Seminari 7

Matematičke metode za informatičare

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FOI, Varaždin

Sadržaj

prvi zadatak

Zadatak 1

a) *U* $\mathbb{R}^3$ *nadopunite do baze skup vektora* $\big\{(5,0,2),\ (0,-5,0)\big\}$.

b) *U* $\mathcal{P}_4(x)$ *nadopunite do baze skup vektora*

$$\big\{6+2x-3x^2-x^3,\ x-7x^3\big\}.$$

Rješenje

a) $\big\{(5,0,2),\ (0,-5,0)\big\}$

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$\alpha \cdot (5,0,2)$

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$$\left.\begin{aligned}5\alpha&=0\\-5\beta&=0\\2\alpha&=0\end{aligned}\right\}\rightsquigarrow\boxed{\begin{aligned}\alpha&=0\\\beta&=0\end{aligned}}$$

Zadani skup vektora je linearno nezavisan u $\mathbb{R}^3$ pa se može nadopuniti do neke baze vektorskog prostora $\mathbb{R}^3$.

$$\mathcal{B}_{\text{kan}}=\{(1,0,0),(0,1,0),(0,0,1)\}$$

$\dim\mathbb{R}^3=3$

$$\{(5,0,2),\ (0,-5,0),\ (1,0,0)\}$$

jedna nadopuna do baze

$$\alpha\cdot(5,0,2)+\beta\cdot(0,-5,0)+\gamma\cdot(1,0,0)=\Theta_{\mathbb{R}^3}$$

$$(5\alpha+\gamma,-5\beta,2\alpha)=(0,0,0)$$

$$\left.\begin{aligned}5\alpha+\gamma&=0\\-5\beta&=0\\2\alpha&=0\end{aligned}\right\}\begin{aligned}&\\&\rightsquigarrow\beta=0\\&\rightsquigarrow\alpha=0\end{aligned}\qquad\gamma=0$$

Rješenje

a) $\{(5,0,2),\ (0,-5,0)\}$

Nadopuna do baze nije jedinstvena.

$$\alpha\cdot(5,0,2)+\beta\cdot(0,-5,0)=\Theta_{\mathbb{R}^3}$$

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b) $\left\{6 + 2x - 3x^2 - x^3,\, x - 7x^3\right\}$

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$\alpha \cdot \left(6 + 2x - 3x^2 - x^3\right)$

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$$(-\alpha-7\beta)x^3$$

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$$\alpha\cdot\left(6+2x-3x^2-x^3\right)+\beta\cdot\left(x-7x^3\right)=\Theta_{\mathcal{P}_4(x)}$$

$$(-\alpha-7\beta)x^3+(-3\alpha)x^2$$

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linearno nezavisni skup vektora, ali nije baza za $\mathcal{P}_4(x)$

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$\dim \mathcal{P}_4(x)=4$

$$\alpha\cdot\left(6+2x-3x^2-x^3\right)+\beta\cdot\left(x-7x^3\right)+\gamma\cdot 1+\delta\cdot x=\Theta_{\mathcal{P}_4(x)}$$

$$(-\alpha-7\beta)x^3+(-3\alpha)x^2+(2\alpha+\beta+\delta)x+(6\alpha+\gamma)=0$$

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jedna nadopuna do baze

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Nadopuna do baze nije jedinstvena.

drugi zadatak

Sustav linearnih jednadžbi

$$x_1 + 2x_2 - x_3 = 5$$
$$5x_1 - 3x_2 + 4x_3 = -8$$

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Zapis pomoću linearne kombinacije vektora

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$$x_1 \cdot \begin{bmatrix} 1 \\ 5 \end{bmatrix} + x_2 \cdot \begin{bmatrix} 2 \\ -3 \end{bmatrix} + x_3 \cdot \begin{bmatrix} -1 \\ 4 \end{bmatrix} = \begin{bmatrix} 5 \\ -8 \end{bmatrix}$$

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Proširena matrica sustava

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$$A_p = \left[\begin{array}{ccc:c} 1 & 2 & -1 & 5 \\ 5 & -3 & 4 & -8 \end{array}\right]$$

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  Sustav linearnih jednadžbi $AX = B$ je rješiv akko $r(A_p) = r(A)$.

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- Kronecker-Capellijev teorem
  Sustav linearnih jednadžbi $AX = B$ je rješiv akko $r(A_p) = r(A)$.
- Posljednji stupac u matrici A_p može se zapisati kao linearna kombinacija preostalih stupaca akko $r(A_p) = r(A)$.

Zadatak 2

U $\mathcal{P}_4(t)$ zadani su polinomi

$$p(t) = t^3 + t^2 + t, \quad q(t) = t^3 - t + 1, \quad r(t) = 2t^3 - t^2 + t - 2.$$

a) *Ispitajte jesu li polinomi p, q i r linearno nezavisni u $\mathcal{P}_4(t)$.*

b) *Može li se polinom $f(t) = t^3 + 3t^2 + 3$ prikazati kao linearna kombinacija polinoma p, q i r?*

c) *Može li se polinom $g(t) = t^3 + 3t^2 + t + 3$ prikazati kao linearna kombinacija polinoma p, q i r?*

Rješenje

- Kanonska baza za $\mathcal{P}_4(t)$:

Rješenje

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$$p(t) = t^3 + t^2 + t$$

Rješenje

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- Kanonska baza za $\mathcal{P}_4(t)$: $\{1, t, t^2, t^3\}$

$$p(t) = t^3 + t^2 + t \quad \longrightarrow \quad p(t) =$$

Rješenje

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- Kanonska baza za $\mathcal{P}_4(t)$: $\{1, t, t^2, t^3\}$

$$p(t) = t^3 + t^2 + t \quad \longrightarrow \quad p(t) = (0, 1, 1, 1)$$

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$$A = \begin{bmatrix} 0 & 1 & & & \\ 1 & -1 & & & \\ 1 & 0 & & & \\ 1 & 1 & & & \end{bmatrix}$$

Rješenje

$$\boxed{\dim \mathcal{P}_4(t) = 4}$$

- Kanonska baza za $\mathcal{P}_4(t)$: $\{1, t, t^2, t^3\}$

$$\begin{aligned}
p(t) &= t^3 + t^2 + t & &\longrightarrow & p(t) &= (0, 1, 1, 1)\\
q(t) &= t^3 - t + 1 & &\longrightarrow & q(t) &= (1, -1, 0, 1)\\
r(t) &= 2t^3 - t^2 + t - 2 & &\longrightarrow & r(t) &= (-2, 1, -1, 2)\\
f(t) &= t^3 + 3t^2 + 3 & &\longrightarrow & f(t) &= (3, 0, 3, 1)\\
g(t) &= t^3 + 3t^2 + t + 3 & &\longrightarrow & g(t) &= (3, 1, 3, 1)
\end{aligned}$$

$$A = \begin{bmatrix} 0 & 1 & -2 \\ 1 & -1 & 1 \\ 1 & 0 & -1 \\ 1 & 1 & 2 \end{bmatrix}$$

Rješenje

$\dim \mathcal{P}_4(t) = 4$

- Kanonska baza za $\mathcal{P}_4(t)$: $\{1, t, t^2, t^3\}$

$$
\begin{aligned}
p(t) = t^3 + t^2 + t &\longrightarrow p(t) = (0, 1, 1, 1) \\
q(t) = t^3 - t + 1 &\longrightarrow q(t) = (1, -1, 0, 1) \\
r(t) = 2t^3 - t^2 + t - 2 &\longrightarrow r(t) = (-2, 1, -1, 2) \\
f(t) = t^3 + 3t^2 + 3 &\longrightarrow f(t) = (3, 0, 3, 1) \\
g(t) = t^3 + 3t^2 + t + 3 &\longrightarrow g(t) = (3, 1, 3, 1)
\end{aligned}
$$

$$
A = \begin{bmatrix} 0 & 1 & -2 & 3 \\ 1 & -1 & 1 & 0 \\ 1 & 0 & -1 & 3 \\ 1 & 1 & 2 & 1 \end{bmatrix}
$$

Rješenje

$$\dim \mathcal{P}_4(t) = 4$$

- Kanonska baza za $\mathcal{P}_4(t)$: $\{1, t, t^2, t^3\}$

$$\begin{aligned}
p(t) &= t^3 + t^2 + t &\longrightarrow\quad p(t) &= (0, 1, 1, 1)\\
q(t) &= t^3 - t + 1 &\longrightarrow\quad q(t) &= (1, -1, 0, 1)\\
r(t) &= 2t^3 - t^2 + t - 2 &\longrightarrow\quad r(t) &= (-2, 1, -1, 2)\\
f(t) &= t^3 + 3t^2 + 3 &\longrightarrow\quad f(t) &= (3, 0, 3, 1)\\
g(t) &= t^3 + 3t^2 + t + 3 &\longrightarrow\quad g(t) &= (3, 1, 3, 1)
\end{aligned}$$

$$A = \begin{bmatrix} 0 & 1 & -2 & 3 & 3 \\ 1 & -1 & 1 & 0 & 1 \\ 1 & 0 & -1 & 3 & 3 \\ 1 & 1 & 2 & 1 & 1 \end{bmatrix}$$

Rješenje

$\dim \mathcal{P}_4(t) = 4$

- Kanonska baza za $\mathcal{P}_4(t)$: $\{1, t, t^2, t^3\}$

$$p(t) = t^3 + t^2 + t \longrightarrow p(t) = (0, 1, 1, 1)$$

$$q(t) = t^3 - t + 1 \longrightarrow q(t) = (1, -1, 0, 1)$$

$$r(t) = 2t^3 - t^2 + t - 2 \longrightarrow r(t) = (-2, 1, -1, 2)$$

$$f(t) = t^3 + 3t^2 + 3 \longrightarrow f(t) = (3, 0, 3, 1)$$

$$g(t) = t^3 + 3t^2 + t + 3 \longrightarrow g(t) = (3, 1, 3, 1)$$

$$A = \left[\begin{array}{ccc:cc} 0 & 1 & -2 & 3 & 3 \\ 1 & -1 & 1 & 0 & 1 \\ 1 & 0 & -1 & 3 & 3 \\ 1 & 1 & 2 & 1 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc:cc} 0 & 1 & -2 & 3 & 3 \\ 1 & -1 & 1 & 0 & 1 \\ 1 & 0 & -1 & 3 & 3 \\ 1 & 1 & 2 & 1 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|cc} 0 & 1 & -2 & 3 & 3 \\ 1 & -1 & 1 & 0 & 1 \\ 1 & 0 & -1 & 3 & 3 \\ 1 & 1 & 2 & 1 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc:cc} 0 & 1 & -2 & 3 & 3 \\ 1 & -1 & 1 & 0 & 1 \\ 1 & 0 & -1 & 3 & 3 \\ 1 & 1 & 2 & 1 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|cc} 0 & 1 & -2 & 3 & 3 \\ 1 & -1 & 1 & 0 & 1 \\ 1 & 0 & -1 & 3 & 3 \\ 1 & 1 & 2 & 1 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|cc} & & & & \\ & & & & \\ & & & & \\ & & & & \end{array}\right]$$

$$\left[\begin{array}{ccc:cc} 0 & 1 & -2 & 3 & 3 \\ 1 & -1 & 1 & 0 & 1 \\ 1 & 0 & -1 & 3 & 3 \\ 1 & 1 & 2 & 1 & 1 \end{array}\right] \sim \left[\begin{array}{ccc:cc} 1 & -1 & 1 & 0 & 1 \\ & & & & \\ & & & & \\ & & & & \end{array}\right]$$

$$\left[\begin{array}{ccc:cc} 0 & 1 & -2 & 3 & 3 \\ 1 & -1 & 1 & 0 & 1 \\ 1 & 0 & -1 & 3 & 3 \\ 1 & 1 & 2 & 1 & 1 \end{array}\right] \sim \left[\begin{array}{ccc:cc} 1 & -1 & 1 & 0 & 1 \\ 0 & 1 & -2 & 3 & 3 \\ & & & & \\ & & & & \end{array}\right]$$

$$\left[\begin{array}{ccc|cc} 0 & 1 & -2 & 3 & 3 \\ 1 & -1 & 1 & 0 & 1 \\ 1 & 0 & -1 & 3 & 3 \\ 1 & 1 & 2 & 1 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|cc} 1 & -1 & 1 & 0 & 1 \\ 0 & 1 & -2 & 3 & 3 \\ 1 & 0 & -1 & 3 & 3 \\ & & & & \end{array}\right]$$

$$\left[\begin{array}{ccc|cc} 0 & 1 & -2 & 3 & 3 \\ 1 & -1 & 1 & 0 & 1 \\ 1 & 0 & -1 & 3 & 3 \\ 1 & 1 & 2 & 1 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|cc} 1 & -1 & 1 & 0 & 1 \\ 0 & 1 & -2 & 3 & 3 \\ 1 & 0 & -1 & 3 & 3 \\ 1 & 1 & 2 & 1 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc:cc} 0 & 1 & -2 & 3 & 3 \\ 1 & -1 & 1 & 0 & 1 \\ 1 & 0 & -1 & 3 & 3 \\ 1 & 1 & 2 & 1 & 1 \end{array}\right] \sim \left[\begin{array}{ccc:cc} 1 & -1 & 1 & 0 & 1 \\ 0 & 1 & -2 & 3 & 3 \\ 1 & 0 & -1 & 3 & 3 \\ 1 & 1 & 2 & 1 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc:cc} 0 & 1 & -2 & 3 & 3 \\ 1 & -1 & 1 & 0 & 1 \\ 1 & 0 & -1 & 3 & 3 \\ 1 & 1 & 2 & 1 & 1 \end{array}\right] \sim \left[\begin{array}{ccc:cc} 1 & -1 & 1 & 0 & 1 \\ 0 & 1 & -2 & 3 & 3 \\ 1 & 0 & -1 & 3 & 3 \\ 1 & 1 & 2 & 1 & 1 \end{array}\right]$$

$$
\left[\begin{array}{ccc:cc} 0 & 1 & -2 & 3 & 3 \\ 1 & -1 & 1 & 0 & 1 \\ 1 & 0 & -1 & 3 & 3 \\ 1 & 1 & 2 & 1 & 1 \end{array}\right] \sim \left[\begin{array}{ccc:cc} 1 & -1 & 1 & 0 & 1 \\ 0 & 1 & -2 & 3 & 3 \\ 1 & 0 & -1 & 3 & 3 \\ 1 & 1 & 2 & 1 & 1 \end{array}\right] / \cdot (-1)
$$

$$\left[\begin{array}{ccc:cc} 0 & 1 & -2 & 3 & 3 \\ 1 & -1 & 1 & 0 & 1 \\ 1 & 0 & -1 & 3 & 3 \\ 1 & 1 & 2 & 1 & 1 \end{array}\right] \sim \left[\begin{array}{ccc:cc} 1 & -1 & 1 & 0 & 1 \\ 0 & 1 & -2 & 3 & 3 \\ 1 & 0 & -1 & 3 & 3 \\ 1 & 1 & 2 & 1 & 1 \end{array}\right] / \cdot (-1)$$

$+$

$$\left[\begin{array}{ccc:cc} 0 & 1 & -2 & 3 & 3 \\ 1 & -1 & 1 & 0 & 1 \\ 1 & 0 & -1 & 3 & 3 \\ 1 & 1 & 2 & 1 & 1 \end{array}\right] \sim \left[\begin{array}{ccc:cc} 1 & -1 & 1 & 0 & 1 \\ 0 & 1 & -2 & 3 & 3 \\ 1 & 0 & -1 & 3 & 3 \\ 1 & 1 & 2 & 1 & 1 \end{array}\right] \begin{array}{l} / \cdot (-1) \, / \cdot (-1) \\ \\ + \\ \\ \end{array}$$

$$\left[\begin{array}{ccc|cc} 0 & 1 & -2 & 3 & 3 \\ 1 & -1 & 1 & 0 & 1 \\ 1 & 0 & -1 & 3 & 3 \\ 1 & 1 & 2 & 1 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|cc} 1 & -1 & 1 & 0 & 1 \\ 0 & 1 & -2 & 3 & 3 \\ 1 & 0 & -1 & 3 & 3 \\ 1 & 1 & 2 & 1 & 1 \end{array}\right] /\cdot(-1) /\cdot(-1)$$

