lambda\vec{r}_D$$

$$\vec{r}_D + \lambda\vec{r}_D = \vec{r}_A + \lambda\vec{r}_B$$

$$(1+\lambda)\vec{r}_D = \vec{r}_A + \lambda\vec{r}_B$$

$\lambda \neq -1$ $$\boxed{\vec{r}_D = \frac{\vec{r}_A + \lambda\vec{r}_B}{1+\lambda}}$$



$A(x_A, y_A, z_A) \qquad B(x_B, y_B, z_B)$

polovište $\rightsquigarrow$ $\lambda = 1$

$$D\left(\frac{x_A+\lambda x_B}{1+\lambda}, \frac{y_A+\lambda y_B}{1+\lambda}, \frac{z_A+\lambda z_B}{1+\lambda}\right) \qquad P\left(\frac{x_A+x_B}{2}, \frac{y_A+y_B}{2}, \frac{z_A+z_B}{2}\right)$$

Beskonačno daleku točku možemo *uhvatiti* s homogenim koordinatama.

Parametrizacija dužine i pravca

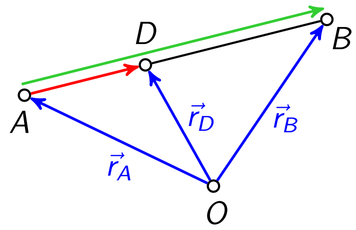
$$\overrightarrow{AD} = \lambda\overrightarrow{AB}$$

$$\vec{r}_D - \vec{r}_A = \lambda(\vec{r}_B - \vec{r}_A)$$

$$\vec{r}_D - \vec{r}_A = \lambda\vec{r}_B - \lambda\vec{r}_A$$

$$\vec{r}_D = \vec{r}_A + \lambda\vec{r}_B - \lambda\vec{r}_A$$

$$\boxed{\vec{r}_D = (1-\lambda)\vec{r}_A + \lambda\vec{r}_B}$$



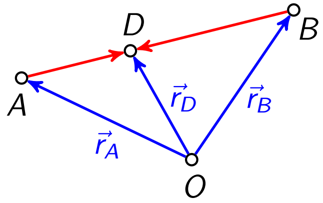
$A(x_A, y_A, z_A) \qquad B(x_B, y_B, z_B)$

$$D\big((1-\lambda)x_A+\lambda x_B,\ (1-\lambda)y_A+\lambda y_B,\ (1-\lambda)z_A+\lambda z_B\big)$$

polovište $\rightsquigarrow$ $\lambda = \frac{1}{2}$ $\qquad P\left(\frac{x_A+x_B}{2}, \frac{y_A+y_B}{2}, \frac{z_A+z_B}{2}\right)$

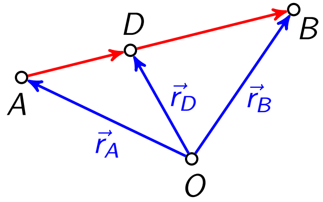
$$\vec{r}_D = \frac{\vec{r}_A - \lambda \vec{r}_B}{1-\lambda}$$

$$\vec{r}_D = \frac{1}{1-\lambda}\vec{r}_A + \frac{-\lambda}{1-\lambda}\vec{r}_B$$

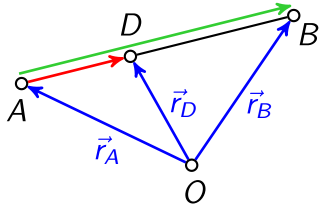


$$\vec{r}_D = \frac{\vec{r}_A + \lambda \vec{r}_B}{1+\lambda}$$

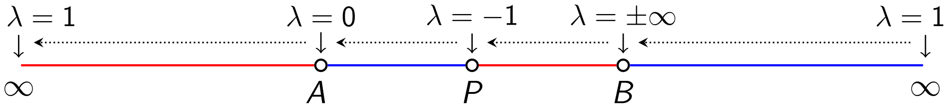
$$\vec{r}_D = \frac{1}{1+\lambda}\vec{r}_A + \frac{\lambda}{1+\lambda}\vec{r}_B$$



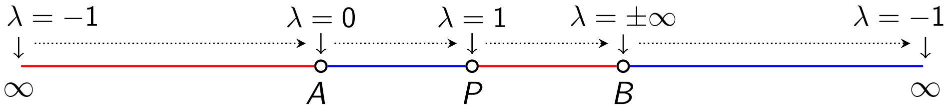
$$\vec{r}_D = (1-\lambda)\vec{r}_A + \lambda \vec{r}_B$$



$$\vec{r}_D = \frac{\vec{r}_A - \lambda \vec{r}_B}{1-\lambda}$$



$$\vec{r}_D = \frac{\vec{r}_A + \lambda \vec{r}_B}{1+\lambda}$$



$$\vec{r}_D = (1-\lambda)\vec{r}_A + \lambda \vec{r}_B$$