+ +

$$\left[\begin{array}{ccc|cc} 0 & 1 & -2 & 3 & 3 \\ 1 & -1 & 1 & 0 & 1 \\ 1 & 0 & -1 & 3 & 3 \\ 1 & 1 & 2 & 1 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|cc} 1 & -1 & 1 & 0 & 1 \\ 0 & 1 & -2 & 3 & 3 \\ 1 & 0 & -1 & 3 & 3 \\ 1 & 1 & 2 & 1 & 1 \end{array}\right] / \cdot (-1) \, / \cdot (-1) \sim$$

$$\sim \left[\begin{array}{ccc|cc} & & & & \\ & & & & \\ & & & & \\ & & & & \end{array}\right]$$

$$\left[\begin{array}{ccc:cc} 0 & 1 & -2 & 3 & 3 \\ 1 & -1 & 1 & 0 & 1 \\ 1 & 0 & -1 & 3 & 3 \\ 1 & 1 & 2 & 1 & 1 \end{array}\right] \sim \left[\begin{array}{ccc:cc} 1 & -1 & 1 & 0 & 1 \\ 0 & 1 & -2 & 3 & 3 \\ 1 & 0 & -1 & 3 & 3 \\ 1 & 1 & 2 & 1 & 1 \end{array}\right] / \cdot (-1) / \cdot (-1) \sim$$

$$\sim \left[\begin{array}{ccc:cc} 1 & -1 & 1 & 0 & 1 \\ & & & & \\ & & & & \\ & & & & \end{array}\right]$$

$$\left[\begin{array}{ccc:cc} 0 & 1 & -2 & 3 & 3 \\ 1 & -1 & 1 & 0 & 1 \\ 1 & 0 & -1 & 3 & 3 \\ 1 & 1 & 2 & 1 & 1 \end{array}\right] \sim \left[\begin{array}{ccc:cc} 1 & -1 & 1 & 0 & 1 \\ 0 & 1 & -2 & 3 & 3 \\ 1 & 0 & -1 & 3 & 3 \\ 1 & 1 & 2 & 1 & 1 \end{array}\right] / \cdot (-1) \; / \cdot (-1) \sim$$

$$\sim \left[\begin{array}{ccc:cc} 1 & -1 & 1 & 0 & 1 \\ 0 & 1 & -2 & 3 & 3 \\ & & & & \\ & & & & \end{array}\right]$$

$$\left[\begin{array}{ccc:cc} 0 & 1 & -2 & 3 & 3 \\ 1 & -1 & 1 & 0 & 1 \\ 1 & 0 & -1 & 3 & 3 \\ 1 & 1 & 2 & 1 & 1 \end{array}\right] \sim \left[\begin{array}{ccc:cc} 1 & -1 & 1 & 0 & 1 \\ 0 & 1 & -2 & 3 & 3 \\ 1 & 0 & -1 & 3 & 3 \\ 1 & 1 & 2 & 1 & 1 \end{array}\right] \begin{array}{l} /\cdot(-1)\,/\cdot(-1) \\ \\ \\ \\ \end{array} \sim$$

$$\sim \left[\begin{array}{ccc:cc} 1 & -1 & 1 & 0 & 1 \\ 0 & 1 & -2 & 3 & 3 \\ 0 & & & & \\ & & & & \end{array}\right]$$

$$\left[\begin{array}{ccc:cc} 0 & 1 & -2 & 3 & 3 \\ 1 & -1 & 1 & 0 & 1 \\ 1 & 0 & -1 & 3 & 3 \\ 1 & 1 & 2 & 1 & 1 \end{array}\right] \sim \left[\begin{array}{ccc:cc} 1 & -1 & 1 & 0 & 1 \\ 0 & 1 & -2 & 3 & 3 \\ 1 & 0 & -1 & 3 & 3 \\ 1 & 1 & 2 & 1 & 1 \end{array}\right] \begin{array}{l} /\cdot(-1)\,/\cdot(-1) \\ \\ + \\ + \end{array} \sim$$

$$\sim \left[\begin{array}{ccc:cc} 1 & -1 & 1 & 0 & 1 \\ 0 & 1 & -2 & 3 & 3 \\ 0 & 1 & & & \\ & & & & \end{array}\right]$$

$$\left[\begin{array}{ccc:cc} 0 & 1 & -2 & 3 & 3 \\ 1 & -1 & 1 & 0 & 1 \\ 1 & 0 & -1 & 3 & 3 \\ 1 & 1 & 2 & 1 & 1 \end{array}\right] \sim \left[\begin{array}{ccc:cc} 1 & -1 & 1 & 0 & 1 \\ 0 & 1 & -2 & 3 & 3 \\ 1 & 0 & -1 & 3 & 3 \\ 1 & 1 & 2 & 1 & 1 \end{array}\right] \begin{array}{l} /\cdot(-1)\,/\cdot(-1) \\ \\ + \\ + \end{array} \sim$$

$$\sim \left[\begin{array}{ccc:cc} 1 & -1 & 1 & 0 & 1 \\ 0 & 1 & -2 & 3 & 3 \\ 0 & 1 & -2 & & \\ & & & & \end{array}\right]$$

$$
\left[\begin{array}{ccc:cc}
0 & 1 & -2 & 3 & 3 \\
1 & -1 & 1 & 0 & 1 \\
1 & 0 & -1 & 3 & 3 \\
1 & 1 & 2 & 1 & 1
\end{array}\right]
\sim
\left[\begin{array}{ccc:cc}
1 & -1 & 1 & 0 & 1 \\
0 & 1 & -2 & 3 & 3 \\
1 & 0 & -1 & 3 & 3 \\
1 & 1 & 2 & 1 & 1
\end{array}\right] / \cdot (-1) \, / \cdot (-1)
\sim
$$

$$
\sim
\left[\begin{array}{ccc:cc}
1 & -1 & 1 & 0 & 1 \\
0 & 1 & -2 & 3 & 3 \\
0 & 1 & -2 & 3 & \\
 & & & &
\end{array}\right]
$$

$$\left[\begin{array}{ccc|cc} 0 & 1 & -2 & 3 & 3 \\ 1 & -1 & 1 & 0 & 1 \\ 1 & 0 & -1 & 3 & 3 \\ 1 & 1 & 2 & 1 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|cc} 1 & -1 & 1 & 0 & 1 \\ 0 & 1 & -2 & 3 & 3 \\ 1 & 0 & -1 & 3 & 3 \\ 1 & 1 & 2 & 1 & 1 \end{array}\right] / \cdot (-1) \, / \cdot (-1) \quad \sim$$

$$\sim \left[\begin{array}{ccc|cc} 1 & -1 & 1 & 0 & 1 \\ 0 & 1 & -2 & 3 & 3 \\ 0 & 1 & -2 & 3 & 2 \\ & & & & \end{array}\right]$$

$$
\left[\begin{array}{ccc|cc}
0 & 1 & -2 & 3 & 3 \\
1 & -1 & 1 & 0 & 1 \\
1 & 0 & -1 & 3 & 3 \\
1 & 1 & 2 & 1 & 1
\end{array}\right]
\sim
\left[\begin{array}{ccc|cc}
1 & -1 & 1 & 0 & 1 \\
0 & 1 & -2 & 3 & 3 \\
1 & 0 & -1 & 3 & 3 \\
1 & 1 & 2 & 1 & 1
\end{array}\right] / \cdot (-1) \; / \cdot (-1)
\sim
$$

$$
\sim
\left[\begin{array}{ccc|cc}
1 & -1 & 1 & 0 & 1 \\
0 & 1 & -2 & 3 & 3 \\
0 & 1 & -2 & 3 & 2 \\
0 & & & &
\end{array}\right]
$$

$$\begin{bmatrix} 0 & 1 & -2 & \vdots & 3 & 3 \\ 1 & -1 & 1 & \vdots & 0 & 1 \\ 1 & 0 & -1 & \vdots & 3 & 3 \\ 1 & 1 & 2 & \vdots & 1 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 & 1 & \vdots & 0 & 1 \\ 0 & 1 & -2 & \vdots & 3 & 3 \\ 1 & 0 & -1 & \vdots & 3 & 3 \\ 1 & 1 & 2 & \vdots & 1 & 1 \end{bmatrix} / \cdot (-1) / \cdot (-1) \sim$$

$$\sim \begin{bmatrix} 1 & -1 & 1 & \vdots & 0 & 1 \\ 0 & 1 & -2 & \vdots & 3 & 3 \\ 0 & 1 & -2 & \vdots & 3 & 2 \\ 0 & 2 & & \vdots & & \end{bmatrix}$$

$$\left[\begin{array}{ccc:cc} 0 & 1 & -2 & 3 & 3 \\ 1 & -1 & 1 & 0 & 1 \\ 1 & 0 & -1 & 3 & 3 \\ 1 & 1 & 2 & 1 & 1 \end{array}\right] \sim \left[\begin{array}{ccc:cc} 1 & -1 & 1 & 0 & 1 \\ 0 & 1 & -2 & 3 & 3 \\ 1 & 0 & -1 & 3 & 3 \\ 1 & 1 & 2 & 1 & 1 \end{array}\right] / \cdot (-1) \, / \cdot (-1) \sim$$

$$\sim \left[\begin{array}{ccc:cc} 1 & -1 & 1 & 0 & 1 \\ 0 & 1 & -2 & 3 & 3 \\ 0 & 1 & -2 & 3 & 2 \\ 0 & 2 & 1 & & \end{array}\right]$$

$$
\left[\begin{array}{ccc:cc}
0 & 1 & -2 & 3 & 3 \\
1 & -1 & 1 & 0 & 1 \\
1 & 0 & -1 & 3 & 3 \\
1 & 1 & 2 & 1 & 1
\end{array}\right]
\sim
\left[\begin{array}{ccc:cc}
1 & -1 & 1 & 0 & 1 \\
0 & 1 & -2 & 3 & 3 \\
1 & 0 & -1 & 3 & 3 \\
1 & 1 & 2 & 1 & 1
\end{array}\right]
\begin{array}{l}
/\cdot(-1)\;/\cdot(-1) \\
\\
+ \\
+
\end{array}
\sim
$$

$$
\sim
\left[\begin{array}{ccc:cc}
1 & -1 & 1 & 0 & 1 \\
0 & 1 & -2 & 3 & 3 \\
0 & 1 & -2 & 3 & 2 \\
0 & 2 & 1 & 1 &
\end{array}\right]
$$

$$\left[\begin{array}{ccc:cc} 0 & 1 & -2 & 3 & 3 \\ 1 & -1 & 1 & 0 & 1 \\ 1 & 0 & -1 & 3 & 3 \\ 1 & 1 & 2 & 1 & 1 \end{array}\right] \sim \left[\begin{array}{ccc:cc} 1 & -1 & 1 & 0 & 1 \\ 0 & 1 & -2 & 3 & 3 \\ 1 & 0 & -1 & 3 & 3 \\ 1 & 1 & 2 & 1 & 1 \end{array}\right] / \cdot (-1) / \cdot (-1) \sim$$

$$\sim \left[\begin{array}{ccc:cc} 1 & -1 & 1 & 0 & 1 \\ 0 & 1 & -2 & 3 & 3 \\ 0 & 1 & -2 & 3 & 2 \\ 0 & 2 & 1 & 1 & 0 \end{array}\right]$$

$$
\left[\begin{array}{ccc|cc} 0 & 1 & -2 & 3 & 3 \\ 1 & -1 & 1 & 0 & 1 \\ 1 & 0 & -1 & 3 & 3 \\ 1 & 1 & 2 & 1 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|cc} 1 & -1 & 1 & 0 & 1 \\ 0 & 1 & -2 & 3 & 3 \\ 1 & 0 & -1 & 3 & 3 \\ 1 & 1 & 2 & 1 & 1 \end{array}\right] / \cdot (-1) \, / \cdot (-1) \sim
$$

$$
\sim \left[\begin{array}{ccc|cc} 1 & -1 & 1 & 0 & 1 \\ 0 & 1 & -2 & 3 & 3 \\ 0 & 1 & -2 & 3 & 2 \\ 0 & 2 & 1 & 1 & 0 \end{array}\right]
$$

$$\left[\begin{array}{ccc:cc} 0 & 1 & -2 & 3 & 3 \\ 1 & -1 & 1 & 0 & 1 \\ 1 & 0 & -1 & 3 & 3 \\ 1 & 1 & 2 & 1 & 1 \end{array}\right] \sim \left[\begin{array}{ccc:cc} 1 & -1 & 1 & 0 & 1 \\ 0 & 1 & -2 & 3 & 3 \\ 1 & 0 & -1 & 3 & 3 \\ 1 & 1 & 2 & 1 & 1 \end{array}\right] /\cdot(-1) \, /\cdot(-1) \sim$$

$$\sim \left[\begin{array}{ccc:cc} 1 & -1 & 1 & 0 & 1 \\ 0 & 1 & -2 & 3 & 3 \\ 0 & 1 & -2 & 3 & 2 \\ 0 & 2 & 1 & 1 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|cc} 0 & 1 & -2 & 3 & 3 \\ 1 & -1 & 1 & 0 & 1 \\ 1 & 0 & -1 & 3 & 3 \\ 1 & 1 & 2 & 1 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|cc} 1 & -1 & 1 & 0 & 1 \\ 0 & 1 & -2 & 3 & 3 \\ 1 & 0 & -1 & 3 & 3 \\ 1 & 1 & 2 & 1 & 1 \end{array}\right] / \cdot (-1) \, / \cdot (-1) \sim$$

$$\sim \left[\begin{array}{ccc|cc} 1 & -1 & 1 & 0 & 1 \\ 0 & 1 & -2 & 3 & 3 \\ 0 & 1 & -2 & 3 & 2 \\ 0 & 2 & 1 & 1 & 0 \end{array}\right] / \cdot (-1)$$

$$\left[\begin{array}{ccc:cc} 0 & 1 & -2 & 3 & 3 \\ 1 & -1 & 1 & 0 & 1 \\ 1 & 0 & -1 & 3 & 3 \\ 1 & 1 & 2 & 1 & 1 \end{array}\right] \sim \left[\begin{array}{ccc:cc} 1 & -1 & 1 & 0 & 1 \\ 0 & 1 & -2 & 3 & 3 \\ 1 & 0 & -1 & 3 & 3 \\ 1 & 1 & 2 & 1 & 1 \end{array}\right] / \cdot (-1) / \cdot (-1) \sim$$

$$\sim \left[\begin{array}{ccc:cc} 1 & -1 & 1 & 0 & 1 \\ 0 & 1 & -2 & 3 & 3 \\ 0 & 1 & -2 & 3 & 2 \\ 0 & 2 & 1 & 1 & 0 \end{array}\right] / \cdot (-1)$$

+ +

+

$$
\left[\begin{array}{ccc:cc}
0 & 1 & -2 & 3 & 3\\
1 & -1 & 1 & 0 & 1\\
1 & 0 & -1 & 3 & 3\\
1 & 1 & 2 & 1 & 1
\end{array}\right]
\sim
\left[\begin{array}{ccc:cc}
1 & -1 & 1 & 0 & 1\\
0 & 1 & -2 & 3 & 3\\
1 & 0 & -1 & 3 & 3\\
1 & 1 & 2 & 1 & 1
\end{array}\right]
\begin{matrix} /\cdot(-1)\,/\cdot(-1) \\ \\ + \quad + \\ \end{matrix}
\sim
$$

$$
\sim
\left[\begin{array}{ccc:cc}
1 & -1 & 1 & 0 & 1\\
0 & 1 & -2 & 3 & 3\\
0 & 1 & -2 & 3 & 2\\
0 & 2 & 1 & 1 & 0
\end{array}\right]
\begin{matrix} \\ /\cdot(-1)\,/\cdot(-2) \\ + \\ \end{matrix}
$$

$$\left[\begin{array}{ccc|cc} 0 & 1 & -2 & 3 & 3 \\ 1 & -1 & 1 & 0 & 1 \\ 1 & 0 & -1 & 3 & 3 \\ 1 & 1 & 2 & 1 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|cc} 1 & -1 & 1 & 0 & 1 \\ 0 & 1 & -2 & 3 & 3 \\ 1 & 0 & -1 & 3 & 3 \\ 1 & 1 & 2 & 1 & 1 \end{array}\right] /\cdot(-1) \, /\cdot(-1) \sim$$

$$\sim \left[\begin{array}{ccc|cc} 1 & -1 & 1 & 0 & 1 \\ 0 & 1 & -2 & 3 & 3 \\ 0 & 1 & -2 & 3 & 2 \\ 0 & 2 & 1 & 1 & 0 \end{array}\right] /\cdot(-1) \, /\cdot(-2)$$

$$\left[\begin{array}{ccc:cc} 0 & 1 & -2 & 3 & 3 \\ 1 & -1 & 1 & 0 & 1 \\ 1 & 0 & -1 & 3 & 3 \\ 1 & 1 & 2 & 1 & 1 \end{array}\right] \sim \left[\begin{array}{ccc:cc} 1 & -1 & 1 & 0 & 1 \\ 0 & 1 & -2 & 3 & 3 \\ 1 & 0 & -1 & 3 & 3 \\ 1 & 1 & 2 & 1 & 1 \end{array}\right] \begin{array}{l} /\cdot(-1)\,/\cdot(-1) \\ \\ \\ \\ \end{array} \sim$$

$$\sim \left[\begin{array}{ccc:cc} 1 & -1 & 1 & 0 & 1 \\ 0 & 1 & -2 & 3 & 3 \\ 0 & 1 & -2 & 3 & 2 \\ 0 & 2 & 1 & 1 & 0 \end{array}\right] \begin{array}{l} \\ /\cdot(-1)\,/\cdot(-2) \\ \\ \\ \end{array} \sim \left[\begin{array}{ccc:cc} & & & & \\ \\ \\ \\ \end{array}\right]$$

$$
\left[\begin{array}{ccc:cc}
0 & 1 & -2 & 3 & 3 \\
1 & -1 & 1 & 0 & 1 \\
1 & 0 & -1 & 3 & 3 \\
1 & 1 & 2 & 1 & 1
\end{array}\right]
\sim
\left[\begin{array}{ccc:cc}
1 & -1 & 1 & 0 & 1 \\
0 & 1 & -2 & 3 & 3 \\
1 & 0 & -1 & 3 & 3 \\
1 & 1 & 2 & 1 & 1
\end{array}\right]
\begin{array}{l}
/\cdot(-1)\,/\cdot(-1) \\
\\
\\
\\
\end{array}
\sim
$$

$$
\sim
\left[\begin{array}{ccc:cc}
1 & -1 & 1 & 0 & 1 \\
0 & 1 & -2 & 3 & 3 \\
0 & 1 & -2 & 3 & 2 \\
0 & 2 & 1 & 1 & 0
\end{array}\right]
\begin{array}{l}
\\
/\cdot(-1)\,/\cdot(-2) \\
\\
\\
\end{array}
\sim
\left[\begin{array}{ccc:cc}
1 & -1 & 1 & 0 & 1 \\
 & & & & \\
 & & & & \\
 & & & &
\end{array}\right]
$$

$$\left[\begin{array}{ccc|cc} 0 & 1 & -2 & 3 & 3 \\ 1 & -1 & 1 & 0 & 1 \\ 1 & 0 & -1 & 3 & 3 \\ 1 & 1 & 2 & 1 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|cc} 1 & -1 & 1 & 0 & 1 \\ 0 & 1 & -2 & 3 & 3 \\ 1 & 0 & -1 & 3 & 3 \\ 1 & 1 & 2 & 1 & 1 \end{array}\right] /\cdot(-1) /\cdot(-1) \sim$$

$$\sim \left[\begin{array}{ccc|cc} 1 & -1 & 1 & 0 & 1 \\ 0 & 1 & -2 & 3 & 3 \\ 0 & 1 & -2 & 3 & 2 \\ 0 & 2 & 1 & 1 & 0 \end{array}\right] /\cdot(-1) /\cdot(-2) \sim \left[\begin{array}{ccc|cc} 1 & -1 & 1 & 0 & 1 \\ 0 & 1 & -2 & 3 & 3 \\ & & & & \\ & & & & \end{array}\right]$$

$$
\left[\begin{array}{ccc|cc}
0 & 1 & -2 & 3 & 3 \\
1 & -1 & 1 & 0 & 1 \\
1 & 0 & -1 & 3 & 3 \\
1 & 1 & 2 & 1 & 1
\end{array}\right]
\sim
\left[\begin{array}{ccc|cc}
1 & -1 & 1 & 0 & 1 \\
0 & 1 & -2 & 3 & 3 \\
1 & 0 & -1 & 3 & 3 \\
1 & 1 & 2 & 1 & 1
\end{array}\right] / \cdot (-1) \, / \cdot (-1)
\sim
$$

$$
\sim
\left[\begin{array}{ccc|cc}
1 & -1 & 1 & 0 & 1 \\
0 & 1 & -2 & 3 & 3 \\
0 & 1 & -2 & 3 & 2 \\
0 & 2 & 1 & 1 & 0
\end{array}\right] / \cdot (-1) \, / \cdot (-2)
\sim
\left[\begin{array}{ccc|cc}
1 & -1 & 1 & 0 & 1 \\
0 & 1 & -2 & 3 & 3 \\
0 & & & & \\
 & & & &
\end{array}\right]
$$

$$\left[\begin{array}{ccc:cc} 0 & 1 & -2 & 3 & 3 \\ 1 & -1 & 1 & 0 & 1 \\ 1 & 0 & -1 & 3 & 3 \\ 1 & 1 & 2 & 1 & 1 \end{array}\right] \sim \left[\begin{array}{ccc:cc} 1 & -1 & 1 & 0 & 1 \\ 0 & 1 & -2 & 3 & 3 \\ 1 & 0 & -1 & 3 & 3 \\ 1 & 1 & 2 & 1 & 1 \end{array}\right] / \cdot (-1) \, / \cdot (-1) \sim$$

$$\sim \left[\begin{array}{ccc:cc} 1 & -1 & 1 & 0 & 1 \\ 0 & 1 & -2 & 3 & 3 \\ 0 & 1 & -2 & 3 & 2 \\ 0 & 2 & 1 & 1 & 0 \end{array}\right] / \cdot (-1) \, / \cdot (-2) \sim \left[\begin{array}{ccc:cc} 1 & -1 & 1 & 0 & 1 \\ 0 & 1 & -2 & 3 & 3 \\ 0 & 0 & & & \\ & & & & \end{array}\right]$$

$$
\left[\begin{array}{ccc|cc}
0 & 1 & -2 & 3 & 3\\
1 & -1 & 1 & 0 & 1\\
1 & 0 & -1 & 3 & 3\\
1 & 1 & 2 & 1 & 1
\end{array}\right]
\sim
\left[\begin{array}{ccc|cc}
1 & -1 & 1 & 0 & 1\\
0 & 1 & -2 & 3 & 3\\
1 & 0 & -1 & 3 & 3\\
1 & 1 & 2 & 1 & 1
\end{array}\right] / \cdot (-1) \,/ \cdot (-1)
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\sim
\left[\begin{array}{ccc|cc}
1 & -1 & 1 & 0 & 1\\
0 & 1 & -2 & 3 & 3\\
0 & 1 & -2 & 3 & 2\\
0 & 2 & 1 & 1 & 0
\end{array}\right] / \cdot (-1) \,/ \cdot (-2)
\sim
\left[\begin{array}{ccc|cc}
1 & -1 & 1 & 0 & 1\\
0 & 1 & -2 & 3 & 3\\
0 & 0 & 0 & & \\
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\end{array}\right]
$$

$$\left[\begin{array}{ccc|cc} 0 & 1 & -2 & 3 & 3 \\ 1 & -1 & 1 & 0 & 1 \\ 1 & 0 & -1 & 3 & 3 \\ 1 & 1 & 2 & 1 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|cc} 1 & -1 & 1 & 0 & 1 \\ 0 & 1 & -2 & 3 & 3 \\ 1 & 0 & -1 & 3 & 3 \\ 1 & 1 & 2 & 1 & 1 \end{array}\right] / \cdot (-1) \; / \cdot (-1) \quad \sim$$

$$\sim \left[\begin{array}{ccc|cc} 1 & -1 & 1 & 0 & 1 \\ 0 & 1 & -2 & 3 & 3 \\ 0 & 1 & -2 & 3 & 2 \\ 0 & 2 & 1 & 1 & 0 \end{array}\right] / \cdot (-1) \; / \cdot (-2) \quad \sim \left[\begin{array}{ccc|cc} 1 & -1 & 1 & 0 & 1 \\ 0 & 1 & -2 & 3 & 3 \\ 0 & 0 & 0 & 0 & \\ & & & & \end{array}\right]$$

$$
\left[\begin{array}{ccc:cc} 0 & 1 & -2 & 3 & 3 \\ 1 & -1 & 1 & 0 & 1 \\ 1 & 0 & -1 & 3 & 3 \\ 1 & 1 & 2 & 1 & 1 \end{array}\right]
\sim
\left[\begin{array}{ccc:cc} 1 & -1 & 1 & 0 & 1 \\ 0 & 1 & -2 & 3 & 3 \\ 1 & 0 & -1 & 3 & 3 \\ 1 & 1 & 2 & 1 & 1 \end{array}\right]
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\left[\begin{array}{ccc:cc} 1 & -1 & 1 & 0 & 1 \\ 0 & 1 & -2 & 3 & 3 \\ 0 & 1 & -2 & 3 & 2 \\ 0 & 2 & 1 & 1 & 0 \end{array}\right]
\begin{array}{l} \\ /\cdot(-1)\,/\cdot(-2) \\ + \\ + \end{array}
\sim
\left[\begin{array}{ccc:cc} 1 & -1 & 1 & 0 & 1 \\ 0 & 1 & -2 & 3 & 3 \\ 0 & 0 & 0 & 0 & -1 \\ & & & & \end{array}\right]
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\left[\begin{array}{ccc:cc} 0 & 1 & -2 & 3 & 3 \\ 1 & -1 & 1 & 0 & 1 \\ 1 & 0 & -1 & 3 & 3 \\ 1 & 1 & 2 & 1 & 1 \end{array}\right] \sim \left[\begin{array}{ccc:cc} 1 & -1 & 1 & 0 & 1 \\ 0 & 1 & -2 & 3 & 3 \\ 1 & 0 & -1 & 3 & 3 \\ 1 & 1 & 2 & 1 & 1 \end{array}\right] / \cdot (-1) \, / \cdot (-1) \sim
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\sim \left[\begin{array}{ccc:cc} 1 & -1 & 1 & 0 & 1 \\ 0 & 1 & -2 & 3 & 3 \\ 0 & 1 & -2 & 3 & 2 \\ 0 & 2 & 1 & 1 & 0 \end{array}\right] / \cdot (-1) \, / \cdot (-2) \sim \left[\begin{array}{ccc:cc} 1 & -1 & 1 & 0 & 1 \\ 0 & 1 & -2 & 3 & 3 \\ 0 & 0 & 0 & 0 & -1 \\ 0 & & & & \end{array}\right]
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\left[\begin{array}{ccc|cc}
0 & 1 & -2 & 3 & 3 \\
1 & -1 & 1 & 0 & 1 \\
1 & 0 & -1 & 3 & 3 \\
1 & 1 & 2 & 1 & 1
\end{array}\right]
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\left[\begin{array}{ccc|cc}
1 & -1 & 1 & 0 & 1 \\
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\end{array}\right] / \cdot (-1) \, / \cdot (-1) \quad \sim
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\left[\begin{array}{ccc|cc}
1 & -1 & 1 & 0 & 1 \\
0 & 1 & -2 & 3 & 3 \\
0 & 1 & -2 & 3 & 2 \\
0 & 2 & 1 & 1 & 0
\end{array}\right] / \cdot (-1) \, / \cdot (-2)
\sim
\left[\begin{array}{ccc|cc}
1 & -1 & 1 & 0 & 1 \\
0 & 1 & -2 & 3 & 3 \\
0 & 0 & 0 & 0 & -1 \\
0 & 0 & & &
\end{array}\right]
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$$
\left[\begin{array}{ccc:cc} 0 & 1 & -2 & 3 & 3 \\ 1 & -1 & 1 & 0 & 1 \\ 1 & 0 & -1 & 3 & 3 \\ 1 & 1 & 2 & 1 & 1 \end{array}\right]
\sim
\left[\begin{array}{ccc:cc} 1 & -1 & 1 & 0 & 1 \\ 0 & 1 & -2 & 3 & 3 \\ 1 & 0 & -1 & 3 & 3 \\ 1 & 1 & 2 & 1 & 1 \end{array}\right] / \cdot (-1) \, / \cdot (-1)
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\left[\begin{array}{ccc:cc} 1 & -1 & 1 & 0 & 1 \\ 0 & 1 & -2 & 3 & 3 \\ 0 & 1 & -2 & 3 & 2 \\ 0 & 2 & 1 & 1 & 0 \end{array}\right] / \cdot (-1) \, / \cdot (-2)
\sim
\left[\begin{array}{ccc:cc} 1 & -1 & 1 & 0 & 1 \\ 0 & 1 & -2 & 3 & 3 \\ 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 5 & & \end{array}\right]
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\left[\begin{array}{ccc|cc}
0 & 1 & -2 & 3 & 3 \\
1 & -1 & 1 & 0 & 1 \\
1 & 0 & -1 & 3 & 3 \\
1 & 1 & 2 & 1 & 1
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\left[\begin{array}{ccc|cc}
1 & -1 & 1 & 0 & 1 \\
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\left[\begin{array}{ccc|cc}
1 & -1 & 1 & 0 & 1 \\
0 & 1 & -2 & 3 & 3 \\
0 & 1 & -2 & 3 & 2 \\
0 & 2 & 1 & 1 & 0
\end{array}\right] \begin{array}{l} \\ / \cdot (-1) \; / \cdot (-2) \\ \\ \\ \end{array}
\sim
\left[\begin{array}{ccc|cc}
1 & -1 & 1 & 0 & 1 \\
0 & 1 & -2 & 3 & 3 \\
0 & 0 & 0 & 0 & -1 \\
0 & 0 & 5 & -5 &
\end{array}\right]
$$

$$\left[\begin{array}{ccc:cc} 0 & 1 & -2 & 3 & 3 \\ 1 & -1 & 1 & 0 & 1 \\ 1 & 0 & -1 & 3 & 3 \\ 1 & 1 & 2 & 1 & 1 \end{array}\right] \sim \left[\begin{array}{ccc:cc} 1 & -1 & 1 & 0 & 1 \\ 0 & 1 & -2 & 3 & 3 \\ 1 & 0 & -1 & 3 & 3 \\ 1 & 1 & 2 & 1 & 1 \end{array}\right] / \cdot (-1) / \cdot (-1) \sim$$

$$\sim \left[\begin{array}{ccc:cc} 1 & -1 & 1 & 0 & 1 \\ 0 & 1 & -2 & 3 & 3 \\ 0 & 1 & -2 & 3 & 2 \\ 0 & 2 & 1 & 1 & 0 \end{array}\right] / \cdot (-1) / \cdot (-2) \sim \left[\begin{array}{ccc:cc} 1 & -1 & 1 & 0 & 1 \\ 0 & 1 & -2 & 3 & 3 \\ 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 5 & -5 & -6 \end{array}\right]$$

$$
\left[\begin{array}{ccc:cc} 0 & 1 & -2 & 3 & 3 \\ 1 & -1 & 1 & 0 & 1 \\ 1 & 0 & -1 & 3 & 3 \\ 1 & 1 & 2 & 1 & 1 \end{array}\right]
\sim
\left[\begin{array}{ccc:cc} 1 & -1 & 1 & 0 & 1 \\ 0 & 1 & -2 & 3 & 3 \\ 1 & 0 & -1 & 3 & 3 \\ 1 & 1 & 2 & 1 & 1 \end{array}\right] / \cdot (-1) \, / \cdot (-1)
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\left[\begin{array}{ccc:cc} 1 & -1 & 1 & 0 & 1 \\ 0 & 1 & -2 & 3 & 3 \\ 0 & 1 & -2 & 3 & 2 \\ 0 & 2 & 1 & 1 & 0 \end{array}\right] / \cdot (-1) \, / \cdot (-2)
\sim
\left[\begin{array}{ccc:cc} 1 & -1 & 1 & 0 & 1 \\ 0 & 1 & -2 & 3 & 3 \\ 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 5 & -5 & -6 \end{array}\right]
$$

$$
\left[\begin{array}{ccc:cc}
0 & 1 & -2 & 3 & 3 \\
1 & -1 & 1 & 0 & 1 \\
1 & 0 & -1 & 3 & 3 \\
1 & 1 & 2 & 1 & 1
\end{array}\right]
\sim
\left[\begin{array}{ccc:cc}
1 & -1 & 1 & 0 & 1 \\
0 & 1 & -2 & 3 & 3 \\
1 & 0 & -1 & 3 & 3 \\
1 & 1 & 2 & 1 & 1
\end{array}\right] / \cdot (-1) \, / \cdot (-1)
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\left[\begin{array}{ccc:cc}
1 & -1 & 1 & 0 & 1 \\
0 & 1 & -2 & 3 & 3 \\
0 & 1 & -2 & 3 & 2 \\
0 & 2 & 1 & 1 & 0
\end{array}\right] / \cdot (-1) \, / \cdot (-2)
\sim
\left[\begin{array}{ccc:cc}
1 & -1 & 1 & 0 & 1 \\
0 & 1 & -2 & 3 & 3 \\
0 & 0 & 0 & 0 & -1 \\
0 & 0 & 5 & -5 & -6
\end{array}\right]
$$

$$\left[\begin{array}{ccc|cc} 0 & 1 & -2 & 3 & 3 \\ 1 & -1 & 1 & 0 & 1 \\ 1 & 0 & -1 & 3 & 3 \\ 1 & 1 & 2 & 1 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|cc} 1 & -1 & 1 & 0 & 1 \\ 0 & 1 & -2 & 3 & 3 \\ 1 & 0 & -1 & 3 & 3 \\ 1 & 1 & 2 & 1 & 1 \end{array}\right] / \cdot (-1) \,/ \cdot (-1) \sim$$

$$\sim \left[\begin{array}{ccc|cc} 1 & -1 & 1 & 0 & 1 \\ 0 & 1 & -2 & 3 & 3 \\ 0 & 1 & -2 & 3 & 2 \\ 0 & 2 & 1 & 1 & 0 \end{array}\right] / \cdot (-1) \,/ \cdot (-2) \sim \left[\begin{array}{ccc|cc} 1 & -1 & 1 & 0 & 1 \\ 0 & 1 & -2 & 3 & 3 \\ 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 5 & -5 & -6 \end{array}\right] \sim$$

$$\sim \left[\begin{array}{ccc|cc} & & & & \\ & & & & \\ & & & & \\ & & & & \end{array}\right]$$

$$
\left[\begin{array}{ccc|cc}
0 & 1 & -2 & 3 & 3 \\
1 & -1 & 1 & 0 & 1 \\
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\end{array}\right]
\sim
\begin{array}{l}
\left[\begin{array}{ccc|cc}
1 & -1 & 1 & 0 & 1 \\
0 & 1 & -2 & 3 & 3 \\
1 & 0 & -1 & 3 & 3 \\
1 & 1 & 2 & 1 & 1
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\left[\begin{array}{ccc|cc}
1 & -1 & 1 & 0 & 1 \\
0 & 1 & -2 & 3 & 3 \\
0 & 1 & -2 & 3 & 2 \\
0 & 2 & 1 & 1 & 0
\end{array}\right]
\begin{array}{l}
\\ /\cdot(-1)\,/\cdot(-2) \\ \\ \\
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\sim
\left[\begin{array}{ccc|cc}
1 & -1 & 1 & 0 & 1 \\
0 & 1 & -2 & 3 & 3 \\
0 & 0 & 0 & 0 & -1 \\
0 & 0 & 5 & -5 & -6
\end{array}\right]
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\left[\begin{array}{ccc|cc}
1 & -1 & 1 & 0 & 1 \\
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$$\left[\begin{array}{ccc|cc} 0 & 1 & -2 & 3 & 3 \\ 1 & -1 & 1 & 0 & 1 \\ 1 & 0 & -1 & 3 & 3 \\ 1 & 1 & 2 & 1 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|cc} 1 & -1 & 1 & 0 & 1 \\ 0 & 1 & -2 & 3 & 3 \\ 1 & 0 & -1 & 3 & 3 \\ 1 & 1 & 2 & 1 & 1 \end{array}\right] /\cdot(-1) \;/\cdot(-1) \quad \sim$$

$$\sim \left[\begin{array}{ccc|cc} 1 & -1 & 1 & 0 & 1 \\ 0 & 1 & -2 & 3 & 3 \\ 0 & 1 & -2 & 3 & 2 \\ 0 & 2 & 1 & 1 & 0 \end{array}\right] /\cdot(-1) \;/\cdot(-2) \sim \left[\begin{array}{ccc|cc} 1 & -1 & 1 & 0 & 1 \\ 0 & 1 & -2 & 3 & 3 \\ 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 5 & -5 & -6 \end{array}\right] \sim$$

$$\sim \left[\begin{array}{ccc|cc} 1 & -1 & 1 & 0 & 1 \\ 0 & 1 & -2 & 3 & 3 \\ & & & & \\ & & & & \end{array}\right]$$

$$
\left[\begin{array}{ccc:cc}
0 & 1 & -2 & 3 & 3 \\
1 & -1 & 1 & 0 & 1 \\
1 & 0 & -1 & 3 & 3 \\
1 & 1 & 2 & 1 & 1
\end{array}\right]
\sim
\left[\begin{array}{ccc:cc}
1 & -1 & 1 & 0 & 1 \\
0 & 1 & -2 & 3 & 3 \\
1 & 0 & -1 & 3 & 3 \\
1 & 1 & 2 & 1 & 1
\end{array}\right] / \cdot (-1) \, / \cdot (-1)
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\left[\begin{array}{ccc:cc}
1 & -1 & 1 & 0 & 1 \\
0 & 1 & -2 & 3 & 3 \\
0 & 1 & -2 & 3 & 2 \\
0 & 2 & 1 & 1 & 0
\end{array}\right] / \cdot (-1) \, / \cdot (-2)
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\left[\begin{array}{ccc:cc}
1 & -1 & 1 & 0 & 1 \\
0 & 1 & -2 & 3 & 3 \\
0 & 0 & 0 & 0 & -1 \\
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\left[\begin{array}{ccc:cc}
1 & -1 & 1 & 0 & 1 \\
0 & 1 & -2 & 3 & 3 \\
0 & 0 & 5 & -5 & -6 \\
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$$
\left[\begin{array}{ccc:cc} 0 & 1 & -2 & 3 & 3 \\ 1 & -1 & 1 & 0 & 1 \\ 1 & 0 & -1 & 3 & 3 \\ 1 & 1 & 2 & 1 & 1 \end{array}\right]
\sim
\left[\begin{array}{ccc:cc} 1 & -1 & 1 & 0 & 1 \\ 0 & 1 & -2 & 3 & 3 \\ 1 & 0 & -1 & 3 & 3 \\ 1 & 1 & 2 & 1 & 1 \end{array}\right] / \cdot (-1) \, / \cdot (-1)
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\left[\begin{array}{ccc:cc} 1 & -1 & 1 & 0 & 1 \\ 0 & 1 & -2 & 3 & 3 \\ 0 & 1 & -2 & 3 & 2 \\ 0 & 2 & 1 & 1 & 0 \end{array}\right] / \cdot (-1) \, / \cdot (-2)
\sim
\left[\begin{array}{ccc:cc} 1 & -1 & 1 & 0 & 1 \\ 0 & 1 & -2 & 3 & 3 \\ 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 5 & -5 & -6 \end{array}\right]
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\left[\begin{array}{ccc:cc} 1 & -1 & 1 & 0 & 1 \\ 0 & 1 & -2 & 3 & 3 \\ 0 & 0 & 5 & -5 & -6 \\ 0 & 0 & 0 & 0 & -1 \end{array}\right]
$$

$$\left[\begin{array}{ccc|cc} 0 & 1 & -2 & 3 & 3 \\ 1 & -1 & 1 & 0 & 1 \\ 1 & 0 & -1 & 3 & 3 \\ 1 & 1 & 2 & 1 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|cc} 1 & -1 & 1 & 0 & 1 \\ 0 & 1 & -2 & 3 & 3 \\ 1 & 0 & -1 & 3 & 3 \\ 1 & 1 & 2 & 1 & 1 \end{array}\right] \begin{array}{l} /\cdot(-1)\,/\cdot(-1) \\ \\ \\ \\ \end{array} \sim$$

$$\sim \left[\begin{array}{ccc|cc} 1 & -1 & 1 & 0 & 1 \\ 0 & 1 & -2 & 3 & 3 \\ 0 & 1 & -2 & 3 & 2 \\ 0 & 2 & 1 & 1 & 0 \end{array}\right] \begin{array}{l} \\ /\cdot(-1)\,/\cdot(-2) \\ \\ \\ \end{array} \sim \left[\begin{array}{ccc|cc} 1 & -1 & 1 & 0 & 1 \\ 0 & 1 & -2 & 3 & 3 \\ 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 5 & -5 & -6 \end{array}\right] \sim$$

$$\sim \left[\begin{array}{ccc|cc} 1 & -1 & 1 & 0 & 1 \\ 0 & 1 & -2 & 3 & 3 \\ 0 & 0 & 5 & -5 & -6 \\ 0 & 0 & 0 & 0 & -1 \end{array}\right]$$

$$\left[\begin{array}{ccc|cc} 0 & 1 & -2 & 3 & 3 \\ 1 & -1 & 1 & 0 & 1 \\ 1 & 0 & -1 & 3 & 3 \\ 1 & 1 & 2 & 1 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|cc} 1 & -1 & 1 & 0 & 1 \\ 0 & 1 & -2 & 3 & 3 \\ 1 & 0 & -1 & 3 & 3 \\ 1 & 1 & 2 & 1 & 1 \end{array}\right] / \cdot (-1) \, / \cdot (-1) \sim$$

$$\sim \left[\begin{array}{ccc|cc} 1 & -1 & 1 & 0 & 1 \\ 0 & 1 & -2 & 3 & 3 \\ 0 & 1 & -2 & 3 & 2 \\ 0 & 2 & 1 & 1 & 0 \end{array}\right] / \cdot (-1) \, / \cdot (-2) \sim \left[\begin{array}{ccc|cc} 1 & -1 & 1 & 0 & 1 \\ 0 & 1 & -2 & 3 & 3 \\ 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 5 & -5 & -6 \end{array}\right] \sim$$

$$\sim \left[\begin{array}{ccc|cc} 1 & -1 & 1 & 0 & 1 \\ 0 & 1 & -2 & 3 & 3 \\ 0 & 0 & 5 & -5 & -6 \\ 0 & 0 & 0 & 0 & -1 \end{array}\right]$$

$$
\left[\begin{array}{ccc:cc}
0 & 1 & -2 & 3 & 3 \\
1 & -1 & 1 & 0 & 1 \\
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1 & 1 & 2 & 1 & 1
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\left[\begin{array}{ccc:cc}
1 & -1 & 1 & 0 & 1 \\
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\left[\begin{array}{ccc:cc}
1 & -1 & 1 & 0 & 1 \\
0 & 1 & -2 & 3 & 3 \\
0 & 1 & -2 & 3 & 2 \\
0 & 2 & 1 & 1 & 0
\end{array}\right] / \cdot (-1) \, / \cdot (-2)
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\left[\begin{array}{ccc:cc}
1 & -1 & 1 & 0 & 1 \\
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0 & 0 & 0 & 0 & -1 \\
0 & 0 & 5 & -5 & -6
\end{array}\right]
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\left[\begin{array}{ccc:cc}
1 & -1 & 1 & 0 & 1 \\
0 & 1 & -2 & 3 & 3 \\
0 & 0 & 5 & -5 & -6 \\
0 & 0 & 0 & 0 & -1
\end{array}\right]
$$

$$\left[\begin{array}{ccc:cc} 0 & 1 & -2 & 3 & 3 \\ 1 & -1 & 1 & 0 & 1 \\ 1 & 0 & -1 & 3 & 3 \\ 1 & 1 & 2 & 1 & 1 \end{array}\right] \sim \left[\begin{array}{ccc:cc} 1 & -1 & 1 & 0 & 1 \\ 0 & 1 & -2 & 3 & 3 \\ 1 & 0 & -1 & 3 & 3 \\ 1 & 1 & 2 & 1 & 1 \end{array}\right] /\cdot(-1) \, /\cdot(-1) \quad \sim$$

$$\sim \left[\begin{array}{ccc:cc} 1 & -1 & 1 & 0 & 1 \\ 0 & 1 & -2 & 3 & 3 \\ 0 & 1 & -2 & 3 & 2 \\ 0 & 2 & 1 & 1 & 0 \end{array}\right] /\cdot(-1) \, /\cdot(-2) \sim \left[\begin{array}{ccc:cc} 1 & -1 & 1 & 0 & 1 \\ 0 & 1 & -2 & 3 & 3 \\ 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 5 & -5 & -6 \end{array}\right] \sim$$

$$\sim \left[\begin{array}{ccc:cc} 1 & -1 & 1 & 0 & 1 \\ 0 & 1 & -2 & 3 & 3 \\ 0 & 0 & 5 & -5 & -6 \\ 0 & 0 & 0 & 0 & -1 \end{array}\right]$$

$$\left[\begin{array}{ccc:cc} 0 & 1 & -2 & 3 & 3 \\ 1 & -1 & 1 & 0 & 1 \\ 1 & 0 & -1 & 3 & 3 \\ 1 & 1 & 2 & 1 & 1 \end{array}\right] \sim \left[\begin{array}{ccc:cc} 1 & -1 & 1 & 0 & 1 \\ 0 & 1 & -2 & 3 & 3 \\ 1 & 0 & -1 & 3 & 3 \\ 1 & 1 & 2 & 1 & 1 \end{array}\right] /\cdot(-1) \, /\cdot(-1) \sim$$

$$\sim \left[\begin{array}{ccc:cc} 1 & -1 & 1 & 0 & 1 \\ 0 & 1 & -2 & 3 & 3 \\ 0 & 1 & -2 & 3 & 2 \\ 0 & 2 & 1 & 1 & 0 \end{array}\right] /\cdot(-1) \, /\cdot(-2) \sim \left[\begin{array}{ccc:cc} 1 & -1 & 1 & 0 & 1 \\ 0 & 1 & -2 & 3 & 3 \\ 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 5 & -5 & -6 \end{array}\right] \sim$$

$$\sim \left[\begin{array}{ccc:cc} 1 & -1 & 1 & 0 & 1 \\ 0 & 1 & -2 & 3 & 3 \\ 0 & 0 & 5 & -5 & -6 \\ 0 & 0 & 0 & 0 & -1 \end{array}\right]$$

$$
\left[\begin{array}{ccc:cc} 0 & 1 & -2 & 3 & 3 \\ 1 & -1 & 1 & 0 & 1 \\ 1 & 0 & -1 & 3 & 3 \\ 1 & 1 & 2 & 1 & 1 \end{array}\right]
\sim
\left[\begin{array}{ccc:cc} 1 & -1 & 1 & 0 & 1 \\ 0 & 1 & -2 & 3 & 3 \\ 1 & 0 & -1 & 3 & 3 \\ 1 & 1 & 2 & 1 & 1 \end{array}\right] / \cdot (-1) \, / \cdot (-1)
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\left[\begin{array}{ccc:cc} 1 & -1 & 1 & 0 & 1 \\ 0 & 1 & -2 & 3 & 3 \\ 0 & 1 & -2 & 3 & 2 \\ 0 & 2 & 1 & 1 & 0 \end{array}\right] / \cdot (-1) \, / \cdot (-2)
\sim
\left[\begin{array}{ccc:cc} 1 & -1 & 1 & 0 & 1 \\ 0 & 1 & -2 & 3 & 3 \\ 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 5 & -5 & -6 \end{array}\right]
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\left[\begin{array}{ccc:cc} 1 & -1 & 1 & 0 & 1 \\ 0 & 1 & -2 & 3 & 3 \\ 0 & 0 & 5 & -5 & -6 \\ 0 & 0 & 0 & 0 & -1 \end{array}\right]
$$

$$A = \left[\begin{array}{ccc:cc} 0 & 1 & -2 & 3 & 3 \\ 1 & -1 & 1 & 0 & 1 \\ 1 & 0 & -1 & 3 & 3 \\ 1 & 1 & 2 & 1 & 1 \end{array}\right] \sim \quad \cdots \quad \sim \left[\begin{array}{ccc:cc} 1 & -1 & 1 & 0 & 1 \\ 0 & 1 & -2 & 3 & 3 \\ 0 & 0 & 5 & -5 & -6 \\ 0 & 0 & 0 & 0 & -1 \end{array}\right]$$

$$A = \begin{array}{c} \begin{array}{ccccc} p & q & r & f & g \end{array} \\ \left[\begin{array}{ccc:cc} 0 & 1 & -2 & 3 & 3 \\ 1 & -1 & 1 & 0 & 1 \\ 1 & 0 & -1 & 3 & 3 \\ 1 & 1 & 2 & 1 & 1 \end{array}\right] \end{array} \sim \quad \cdots \quad \sim \left[\begin{array}{ccc:cc} 1 & -1 & 1 & 0 & 1 \\ 0 & 1 & -2 & 3 & 3 \\ 0 & 0 & 5 & -5 & -6 \\ 0 & 0 & 0 & 0 & -1 \end{array}\right]$$

$$A = \begin{array}{ccccc} p & q & r & f & g \\ \end{array}$$

$$A = \left[\begin{array}{ccc:cc} 0 & 1 & -2 & 3 & 3 \\ 1 & -1 & 1 & 0 & 1 \\ 1 & 0 & -1 & 3 & 3 \\ 1 & 1 & 2 & 1 & 1 \end{array}\right] \sim \quad \cdots \quad \sim \left[\begin{array}{ccc:cc} 1 & -1 & 1 & 0 & 1 \\ 0 & 1 & -2 & 3 & 3 \\ 0 & 0 & 5 & -5 & -6 \\ 0 & 0 & 0 & 0 & -1 \end{array}\right]$$

a) Polinomi p, q i r su linearno nezavisni u $\mathcal{P}_4(t)$

$$
\begin{array}{ccccc}
p & q & r & f & g \\
\downarrow & \downarrow & \downarrow & \downarrow & \downarrow
\end{array}
$$

$$
A = \left[\begin{array}{ccc:cc}
0 & 1 & -2 & 3 & 3 \\
1 & -1 & 1 & 0 & 1 \\
1 & 0 & -1 & 3 & 3 \\
1 & 1 & 2 & 1 & 1
\end{array}\right] \sim \quad \cdots \quad \sim \left[\begin{array}{ccc:cc}
1 & -1 & 1 & 0 & 1 \\
0 & 1 & -2 & 3 & 3 \\
0 & 0 & 5 & -5 & -6 \\
0 & 0 & 0 & 0 & -1
\end{array}\right]
$$

a) Polinomi p, q i r su linearno nezavisni u $\mathcal{P}_4(t)$ pa je $\mathcal{L}(p, q, r)$ potprostor dimenzije 3 u vektorskom prostoru $\mathcal{P}_4(t)$.

$$A = \begin{array}{c} \begin{array}{ccccc} p & q & r & f & g \end{array} \\ \left[\begin{array}{ccc:cc} 0 & 1 & -2 & 3 & 3 \\ 1 & -1 & 1 & 0 & 1 \\ 1 & 0 & -1 & 3 & 3 \\ 1 & 1 & 2 & 1 & 1 \end{array}\right] \end{array} \sim \quad \cdots \quad \sim \left[\begin{array}{ccc:cc} 1 & -1 & 1 & 0 & 1 \\ 0 & 1 & -2 & 3 & 3 \\ 0 & 0 & 5 & -5 & -6 \\ 0 & 0 & 0 & 0 & -1 \end{array}\right]$$

a) Polinomi p, q i r su linearno nezavisni u $\mathcal{P}_4(t)$ pa je $\mathcal{L}(p, q, r)$ potprostor dimenzije 3 u vektorskom prostoru $\mathcal{P}_4(t)$.

b) Polinom f se može prikazati kao linearna kombinacija polinoma p, q i r.

$$A = \begin{array}{c} \begin{array}{ccccc} p & q & r & f & g \end{array} \\ \left[\begin{array}{ccc|cc} 0 & 1 & -2 & 3 & 3 \\ 1 & -1 & 1 & 0 & 1 \\ 1 & 0 & -1 & 3 & 3 \\ 1 & 1 & 2 & 1 & 1 \end{array}\right] \end{array} \sim \quad \cdots \quad \sim \left[\begin{array}{ccc|cc} 1 & -1 & 1 & 0 & 1 \\ 0 & 1 & -2 & 3 & 3 \\ 0 & 0 & 5 & -5 & -6 \\ 0 & 0 & 0 & 0 & -1 \end{array}\right]$$

a) Polinomi p, q i r su linearno nezavisni u $\mathcal{P}_4(t)$ pa je $\mathcal{L}(p, q, r)$ potprostor dimenzije 3 u vektorskom prostoru $\mathcal{P}_4(t)$.

b) Polinom f se može prikazati kao linearna kombinacija polinoma p, q i r. Drugim riječima, $f \in \mathcal{L}(p, q, r)$.

$$A = \begin{array}{c} \begin{array}{ccccc} p & q & r & f & g \end{array} \\ \left[\begin{array}{ccc:cc} 0 & 1 & -2 & 3 & 3 \\ 1 & -1 & 1 & 0 & 1 \\ 1 & 0 & -1 & 3 & 3 \\ 1 & 1 & 2 & 1 & 1 \end{array}\right] \end{array} \sim \quad \cdots \quad \sim \left[\begin{array}{ccc:cc} 1 & -1 & 1 & 0 & 1 \\ 0 & 1 & -2 & 3 & 3 \\ 0 & 0 & 5 & -5 & -6 \\ 0 & 0 & 0 & 0 & -1 \end{array}\right]$$

a) Polinomi p, q i r su linearno nezavisni u $\mathcal{P}_4(t)$ pa je $\mathcal{L}(p, q, r)$ potprostor dimenzije 3 u vektorskom prostoru $\mathcal{P}_4(t)$.

b) Polinom f se može prikazati kao linearna kombinacija polinoma p, q i r. Drugim riječima, $f \in \mathcal{L}(p, q, r)$.

c) Polinom g se ne može prikazati kao linearna kombinacija polinoma p, q i r.

$$A = \begin{array}{c} \begin{array}{ccccc} p & q & r & f & g \end{array} \\ \left[\begin{array}{ccc|cc} 0 & 1 & -2 & 3 & 3 \\ 1 & -1 & 1 & 0 & 1 \\ 1 & 0 & -1 & 3 & 3 \\ 1 & 1 & 2 & 1 & 1 \end{array}\right] \end{array} \sim \quad \cdots \quad \sim \left[\begin{array}{ccc|cc} 1 & -1 & 1 & 0 & 1 \\ 0 & 1 & -2 & 3 & 3 \\ 0 & 0 & 5 & -5 & -6 \\ 0 & 0 & 0 & 0 & -1 \end{array}\right]$$

a) Polinomi p, q i r su linearno nezavisni u $\mathcal{P}_4(t)$ pa je $\mathcal{L}(p, q, r)$ potprostor dimenzije 3 u vektorskom prostoru $\mathcal{P}_4(t)$.

b) Polinom f se može prikazati kao linearna kombinacija polinoma p, q i r. Drugim riječima, $f \in \mathcal{L}(p, q, r)$.

c) Polinom g se ne može prikazati kao linearna kombinacija polinoma p, q i r. Drugim riječima, $g \notin \mathcal{L}(p, q, r)$.

$$\boxed{r(A) = 4}$$

$$\begin{array}{rc} & \begin{array}{ccccc} p & q & r & f & g \end{array} \\ A = & \left[\begin{array}{ccc:cc} 0 & 1 & -2 & 3 & 3 \\ 1 & -1 & 1 & 0 & 1 \\ 1 & 0 & -1 & 3 & 3 \\ 1 & 1 & 2 & 1 & 1 \end{array}\right] \end{array} \sim \quad \cdots \quad \sim \left[\begin{array}{ccc:cc} 1 & -1 & 1 & 0 & 1 \\ 0 & 1 & -2 & 3 & 3 \\ 0 & 0 & 5 & -5 & -6 \\ 0 & 0 & 0 & 0 & -1 \end{array}\right]$$

a) Polinomi p, q i r su linearno nezavisni u $\mathcal{P}_4(t)$ pa je $\mathcal{L}(p, q, r)$ potprostor dimenzije 3 u vektorskom prostoru $\mathcal{P}_4(t)$.

b) Polinom f se može prikazati kao linearna kombinacija polinoma p, q i r. Drugim riječima, $f \in \mathcal{L}(p, q, r)$.

c) Polinom g se ne može prikazati kao linearna kombinacija polinoma p, q i r. Drugim riječima, $g \notin \mathcal{L}(p, q, r)$.

$$\begin{array}{ccccc} p & q & r & f & g \end{array}$$

$\boxed{r(A) = 4}$

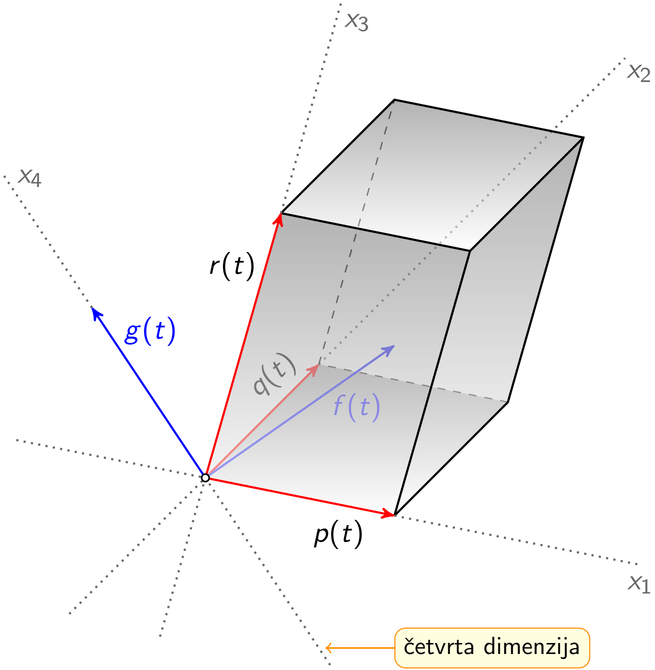
$\{p, q, r, g\}$ je jedna baza za vektorski prostor $\mathcal{P}_4(t)$.

$$A = \left[\begin{array}{ccc:cc} 0 & 1 & -2 & 3 & 3 \\ 1 & -1 & 1 & 0 & 1 \\ 1 & 0 & -1 & 3 & 3 \\ 1 & 1 & 2 & 1 & 1 \end{array}\right] \sim \quad \cdots \quad \sim \left[\begin{array}{ccc:cc} 1 & -1 & 1 & 0 & 1 \\ 0 & 1 & -2 & 3 & 3 \\ 0 & 0 & 5 & -5 & -6 \\ 0 & 0 & 0 & 0 & -1 \end{array}\right]$$

a) Polinomi p, q i r su linearno nezavisni u $\mathcal{P}_4(t)$ pa je $\mathcal{L}(p, q, r)$ potprostor dimenzije 3 u vektorskom prostoru $\mathcal{P}_4(t)$.

b) Polinom f se može prikazati kao linearna kombinacija polinoma p, q i r. Drugim riječima, $f \in \mathcal{L}(p, q, r)$.

c) Polinom g se ne može prikazati kao linearna kombinacija polinoma p, q i r. Drugim riječima, $g \notin \mathcal{L}(p, q, r)$.


x_3
x_2
x_4
$r(t)$
$g(t)$
$q(t)$
$f(t)$
$p(t)$
x_1
četvrta dimenzija

treći zadatak

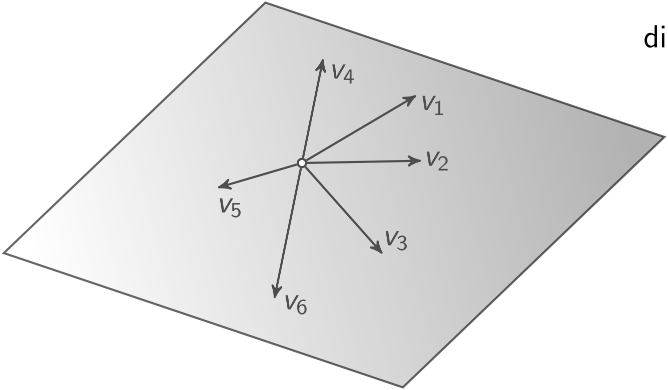
Skup izvodnica za potprostor

$$\mathcal{L}(v_1, v_2, v_3, v_4, v_5, v_6) = \mathcal{L}(v_1, v_2, v_4) = \mathcal{L}(v_1, v_2) = \mathcal{L}(v_3, v_6) = \cdots$$

$$\mathcal{L}(v_1, v_2, v_3, v_4, v_5, v_6) \neq \mathcal{L}(v_4, v_6)$$

$$\dim \mathcal{L}(v_1, v_2, v_3, v_4, v_5, v_6) = 2$$

$$\dim \mathcal{L}(v_4, v_6) = 1$$



Zadatak 3

Neka je W potprostor od $\mathbb{R}^5$ razapet s vektorima

$$u_1 = (1, 2, -1, 3, 4),\ u_2 = (2, 4, -2, 6, 8),\ u_3 = (1, 3, 2, 2, 6),$$
$$u_4 = (1, 4, 5, 1, 8),\ u_5 = (2, 7, 3, 3, 9).$$

Odredite jednu bazu i dimenziju vektorskog prostora W.

Zadatak 3

Neka je W potprostor od $\mathbb{R}^5$ razapet s vektorima

$$u_1 = (1, 2, -1, 3, 4),\ \ u_2 = (2, 4, -2, 6, 8),\ \ u_3 = (1, 3, 2, 2, 6),$$

$$u_4 = (1, 4, 5, 1, 8),\ \ u_5 = (2, 7, 3, 3, 9).$$

Odredite jednu bazu i dimenziju vektorskog prostora W.

Rješenje

$$A = \begin{bmatrix} \quad & \quad & \quad & \quad & \quad \\ & & & & \\ & & & & \\ & & & & \\ & & & & \end{bmatrix}$$

Zadatak 3

Neka je W potprostor od $\mathbb{R}^5$ razapet s vektorima

$$u_1 = (1, 2, -1, 3, 4),\ \ u_2 = (2, 4, -2, 6, 8),\ \ u_3 = (1, 3, 2, 2, 6),$$
$$u_4 = (1, 4, 5, 1, 8),\ \ u_5 = (2, 7, 3, 3, 9).$$

Odredite jednu bazu i dimenziju vektorskog prostora W.

Rješenje

$$A = \begin{bmatrix} 1 & & & & \\ 2 & & & & \\ -1 & & & & \\ 3 & & & & \\ 4 & & & & \end{bmatrix}$$

Zadatak 3

Neka je W potprostor od $\mathbb{R}^5$ razapet s vektorima

$$u_1 = (1,2,-1,3,4),\ \ u_2 = (2,4,-2,6,8),\ \ u_3 = (1,3,2,2,6),$$
$$u_4 = (1,4,5,1,8),\ \ u_5 = (2,7,3,3,9).$$

Odredite jednu bazu i dimenziju vektorskog prostora W.

Rješenje

$$A = \begin{bmatrix} 1 & 2 & & & \\ 2 & 4 & & & \\ -1 & -2 & & & \\ 3 & 6 & & & \\ 4 & 8 & & & \end{bmatrix}$$

Zadatak 3

Neka je W potprostor od $\mathbb{R}^5$ razapet s vektorima

$$u_1 = (1, 2, -1, 3, 4),\ \ u_2 = (2, 4, -2, 6, 8),\ \ u_3 = (1, 3, 2, 2, 6),$$
$$u_4 = (1, 4, 5, 1, 8),\ \ u_5 = (2, 7, 3, 3, 9).$$

Odredite jednu bazu i dimenziju vektorskog prostora W.

Rješenje

$$A = \begin{bmatrix} 1 & 2 & 1 & & \\ 2 & 4 & 3 & & \\ -1 & -2 & 2 & & \\ 3 & 6 & 2 & & \\ 4 & 8 & 6 & & \end{bmatrix}$$

Zadatak 3

Neka je W potprostor od $\mathbb{R}^5$ razapet s vektorima

$$u_1 = (1, 2, -1, 3, 4),\ u_2 = (2, 4, -2, 6, 8),\ u_3 = (1, 3, 2, 2, 6),$$

$$u_4 = (1, 4, 5, 1, 8),\ u_5 = (2, 7, 3, 3, 9).$$

Odredite jednu bazu i dimenziju vektorskog prostora W.

Rješenje

$$A = \begin{bmatrix} 1 & 2 & 1 & 1 & \\ 2 & 4 & 3 & 4 & \\ -1 & -2 & 2 & 5 & \\ 3 & 6 & 2 & 1 & \\ 4 & 8 & 6 & 8 & \end{bmatrix}$$

Zadatak 3

Neka je W potprostor od $\mathbb{R}^5$ razapet s vektorima

$$u_1 = (1, 2, -1, 3, 4),\ u_2 = (2, 4, -2, 6, 8),\ u_3 = (1, 3, 2, 2, 6),$$

$$u_4 = (1, 4, 5, 1, 8),\ u_5 = (2, 7, 3, 3, 9).$$

Odredite jednu bazu i dimenziju vektorskog prostora W.

Rješenje

$$A = \begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 2 & 4 & 3 & 4 & 7 \\ -1 & -2 & 2 & 5 & 3 \\ 3 & 6 & 2 & 1 & 3 \\ 4 & 8 & 6 & 8 & 9 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 2 & 4 & 3 & 4 & 7 \\ -1 & -2 & 2 & 5 & 3 \\ 3 & 6 & 2 & 1 & 3 \\ 4 & 8 & 6 & 8 & 9 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 2 & 4 & 3 & 4 & 7 \\ -1 & -2 & 2 & 5 & 3 \\ 3 & 6 & 2 & 1 & 3 \\ 4 & 8 & 6 & 8 & 9 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 2 & 4 & 3 & 4 & 7 \\ -1 & -2 & 2 & 5 & 3 \\ 3 & 6 & 2 & 1 & 3 \\ 4 & 8 & 6 & 8 & 9 \end{bmatrix} / \cdot (-2)$$

$$\begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 2 & 4 & 3 & 4 & 7 \\ -1 & -2 & 2 & 5 & 3 \\ 3 & 6 & 2 & 1 & 3 \\ 4 & 8 & 6 & 8 & 9 \end{bmatrix} / \cdot (-2) \quad +$$

$$\begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 2 & 4 & 3 & 4 & 7 \\ -1 & -2 & 2 & 5 & 3 \\ 3 & 6 & 2 & 1 & 3 \\ 4 & 8 & 6 & 8 & 9 \end{bmatrix} \begin{matrix} / \cdot (-2) \ / \cdot 1 \\ + \\ \\ \\ \\ \end{matrix}$$

$$\begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 2 & 4 & 3 & 4 & 7 \\ -1 & -2 & 2 & 5 & 3 \\ 3 & 6 & 2 & 1 & 3 \\ 4 & 8 & 6 & 8 & 9 \end{bmatrix} / \cdot (-2) \; / \cdot 1$$

$$\begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 2 & 4 & 3 & 4 & 7 \\ -1 & -2 & 2 & 5 & 3 \\ 3 & 6 & 2 & 1 & 3 \\ 4 & 8 & 6 & 8 & 9 \end{bmatrix} / \cdot (-2) \;/ \cdot 1 \;/ \cdot (-3)$$

$+$ $+$

$$\begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 2 & 4 & 3 & 4 & 7 \\ -1 & -2 & 2 & 5 & 3 \\ 3 & 6 & 2 & 1 & 3 \\ 4 & 8 & 6 & 8 & 9 \end{bmatrix} / \cdot (-2) \; / \cdot 1 \; / \cdot (-3)$$

$+$ $+$ $+$

$$\begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 2 & 4 & 3 & 4 & 7 \\ -1 & -2 & 2 & 5 & 3 \\ 3 & 6 & 2 & 1 & 3 \\ 4 & 8 & 6 & 8 & 9 \end{bmatrix} / \cdot (-2) \; / \cdot 1 \; / \cdot (-3) \; / \cdot (-4)$$

$+$ $+$ $+$

$$\begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 2 & 4 & 3 & 4 & 7 \\ -1 & -2 & 2 & 5 & 3 \\ 3 & 6 & 2 & 1 & 3 \\ 4 & 8 & 6 & 8 & 9 \end{bmatrix} \begin{matrix} / \cdot (-2) \;/ \cdot 1 \;/ \cdot (-3) \;/ \cdot (-4) \\ + \\ + \\ + \\ + \end{matrix}$$

$$\begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 2 & 4 & 3 & 4 & 7 \\ -1 & -2 & 2 & 5 & 3 \\ 3 & 6 & 2 & 1 & 3 \\ 4 & 8 & 6 & 8 & 9 \end{bmatrix} \begin{matrix} /\cdot(-2) \; /\cdot 1 \; /\cdot(-3) \; /\cdot(-4) \\ + \\ + \\ + \\ + \end{matrix} \sim$$

$$\sim \begin{bmatrix} & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 2 & 4 & 3 & 4 & 7 \\ -1 & -2 & 2 & 5 & 3 \\ 3 & 6 & 2 & 1 & 3 \\ 4 & 8 & 6 & 8 & 9 \end{bmatrix} / \cdot (-2) \; / \cdot 1 \; / \cdot (-3) \; / \cdot (-4) \qquad \sim$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ & & & & \\ & & & & \\ & & & & \\ & & & & \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 2 & 4 & 3 & 4 & 7 \\ -1 & -2 & 2 & 5 & 3 \\ 3 & 6 & 2 & 1 & 3 \\ 4 & 8 & 6 & 8 & 9 \end{bmatrix} \begin{matrix} / \cdot (-2) \; / \cdot 1 \; / \cdot (-3) \; / \cdot (-4) \\ + \\ + \\ + \\ + \end{matrix} \sim$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 0 & & & & \\ & & & & \\ & & & & \\ & & & & \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 2 & 4 & 3 & 4 & 7 \\ -1 & -2 & 2 & 5 & 3 \\ 3 & 6 & 2 & 1 & 3 \\ 4 & 8 & 6 & 8 & 9 \end{bmatrix} / \cdot (-2) \; / \cdot 1 \; / \cdot (-3) \; / \cdot (-4) \qquad \sim$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 0 & 0 & & & \\ & & & & \\ & & & & \\ & & & & \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 2 & 4 & 3 & 4 & 7 \\ -1 & -2 & 2 & 5 & 3 \\ 3 & 6 & 2 & 1 & 3 \\ 4 & 8 & 6 & 8 & 9 \end{bmatrix} /\cdot(-2)\ /\cdot 1\ /\cdot(-3)\ /\cdot(-4) \quad \sim$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 0 & 0 & 1 & & \\ & & & & \\ & & & & \\ & & & & \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 2 & 4 & 3 & 4 & 7 \\ -1 & -2 & 2 & 5 & 3 \\ 3 & 6 & 2 & 1 & 3 \\ 4 & 8 & 6 & 8 & 9 \end{bmatrix} /\cdot(-2)\ /\cdot 1\ /\cdot(-3)\ /\cdot(-4) \sim$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 0 & 0 & 1 & 2 & \\ & & & & \\ & & & & \\ & & & & \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 2 & 4 & 3 & 4 & 7 \\ -1 & -2 & 2 & 5 & 3 \\ 3 & 6 & 2 & 1 & 3 \\ 4 & 8 & 6 & 8 & 9 \end{bmatrix} /\cdot(-2) \; /\cdot 1 \; /\cdot(-3) \; /\cdot(-4) \quad \sim$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 0 & 0 & 1 & 2 & 3 \\ & & & & \\ & & & & \\ & & & & \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 2 & 4 & 3 & 4 & 7 \\ -1 & -2 & 2 & 5 & 3 \\ 3 & 6 & 2 & 1 & 3 \\ 4 & 8 & 6 & 8 & 9 \end{bmatrix} \begin{matrix} /\cdot(-2) \; /\cdot 1 \; /\cdot(-3) \; /\cdot(-4) \\ + \\ + \\ + \\ + \end{matrix} \sim$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & & & & \\ & & & & \\ & & & & \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 2 & 4 & 3 & 4 & 7 \\ -1 & -2 & 2 & 5 & 3 \\ 3 & 6 & 2 & 1 & 3 \\ 4 & 8 & 6 & 8 & 9 \end{bmatrix} / \cdot (-2) \; / \cdot 1 \; / \cdot (-3) \; / \cdot (-4) \quad \sim$$

$+$ $+$ $+$ $+$

$$\sim \begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & & & \\ & & & & \\ & & & & \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 2 & 4 & 3 & 4 & 7 \\ -1 & -2 & 2 & 5 & 3 \\ 3 & 6 & 2 & 1 & 3 \\ 4 & 8 & 6 & 8 & 9 \end{bmatrix} / \cdot (-2) \; / \cdot 1 \; / \cdot (-3) \; / \cdot (-4) \quad \sim$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 3 & & \\ & & & & \\ & & & & \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 2 & 4 & 3 & 4 & 7 \\ -1 & -2 & 2 & 5 & 3 \\ 3 & 6 & 2 & 1 & 3 \\ 4 & 8 & 6 & 8 & 9 \end{bmatrix} /\cdot(-2) \; /\cdot 1 \; /\cdot(-3) \; /\cdot(-4) \quad \sim$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 3 & 6 & \\ & & & & \\ & & & & \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 2 & 4 & 3 & 4 & 7 \\ -1 & -2 & 2 & 5 & 3 \\ 3 & 6 & 2 & 1 & 3 \\ 4 & 8 & 6 & 8 & 9 \end{bmatrix} \begin{matrix} / \cdot (-2) \; / \cdot 1 \; / \cdot (-3) \; / \cdot (-4) \\ + \\ + \\ + \\ + \end{matrix} \sim$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 3 & 6 & 5 \\ & & & & \\ & & & & \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 2 & 4 & 3 & 4 & 7 \\ -1 & -2 & 2 & 5 & 3 \\ 3 & 6 & 2 & 1 & 3 \\ 4 & 8 & 6 & 8 & 9 \end{bmatrix} / \cdot (-2) \; / \cdot 1 \; / \cdot (-3) \; / \cdot (-4) \qquad \sim$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 3 & 6 & 5 \\ 0 & & & & \\ & & & & \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 2 & 4 & 3 & 4 & 7 \\ -1 & -2 & 2 & 5 & 3 \\ 3 & 6 & 2 & 1 & 3 \\ 4 & 8 & 6 & 8 & 9 \end{bmatrix} / \cdot (-2) \; / \cdot 1 \; / \cdot (-3) \; / \cdot (-4) \quad \sim$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 3 & 6 & 5 \\ 0 & 0 & & & \\ & & & & \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 2 & 4 & 3 & 4 & 7 \\ -1 & -2 & 2 & 5 & 3 \\ 3 & 6 & 2 & 1 & 3 \\ 4 & 8 & 6 & 8 & 9 \end{bmatrix} / \cdot (-2) \ / \cdot 1 \ / \cdot (-3) \ / \cdot (-4) \quad \sim$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 3 & 6 & 5 \\ 0 & 0 & -1 & & \\ & & & & \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 2 & 4 & 3 & 4 & 7 \\ -1 & -2 & 2 & 5 & 3 \\ 3 & 6 & 2 & 1 & 3 \\ 4 & 8 & 6 & 8 & 9 \end{bmatrix} /\cdot(-2) \; /\cdot 1 \; /\cdot(-3) \; /\cdot(-4) \quad + \quad + \quad + \quad + \qquad \sim$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 3 & 6 & 5 \\ 0 & 0 & -1 & -2 & \\ & & & & \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 2 & 4 & 3 & 4 & 7 \\ -1 & -2 & 2 & 5 & 3 \\ 3 & 6 & 2 & 1 & 3 \\ 4 & 8 & 6 & 8 & 9 \end{bmatrix} \begin{matrix} /\cdot(-2) \ /\cdot 1 \ /\cdot(-3) \ /\cdot(-4) \\ + \\ + \\ + \\ + \end{matrix} \sim$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 3 & 6 & 5 \\ 0 & 0 & -1 & -2 & -3 \\ & & & & \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 2 & 4 & 3 & 4 & 7 \\ -1 & -2 & 2 & 5 & 3 \\ 3 & 6 & 2 & 1 & 3 \\ 4 & 8 & 6 & 8 & 9 \end{bmatrix} / \cdot (-2) \; / \cdot 1 \; / \cdot (-3) \; / \cdot (-4) \quad \sim$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 3 & 6 & 5 \\ 0 & 0 & -1 & -2 & -3 \\ 0 & & & & \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 2 & 4 & 3 & 4 & 7 \\ -1 & -2 & 2 & 5 & 3 \\ 3 & 6 & 2 & 1 & 3 \\ 4 & 8 & 6 & 8 & 9 \end{bmatrix} \begin{array}{l} / \cdot (-2) \; / \cdot 1 \; / \cdot (-3) \; / \cdot (-4) \\ + \\ + \\ + \\ + \end{array} \sim$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 3 & 6 & 5 \\ 0 & 0 & -1 & -2 & -3 \\ 0 & 0 & & & \end{bmatrix}$$

$$
\begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 2 & 4 & 3 & 4 & 7 \\ -1 & -2 & 2 & 5 & 3 \\ 3 & 6 & 2 & 1 & 3 \\ 4 & 8 & 6 & 8 & 9 \end{bmatrix} \begin{array}{l} /\cdot(-2)\ /\cdot 1\ /\cdot(-3)\ /\cdot(-4) \\ \\ \\ \\ \\ \end{array} \sim
$$

$$
\sim \begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 3 & 6 & 5 \\ 0 & 0 & -1 & -2 & -3 \\ 0 & 0 & 2 & & \end{bmatrix}
$$

$$\begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 2 & 4 & 3 & 4 & 7 \\ -1 & -2 & 2 & 5 & 3 \\ 3 & 6 & 2 & 1 & 3 \\ 4 & 8 & 6 & 8 & 9 \end{bmatrix} / \cdot (-2) \; / \cdot 1 \; / \cdot (-3) \; / \cdot (-4) \quad \sim$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 3 & 6 & 5 \\ 0 & 0 & -1 & -2 & -3 \\ 0 & 0 & 2 & 4 & \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 2 & 4 & 3 & 4 & 7 \\ -1 & -2 & 2 & 5 & 3 \\ 3 & 6 & 2 & 1 & 3 \\ 4 & 8 & 6 & 8 & 9 \end{bmatrix} / \cdot (-2) \ / \cdot 1 \ / \cdot (-3) \ / \cdot (-4) \quad \sim$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 3 & 6 & 5 \\ 0 & 0 & -1 & -2 & -3 \\ 0 & 0 & 2 & 4 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 2 & 4 & 3 & 4 & 7 \\ -1 & -2 & 2 & 5 & 3 \\ 3 & 6 & 2 & 1 & 3 \\ 4 & 8 & 6 & 8 & 9 \end{bmatrix} / \cdot (-2) \; / \cdot 1 \; / \cdot (-3) \; / \cdot (-4) \qquad + \quad + \quad + \quad + \qquad \sim$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 3 & 6 & 5 \\ 0 & 0 & -1 & -2 & -3 \\ 0 & 0 & 2 & 4 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 2 & 4 & 3 & 4 & 7 \\ -1 & -2 & 2 & 5 & 3 \\ 3 & 6 & 2 & 1 & 3 \\ 4 & 8 & 6 & 8 & 9 \end{bmatrix} \begin{matrix} /\cdot(-2) \; /\cdot 1 \; /\cdot(-3) \; /\cdot(-4) \\ + \\ + \\ + \\ + \end{matrix} \sim$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 3 & 6 & 5 \\ 0 & 0 & -1 & -2 & -3 \\ 0 & 0 & 2 & 4 & 1 \end{bmatrix} \begin{matrix} \\ /\cdot(-3) \\ \\ \\ \\ \end{matrix}$$

$$\begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 2 & 4 & 3 & 4 & 7 \\ -1 & -2 & 2 & 5 & 3 \\ 3 & 6 & 2 & 1 & 3 \\ 4 & 8 & 6 & 8 & 9 \end{bmatrix} \begin{matrix} / \cdot (-2) \ / \cdot 1 \ / \cdot (-3) \ / \cdot (-4) \\ + \quad + \quad + \quad + \\ \\ \\ \\ \end{matrix} \sim$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 3 & 6 & 5 \\ 0 & 0 & -1 & -2 & -3 \\ 0 & 0 & 2 & 4 & 1 \end{bmatrix} \begin{matrix} \\ / \cdot (-3) \\ + \\ \\ \\ \end{matrix}$$

$$\begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 2 & 4 & 3 & 4 & 7 \\ -1 & -2 & 2 & 5 & 3 \\ 3 & 6 & 2 & 1 & 3 \\ 4 & 8 & 6 & 8 & 9 \end{bmatrix} \begin{matrix} /\cdot(-2)\ /\cdot 1\ /\cdot(-3)\ /\cdot(-4) \\ + \\ + \\ + \\ + \end{matrix} \sim$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 3 & 6 & 5 \\ 0 & 0 & -1 & -2 & -3 \\ 0 & 0 & 2 & 4 & 1 \end{bmatrix} \begin{matrix} \\ /\cdot(-3)\ /\cdot 1 \\ + \\ \\ \\ \end{matrix}$$

$$\begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 2 & 4 & 3 & 4 & 7 \\ -1 & -2 & 2 & 5 & 3 \\ 3 & 6 & 2 & 1 & 3 \\ 4 & 8 & 6 & 8 & 9 \end{bmatrix} \begin{matrix} / \cdot (-2) \ / \cdot 1 \ / \cdot (-3) \ / \cdot (-4) \\ + \\ + \\ + \\ + \end{matrix} \sim$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 3 & 6 & 5 \\ 0 & 0 & -1 & -2 & -3 \\ 0 & 0 & 2 & 4 & 1 \end{bmatrix} \begin{matrix} \\ / \cdot (-3) \ / \cdot 1 \\ + \\ + \\ \\ \end{matrix}$$

$$\begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 2 & 4 & 3 & 4 & 7 \\ -1 & -2 & 2 & 5 & 3 \\ 3 & 6 & 2 & 1 & 3 \\ 4 & 8 & 6 & 8 & 9 \end{bmatrix} \begin{matrix} /\cdot(-2)\ /\cdot 1\ /\cdot(-3)\ /\cdot(-4) \\ + \\ + \\ + \\ + \end{matrix} \sim$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 3 & 6 & 5 \\ 0 & 0 & -1 & -2 & -3 \\ 0 & 0 & 2 & 4 & 1 \end{bmatrix} \begin{matrix} \\ /\cdot(-3)\ /\cdot 1\ /\cdot(-2) \\ + \\ + \\ \\ \end{matrix}$$

$$\begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 2 & 4 & 3 & 4 & 7 \\ -1 & -2 & 2 & 5 & 3 \\ 3 & 6 & 2 & 1 & 3 \\ 4 & 8 & 6 & 8 & 9 \end{bmatrix} \begin{matrix} /\cdot(-2) \; /\cdot 1 \; /\cdot(-3) \; /\cdot(-4) \\ + \\ + \\ + \\ + \end{matrix} \sim$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 3 & 6 & 5 \\ 0 & 0 & -1 & -2 & -3 \\ 0 & 0 & 2 & 4 & 1 \end{bmatrix} \begin{matrix} \\ /\cdot(-3) \; /\cdot 1 \; /\cdot(-2) \\ + \\ + \\ + \end{matrix}$$

$$\begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 2 & 4 & 3 & 4 & 7 \\ -1 & -2 & 2 & 5 & 3 \\ 3 & 6 & 2 & 1 & 3 \\ 4 & 8 & 6 & 8 & 9 \end{bmatrix} /\cdot(-2) \ /\cdot 1 \ /\cdot(-3) \ /\cdot(-4) \quad + \quad + \quad + \quad + \qquad \sim$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 3 & 6 & 5 \\ 0 & 0 & -1 & -2 & -3 \\ 0 & 0 & 2 & 4 & 1 \end{bmatrix} /\cdot(-3) \ /\cdot 1 \ /\cdot(-2) \quad + \quad + \quad + \qquad \sim$$

$$\sim \begin{bmatrix} \\ \\ \\ \\ \\ \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 2 & 4 & 3 & 4 & 7 \\ -1 & -2 & 2 & 5 & 3 \\ 3 & 6 & 2 & 1 & 3 \\ 4 & 8 & 6 & 8 & 9 \end{bmatrix} \begin{array}{l} /\cdot(-2)\ /\cdot 1\ /\cdot(-3)\ /\cdot(-4) \\ + \\ + \\ + \\ + \end{array} \sim$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 3 & 6 & 5 \\ 0 & 0 & -1 & -2 & -3 \\ 0 & 0 & 2 & 4 & 1 \end{bmatrix} \begin{array}{l} \\ /\cdot(-3)\ /\cdot 1\ /\cdot(-2) \\ + \\ + \\ + \end{array} \sim$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ & & & & \\ & & & & \\ & & & & \\ & & & & \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 2 & 4 & 3 & 4 & 7 \\ -1 & -2 & 2 & 5 & 3 \\ 3 & 6 & 2 & 1 & 3 \\ 4 & 8 & 6 & 8 & 9 \end{bmatrix} /\cdot(-2)\ /\cdot 1\ /\cdot(-3)\ /\cdot(-4) \quad \sim$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 3 & 6 & 5 \\ 0 & 0 & -1 & -2 & -3 \\ 0 & 0 & 2 & 4 & 1 \end{bmatrix} /\cdot(-3)\ /\cdot 1\ /\cdot(-2) \quad \sim$$

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$$\begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 2 & 4 & 3 & 4 & 7 \\ -1 & -2 & 2 & 5 & 3 \\ 3 & 6 & 2 & 1 & 3 \\ 4 & 8 & 6 & 8 & 9 \end{bmatrix} \begin{matrix} /\cdot(-2)\ /\cdot 1\ /\cdot(-3)\ /\cdot(-4) \\ + \\ + \\ + \\ + \end{matrix} \sim$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 3 & 6 & 5 \\ 0 & 0 & -1 & -2 & -3 \\ 0 & 0 & 2 & 4 & 1 \end{bmatrix} \begin{matrix} \\ /\cdot(-3)\ /\cdot 1\ /\cdot(-2) \\ + \\ + \\ + \end{matrix} \sim$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & & & & \\ & & & & \\ & & & & \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 2 & 4 & 3 & 4 & 7 \\ -1 & -2 & 2 & 5 & 3 \\ 3 & 6 & 2 & 1 & 3 \\ 4 & 8 & 6 & 8 & 9 \end{bmatrix} \begin{matrix} /\cdot(-2)\ /\cdot 1\ /\cdot(-3)\ /\cdot(-4) \\ \\ \\ \\ \\ \end{matrix} \sim$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 3 & 6 & 5 \\ 0 & 0 & -1 & -2 & -3 \\ 0 & 0 & 2 & 4 & 1 \end{bmatrix} \begin{matrix} \\ /\cdot(-3)\ /\cdot 1\ /\cdot(-2) \\ \\ \\ \\ \end{matrix} \sim$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & & & \\ & & & & \\ & & & & \end{bmatrix}$$

$$
\begin{bmatrix}
1 & 2 & 1 & 1 & 2 \\
2 & 4 & 3 & 4 & 7 \\
-1 & -2 & 2 & 5 & 3 \\
3 & 6 & 2 & 1 & 3 \\
4 & 8 & 6 & 8 & 9
\end{bmatrix}
\begin{matrix} /\cdot(-2) \; /\cdot 1 \; /\cdot(-3) \; /\cdot(-4) \\ + \\ + \\ + \\ + \end{matrix}
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\begin{bmatrix}
1 & 2 & 1 & 1 & 2 \\
0 & 0 & 1 & 2 & 3 \\
0 & 0 & 3 & 6 & 5 \\
0 & 0 & -1 & -2 & -3 \\
0 & 0 & 2 & 4 & 1
\end{bmatrix}
\begin{matrix} \\ /\cdot(-3) \; /\cdot 1 \; /\cdot(-2) \\ + \\ + \\ + \end{matrix}
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\begin{bmatrix}
1 & 2 & 1 & 1 & 2 \\
0 & 0 & 1 & 2 & 3 \\
0 & 0 & 0 & & \\
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\end{bmatrix}
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$$\begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 2 & 4 & 3 & 4 & 7 \\ -1 & -2 & 2 & 5 & 3 \\ 3 & 6 & 2 & 1 & 3 \\ 4 & 8 & 6 & 8 & 9 \end{bmatrix} / \cdot (-2) \; / \cdot 1 \; / \cdot (-3) \; / \cdot (-4) \quad \sim$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 3 & 6 & 5 \\ 0 & 0 & -1 & -2 & -3 \\ 0 & 0 & 2 & 4 & 1 \end{bmatrix} / \cdot (-3) \; / \cdot 1 \; / \cdot (-2) \quad \sim$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 & \\ & & & & \\ & & & & \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 2 & 4 & 3 & 4 & 7 \\ -1 & -2 & 2 & 5 & 3 \\ 3 & 6 & 2 & 1 & 3 \\ 4 & 8 & 6 & 8 & 9 \end{bmatrix} /\cdot(-2) \ /\cdot 1 \ /\cdot(-3) \ /\cdot(-4) \quad \sim$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 3 & 6 & 5 \\ 0 & 0 & -1 & -2 & -3 \\ 0 & 0 & 2 & 4 & 1 \end{bmatrix} \begin{matrix} \\ /\cdot(-3) \ /\cdot 1 \ /\cdot(-2) \\ \\ \\ \\ \end{matrix} \quad \sim$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 & -4 \\ & & & & \\ & & & & \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 2 & 4 & 3 & 4 & 7 \\ -1 & -2 & 2 & 5 & 3 \\ 3 & 6 & 2 & 1 & 3 \\ 4 & 8 & 6 & 8 & 9 \end{bmatrix} / \cdot (-2) \; / \cdot 1 \; / \cdot (-3) \; / \cdot (-4) \quad \sim$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 3 & 6 & 5 \\ 0 & 0 & -1 & -2 & -3 \\ 0 & 0 & 2 & 4 & 1 \end{bmatrix} \begin{matrix} \\ / \cdot (-3) \; / \cdot 1 \; / \cdot (-2) \\ \\ \\ \\ \end{matrix} \quad \sim$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 & -4 \\ 0 & & & & \\ & & & & \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 2 & 4 & 3 & 4 & 7 \\ -1 & -2 & 2 & 5 & 3 \\ 3 & 6 & 2 & 1 & 3 \\ 4 & 8 & 6 & 8 & 9 \end{bmatrix} /\cdot(-2)\ /\cdot 1\ /\cdot(-3)\ /\cdot(-4) \quad \sim$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 3 & 6 & 5 \\ 0 & 0 & -1 & -2 & -3 \\ 0 & 0 & 2 & 4 & 1 \end{bmatrix} /\cdot(-3)\ /\cdot 1\ /\cdot(-2) \quad \sim$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 & -4 \\ 0 & 0 & & & \\ & & & & \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 2 & 4 & 3 & 4 & 7 \\ -1 & -2 & 2 & 5 & 3 \\ 3 & 6 & 2 & 1 & 3 \\ 4 & 8 & 6 & 8 & 9 \end{bmatrix} /\cdot(-2)\ /\cdot 1\ /\cdot(-3)\ /\cdot(-4) \quad \sim$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 3 & 6 & 5 \\ 0 & 0 & -1 & -2 & -3 \\ 0 & 0 & 2 & 4 & 1 \end{bmatrix} \begin{array}{l} \\ /\cdot(-3)\ /\cdot 1\ /\cdot(-2) \\ \\ \\ \\ \end{array} \quad \sim$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 & -4 \\ 0 & 0 & 0 & & \\ & & & & \end{bmatrix}$$

$$
\begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 2 & 4 & 3 & 4 & 7 \\ -1 & -2 & 2 & 5 & 3 \\ 3 & 6 & 2 & 1 & 3 \\ 4 & 8 & 6 & 8 & 9 \end{bmatrix} \begin{matrix} /\cdot(-2) \; /\cdot 1 \; /\cdot(-3) \; /\cdot(-4) \end{matrix} \quad \sim
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$$
\sim \begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 3 & 6 & 5 \\ 0 & 0 & -1 & -2 & -3 \\ 0 & 0 & 2 & 4 & 1 \end{bmatrix} \begin{matrix} \\ /\cdot(-3) \; /\cdot 1 \; /\cdot(-2) \\ \\ \\ \\ \end{matrix} \quad \sim
$$

$$
\sim \begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 & -4 \\ 0 & 0 & 0 & 0 & \\ & & & & \end{bmatrix}
$$

$$\begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 2 & 4 & 3 & 4 & 7 \\ -1 & -2 & 2 & 5 & 3 \\ 3 & 6 & 2 & 1 & 3 \\ 4 & 8 & 6 & 8 & 9 \end{bmatrix} \begin{matrix} / \cdot (-2) \; / \cdot 1 \; / \cdot (-3) \; / \cdot (-4) \\ + \\ + \\ + \\ + \end{matrix} \sim$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 3 & 6 & 5 \\ 0 & 0 & -1 & -2 & -3 \\ 0 & 0 & 2 & 4 & 1 \end{bmatrix} \begin{matrix} \\ / \cdot (-3) \; / \cdot 1 \; / \cdot (-2) \\ + \\ + \\ + \end{matrix} \sim$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 & -4 \\ 0 & 0 & 0 & 0 & 0 \\ & & & & \end{bmatrix}$$

$$
\begin{bmatrix}
1 & 2 & 1 & 1 & 2 \\
2 & 4 & 3 & 4 & 7 \\
-1 & -2 & 2 & 5 & 3 \\
3 & 6 & 2 & 1 & 3 \\
4 & 8 & 6 & 8 & 9
\end{bmatrix}
\begin{array}{l}
/\cdot(-2)\ /\cdot 1\ /\cdot(-3)\ /\cdot(-4) \\
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\end{array}
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\sim
\begin{bmatrix}
1 & 2 & 1 & 1 & 2 \\
0 & 0 & 1 & 2 & 3 \\
0 & 0 & 3 & 6 & 5 \\
0 & 0 & -1 & -2 & -3 \\
0 & 0 & 2 & 4 & 1
\end{bmatrix}
\begin{array}{l}
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/\cdot(-3)\ /\cdot 1\ /\cdot(-2) \\
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\begin{bmatrix}
1 & 2 & 1 & 1 & 2 \\
0 & 0 & 1 & 2 & 3 \\
0 & 0 & 0 & 0 & -4 \\
0 & 0 & 0 & 0 & 0 \\
0 & & & &
\end{bmatrix}
$$

$$
\begin{bmatrix}
1 & 2 & 1 & 1 & 2 \\
2 & 4 & 3 & 4 & 7 \\
-1 & -2 & 2 & 5 & 3 \\
3 & 6 & 2 & 1 & 3 \\
4 & 8 & 6 & 8 & 9
\end{bmatrix}
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/\cdot(-2)\ /\cdot 1\ /\cdot(-3)\ /\cdot(-4) \\ \\ \\ \\ \\
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\begin{bmatrix}
1 & 2 & 1 & 1 & 2 \\
0 & 0 & 1 & 2 & 3 \\
0 & 0 & 3 & 6 & 5 \\
0 & 0 & -1 & -2 & -3 \\
0 & 0 & 2 & 4 & 1
\end{bmatrix}
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\\ /\cdot(-3)\ /\cdot 1\ /\cdot(-2) \\ \\ \\ \\
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\begin{bmatrix}
1 & 2 & 1 & 1 & 2 \\
0 & 0 & 1 & 2 & 3 \\
0 & 0 & 0 & 0 & -4 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & & &
\end{bmatrix}
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\begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 2 & 4 & 3 & 4 & 7 \\ -1 & -2 & 2 & 5 & 3 \\ 3 & 6 & 2 & 1 & 3 \\ 4 & 8 & 6 & 8 & 9 \end{bmatrix} / \cdot (-2) \; / \cdot 1 \; / \cdot (-3) \; / \cdot (-4) \qquad \sim
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\sim \begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 3 & 6 & 5 \\ 0 & 0 & -1 & -2 & -3 \\ 0 & 0 & 2 & 4 & 1 \end{bmatrix} / \cdot (-3) \; / \cdot 1 \; / \cdot (-2) \qquad \sim
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\sim \begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 & -4 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & & \end{bmatrix}
$$

$$
\begin{bmatrix}
1 & 2 & 1 & 1 & 2 \\
2 & 4 & 3 & 4 & 7 \\
-1 & -2 & 2 & 5 & 3 \\
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4 & 8 & 6 & 8 & 9
\end{bmatrix}
/\cdot(-2)\ /\cdot 1\ /\cdot(-3)\ /\cdot(-4)
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\begin{bmatrix}
1 & 2 & 1 & 1 & 2 \\
0 & 0 & 1 & 2 & 3 \\
0 & 0 & 3 & 6 & 5 \\
0 & 0 & -1 & -2 & -3 \\
0 & 0 & 2 & 4 & 1
\end{bmatrix}
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\begin{bmatrix}
1 & 2 & 1 & 1 & 2 \\
0 & 0 & 1 & 2 & 3 \\
0 & 0 & 0 & 0 & -4 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 &
\end{bmatrix}
$$

$$\begin{bmatrix}1 & 2 & 1 & 1 & 2\\ 2 & 4 & 3 & 4 & 7\\ -1 & -2 & 2 & 5 & 3\\ 3 & 6 & 2 & 1 & 3\\ 4 & 8 & 6 & 8 & 9\end{bmatrix} /\cdot(-2)\ /\cdot 1\ /\cdot(-3)\ /\cdot(-4) \qquad \sim$$

$$\sim \begin{bmatrix}1 & 2 & 1 & 1 & 2\\ 0 & 0 & 1 & 2 & 3\\ 0 & 0 & 3 & 6 & 5\\ 0 & 0 & -1 & -2 & -3\\ 0 & 0 & 2 & 4 & 1\end{bmatrix} \begin{matrix} \\ /\cdot(-3)\ /\cdot 1\ /\cdot(-2) \\ \\ \\ \\ \end{matrix} \qquad \sim$$

$$\sim \begin{bmatrix}1 & 2 & 1 & 1 & 2\\ 0 & 0 & 1 & 2 & 3\\ 0 & 0 & 0 & 0 & -4\\ 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & -5\end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 2 & 4 & 3 & 4 & 7 \\ -1 & -2 & 2 & 5 & 3 \\ 3 & 6 & 2 & 1 & 3 \\ 4 & 8 & 6 & 8 & 9 \end{bmatrix} / \cdot (-2) \; / \cdot 1 \; / \cdot (-3) \; / \cdot (-4) \qquad \sim$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 3 & 6 & 5 \\ 0 & 0 & -1 & -2 & -3 \\ 0 & 0 & 2 & 4 & 1 \end{bmatrix} / \cdot (-3) \; / \cdot 1 \; / \cdot (-2) \qquad \sim$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 & -4 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 2 & 4 & 3 & 4 & 7 \\ -1 & -2 & 2 & 5 & 3 \\ 3 & 6 & 2 & 1 & 3 \\ 4 & 8 & 6 & 8 & 9 \end{bmatrix} \Big/ \cdot (-2) \;\Big/ \cdot 1 \;\Big/ \cdot (-3) \;\Big/ \cdot (-4) \quad \sim$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 3 & 6 & 5 \\ 0 & 0 & -1 & -2 & -3 \\ 0 & 0 & 2 & 4 & 1 \end{bmatrix} \Big/ \cdot (-3) \;\Big/ \cdot 1 \;\Big/ \cdot (-2) \quad \sim$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 & -4 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -5 \end{bmatrix} \Big/ \cdot \frac{-5}{4}$$

$$\begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 2 & 4 & 3 & 4 & 7 \\ -1 & -2 & 2 & 5 & 3 \\ 3 & 6 & 2 & 1 & 3 \\ 4 & 8 & 6 & 8 & 9 \end{bmatrix} \begin{matrix} /\cdot(-2)\ /\cdot 1\ /\cdot(-3)\ /\cdot(-4) \\ + \\ + \\ + \\ + \end{matrix} \sim$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 3 & 6 & 5 \\ 0 & 0 & -1 & -2 & -3 \\ 0 & 0 & 2 & 4 & 1 \end{bmatrix} \begin{matrix} \\ /\cdot(-3)\ /\cdot 1\ /\cdot(-2) \\ + \\ + \\ + \end{matrix} \sim$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 & -4 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -5 \end{bmatrix} \begin{matrix} \\ \\ /\cdot \frac{-5}{4} \\ \\ + \end{matrix}$$

$$\begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 2 & 4 & 3 & 4 & 7 \\ -1 & -2 & 2 & 5 & 3 \\ 3 & 6 & 2 & 1 & 3 \\ 4 & 8 & 6 & 8 & 9 \end{bmatrix} \begin{matrix} / \cdot (-2) \; / \cdot 1 \; / \cdot (-3) \; / \cdot (-4) \\ + \\ + \\ + \\ + \end{matrix} \sim$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 3 & 6 & 5 \\ 0 & 0 & -1 & -2 & -3 \\ 0 & 0 & 2 & 4 & 1 \end{bmatrix} \begin{matrix} \\ / \cdot (-3) \; / \cdot 1 \; / \cdot (-2) \\ + \\ + \\ + \end{matrix} \sim$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 & -4 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -5 \end{bmatrix} \begin{matrix} \\ \\ / \cdot \frac{-5}{4} \\ \\ + \end{matrix} \sim \begin{bmatrix} & & & & \\ \\ \\ \\ \\ \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 2 & 4 & 3 & 4 & 7 \\ -1 & -2 & 2 & 5 & 3 \\ 3 & 6 & 2 & 1 & 3 \\ 4 & 8 & 6 & 8 & 9 \end{bmatrix} /\cdot(-2) \ /\cdot 1 \ /\cdot(-3) \ /\cdot(-4) \qquad \sim$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 3 & 6 & 5 \\ 0 & 0 & -1 & -2 & -3 \\ 0 & 0 & 2 & 4 & 1 \end{bmatrix} /\cdot(-3) \ /\cdot 1 \ /\cdot(-2) \qquad \sim$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 & -4 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -5 \end{bmatrix} \Big/\cdot \frac{-5}{4} \quad \sim \begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ & & & & \\ & & & & \\ & & & & \\ & & & & \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 2 & 4 & 3 & 4 & 7 \\ -1 & -2 & 2 & 5 & 3 \\ 3 & 6 & 2 & 1 & 3 \\ 4 & 8 & 6 & 8 & 9 \end{bmatrix} /\cdot(-2)\ /\cdot 1\ /\cdot(-3)\ /\cdot(-4) \qquad \sim$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 3 & 6 & 5 \\ 0 & 0 & -1 & -2 & -3 \\ 0 & 0 & 2 & 4 & 1 \end{bmatrix} /\cdot(-3)\ /\cdot 1\ /\cdot(-2) \qquad \sim$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 & -4 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -5 \end{bmatrix} \Big/\cdot \frac{-5}{4} \quad \sim \begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 0 & 0 & 1 & 2 & 3 \\ & & & & \\ & & & & \\ & & & & \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 2 & 4 & 3 & 4 & 7 \\ -1 & -2 & 2 & 5 & 3 \\ 3 & 6 & 2 & 1 & 3 \\ 4 & 8 & 6 & 8 & 9 \end{bmatrix} / \cdot (-2) \; / \cdot 1 \; / \cdot (-3) \; / \cdot (-4) \quad \sim$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 3 & 6 & 5 \\ 0 & 0 & -1 & -2 & -3 \\ 0 & 0 & 2 & 4 & 1 \end{bmatrix} / \cdot (-3) \; / \cdot 1 \; / \cdot (-2) \quad \sim$$

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$$
\left[\begin{array}{ccccc}
1 & 2 & 1 & 1 & 2 \\
2 & 4 & 3 & 4 & 7 \\
-1 & -2 & 2 & 5 & 3 \\
3 & 6 & 2 & 1 & 3 \\
4 & 8 & 6 & 8 & 9
\end{array}\right]
\begin{array}{l}
/\cdot(-2)\ /\cdot 1\ /\cdot(-3)\ /\cdot(-4) \\ \\ \\ \\ \\
\end{array}
\sim
$$

$$
\sim
\left[\begin{array}{ccccc}
1 & 2 & 1 & 1 & 2 \\
0 & 0 & 1 & 2 & 3 \\
0 & 0 & 3 & 6 & 5 \\
0 & 0 & -1 & -2 & -3 \\
0 & 0 & 2 & 4 & 1
\end{array}\right]
\begin{array}{l}
\\ /\cdot(-3)\ /\cdot 1\ /\cdot(-2) \\ \\ \\ \\
\end{array}
\sim
$$

$$
\sim
\left[\begin{array}{ccccc}
1 & 2 & 1 & 1 & 2 \\
0 & 0 & 1 & 2 & 3 \\
0 & 0 & 0 & 0 & -4 \\
0 & 0 & 0 & 0 & 0 \\
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\end{array}\right]
\begin{array}{l}
\\ \\ \Big/\cdot\frac{-5}{4} \\ \\ \\
\end{array}
\sim
\left[\begin{array}{ccccc}
1 & 2 & 1 & 1 & 2 \\
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$$\sim \begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 3 & 6 & 5 \\ 0 & 0 & -1 & -2 & -3 \\ 0 & 0 & 2 & 4 & 1 \end{bmatrix} \begin{matrix} \\ / \cdot (-3) \ / \cdot 1 \ / \cdot (-2) \\ + \\ + \\ + \end{matrix} \sim$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 & -4 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -5 \end{bmatrix} \begin{matrix} \\ \\ / \cdot \frac{-5}{4} \\ \\ + \end{matrix} \sim \begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 & -4 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

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$$A = \begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 2 & 4 & 3 & 4 & 7 \\ -1 & -2 & 2 & 5 & 3 \\ 3 & 6 & 2 & 1 & 3 \\ 4 & 8 & 6 & 8 & 9 \end{bmatrix} \sim \quad \cdots \quad \sim \begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 & -4 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$A = \begin{array}{c} \begin{array}{ccccc} u_1 & u_2 & u_3 & u_4 & u_5 \end{array} \\ \begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 2 & 4 & 3 & 4 & 7 \\ -1 & -2 & 2 & 5 & 3 \\ 3 & 6 & 2 & 1 & 3 \\ 4 & 8 & 6 & 8 & 9 \end{bmatrix} \end{array} \sim \quad \cdots \quad \sim \begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 & -4 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\boxed{r(A) = 3}$$

$$A = \begin{array}{c} \begin{matrix} u_1 & \quad u_2 & \quad u_3 & u_4 & u_5 \end{matrix} \\ \begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 2 & 4 & 3 & 4 & 7 \\ -1 & -2 & 2 & 5 & 3 \\ 3 & 6 & 2 & 1 & 3 \\ 4 & 8 & 6 & 8 & 9 \end{bmatrix} \end{array} \sim \quad \cdots \quad \sim \begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 & -4 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

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$$\mathcal{B}_W = \{u_1, u_3, u_5\}$$

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$$\mathcal{B}_W = \left\{ u_1, u_3, u_5 \right\} \qquad \dim W = 3$$

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$$\mathcal{B}_W = \{u_1, u_3, u_5\} \qquad\qquad \dim W = 3$$

$$W = \mathcal{L}(u_1, u_2, u_3, u_4, u_5)$$

$\boxed{r(A) = 3}$

$$A = \begin{array}{ccccc} u_1 & u_2 & u_3 & u_4 & u_5 \\ \left[\begin{array}{c} 1 \\ 2 \\ -1 \\ 3 \\ 4 \end{array}\right. & \begin{array}{c} 2 \\ 4 \\ -2 \\ 6 \\ 8 \end{array} & \begin{array}{c} 1 \\ 3 \\ 2 \\ 2 \\ 6 \end{array} & \begin{array}{c} 1 \\ 4 \\ 5 \\ 1 \\ 8 \end{array} & \left.\begin{array}{c} 2 \\ 7 \\ 3 \\ 3 \\ 9 \end{array}\right] \end{array} \sim \quad \cdots \quad \sim \begin{bmatrix} 1 & 2 & 1 & 1 & 2 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 & -4 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

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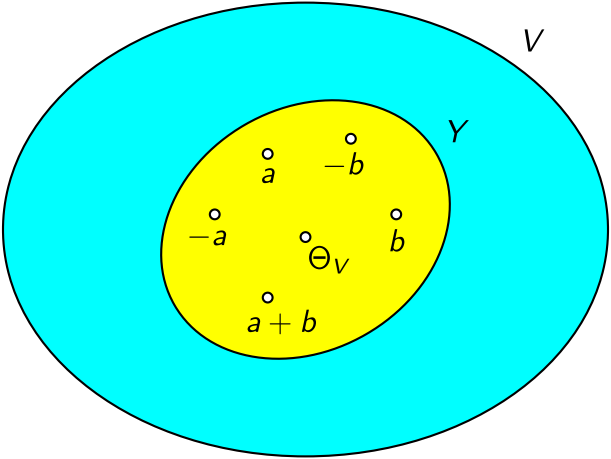
četvrti zadatak

Potprostor vektorskog prostora



Karakterizacija vektorskog potprostora

Neka je V vektorski prostor nad poljem F. Neprazan podskup $Y \subseteq V$ je potprostor od V akko za svaki izbor $a, b \in Y$ i $\alpha, \beta \in F$ vrijedi $\alpha a + \beta b \in Y$.

Linearni omotač skupa

Neka je V vektorski prostor nad poljem F, a $S \subseteq V$ bilo koji podskup. Tada je $\mathcal{L}(S)$ najmanji potprostor od V koji sadrži skup S.

Zadatak 4

Zadani su sljedeći podskupovi od $M_2(\mathbb{R})$:

$$U = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} : c \geqslant 0 \right\}, \; V = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} : c + 2d = 0, \; a + b - 2c = 0 \right\}$$

a) *Dokažite da U nije potprostor od $M_2(\mathbb{R})$.*

b) *Dokažite da je V potprostor od $M_2(\mathbb{R})$ i odredite mu neku bazu i dimenziju.*

Rješenje

a)

$$U = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} : c \geqslant 0 \right\}$$

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$$A = \begin{bmatrix} 2 & -3 \\ 5 & 1 \end{bmatrix}$$

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$$A = \begin{bmatrix} 2 & -3 \\ 5 & 1 \end{bmatrix} \in U \qquad\qquad -2A =$$

≥ 0

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Skup U nije zatvoren na uzimanje linearnih kombinacija svojih elemenata pa stoga U nije potprostor od $M_2(\mathbb{R})$.

Rješenje

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$$U = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} : c \geqslant 0 \right\}$$

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Skup U nije zatvoren na uzimanje linearnih kombinacija svojih elemenata pa stoga U nije potprostor od $M_2(\mathbb{R})$.

$$U \not\leq M_2(\mathbb{R})$$

b)

$$V = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} : c + 2d = 0,\ a + b - 2c = 0 \right\}$$

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$A \in V$

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$$A \in V \;\Rightarrow\; A = \begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix},$$

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$B \in V$

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a	b	c	d	

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$$\begin{array}{cccc|c} a & b & c & d & \\ \hline 0 & 0 & 1 & 2 & 0 \\ 1 & 1 & -2 & 0 & 0 \end{array}$$

$$\left.\begin{aligned} c + 2d &= 0 \\ a + b - 2c &= 0 \end{aligned}\right\}$$

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0	0	1	2	0
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$$\left.\begin{aligned} c + 2d &= 0 \\ a + b - 2c &= 0 \end{aligned}\right\} \rightsquigarrow d = -\frac{1}{2}c$$

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$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} =$$

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$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} -b + 2c & b \\ c & -\frac{1}{2}c \end{bmatrix} = b \cdot \begin{bmatrix} -1 & \\ & \end{bmatrix}$$

b)

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0	0	1	2	0
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$$\mathcal{B}_V = \left\{ \begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 2 & 0 \\ 1 & -\frac{1}{2} \end{bmatrix} \right\}$$

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$$\dim V = 2$$